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Time and gravitational field

Research article

Ilija Barukčić[1]

[1] Internist, Horandstrasse, 26441 Jever, Germany

* **Correspondence:** E-Mail: Barukcic@t-online.de;
Tel: +49-4466-333; Fax: +49-4466-333.

Abstract

Background:

Our investigations of what exists might seduce us to have little to say about an objective reality, in which **nothing exists** .

Methods:

The usual rules of tensor algebra have been used.

Results:

Time is equivalent to gravitational field. The four basic field of nature are geometrized from another point of view. The geometrical structure of the fourth basic field of nature is identified. There is some evidence that **pure non locality** is a feature of an objective reality in one space-time dimension.

Conclusion:

Theoretically, the beginning of our world out of an **empty negative** appears to be possible.

Keywords: Energy; Time; Space; Cause; Effect; Causal relationship k; Causality; Causation

1. Introduction

I have already tried to provide essential answers about the fundamental relationship between energy, time and space in numerous publications of mine. I come back to this topic again, because my former presentation of this subject does not satisfy more. In addition, new theoretical approaches have arisen to proof these relationships from another and different point of view. While following the time-honoured principle of going step by step **from the known to the unknown**, a new and more differentiated focus on widely discussed notions like energy, time and space might be of help to widen our own view on these entities.

Energy,

pure energy as such, existing independently and outside of human mind an consciousness, objectively and real is an energy without any further determination, an energy which is in its own self equal only to itself. In point of fact, the other side of pure energy is that pure energy is also not unequal with respect to another. Pure energy has no difference within itself and pure energy has no difference outwardly. If anything concrete or any determination or content could be identified in pure energy as distinct, or if pure energy were posited by such a determination as distinct from an other, pure energy would thereby fail to hold fast to its purity. In last consequence, pure energy is equally pure emptiness and at the end indeterminateness as such. However, it is not only energy that determines objective reality. Time itself is given too. Historically, investigations into the nature of time and discussions of various issues related to time have had an important impact on science as early as a very long time ago.

Time,

pure time, is similar to pure energy just simple equality with itself, complete emptiness, complete absence of any determination and content, a negation which is equally devoid of any reference. Pure time is the lack of all distinction within itself. No wonder that pure time is the same determination or rather the absence of any determination, and thus altogether the same as what pure energy is. As outlined in view words before, pure energy and pure time are therefore the same. It is noteworthy and necessary to consider that neither energy nor time, but rather that energy has passed over into time and that time has passes over into energy. In spite of all equality and besides of all, it is important to recognize that pure energy and pure time are at the end not without any distinction. At the end, it is more likely that pure energy and pure time are not the same. The principal question is how it is possible that pure energy and pure time are identical and yet also different too? Pure energy and pure time are absolutely distinct even if equally unseparated and inseparable. Each of both, each of pure energy and pure time, immediately vanishes into its own opposite.

Space,

is this movement of the immediate vanishing of pure energy into pure time or of the one into its own other and vice versa. However, such an understanding of the relationship between energy and time as stated before is not without deeper issues and without great concern. In contrast to the previously outlined and according to **the first law of thermodynamics** (see Clausius, 1867, du Châtelet, 1740) energy can be transformed from one energy to another, but can be neither destroyed nor created. However, time itself is not energy, it is the other of energy. Under conditions where energy passes over into time or time into energy our impression solidifies that the first law of thermodynamics is threatened or even violated. The question therefore arises again for ourselves, how can we discover with confidence that which is the truth but still hidden to us? How can we enlighten the epistemiological darkness? No logical alternative is available, space as the unity and the struggle between energy and time is a movement in which these two, pure energy and pure time, are distinguished too. However, it would be necessary to consider that it is this distinction which immediately dissolved itself too. Authors customary oppose time to energy and vice versa in an inappropriate way. It is to be considered that energy as an already determined and self-organised entity distinguishes itself from another energy. In other words, the time which is opposed to energy is also the time of a certain or concrete energy, a determinate time. Here, time should be viewed in its simplicity as pure time. Pure time is non-energy and as such deemed to oppose pure energy. In point of fact, in pure time as non-energy there is contained the reference to pure energy too. In other words, we have reason to suppose that non-energy is both, pure time and equally its own negation, its own other, pure energy. At the end all, pure time and pure energy are united in space.

2. Material and methods

Scientific knowledge and objective reality are more than only interrelated. It cannot be repeated often enough that objective reality or processes of objective reality is the foundation of any scientific knowledge. In point of fact, seen by light, grey is never merely simply grey. In general, human experience teaches us that a high mountain can be conquered by different paths.

2.1. Material

In general, it is appropriate to ensure as much as possible a broader consideration of a research question and to take into account the different facets and viewpoints of an issue investigated in order to reach a goal.

2.2. Methods

Definitions should help us to provide and assure a systematic approach to a scientific issue. It also goes without the need of further saying that a definition as such need to be logically consistent and correct.

2.2.1. Basic definitions of special theory of relativity

Definition 2.1 (Energy).

Let E denote energy (see Einstein, 1905b) which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$E = M \times c^2 \tag{1}$$

where M is the matter and c is the speed of the light in vacuum.

Definition 2.2 (Matter).

Let M denote matter which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. In our understanding of the matter we follow Einstein's explanations very closely.

"... '**Materie**'bezeichnet ... nicht nur die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. " (Einstein, 1916, p. 802/803)

In broken English, '**matter** denotes ... not only **matter in the ordinary sense**, but also the **electromagnetic field**. 'It is worth noting that the equivalence of matter (M) and energy (E) lies at the core of today's physics and has been described by Einstein as follows:

"Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um L/V^2 ... Die Masse eines Körpers ist ein Maß für dessen Energieinhalt "

(see also Einstein, 1905c, p. 641)

In general it is

$$M \equiv \frac{E}{c^2} \tag{2}$$

(see also Einstein, 1905c, p. 641)

where M denotes the matter(see also Tolman, 1912) and c is the speed of the light in vacuum. In other words, Einstein is demanding the equivalence of matter and energy as the most important upshot of his special theory of relativity.

"Eines der wichtigsten Resultate der Relativitätstheorie ist die Erkenntnis,

daß jegliche Energie E eine ihr proportionale Trägheit (E/c²) besitzt"

(see also Einstein, 1912, p. 1062)

Definition 2.3 (**Anti energy**).

Let $\underline{E}$ denote non-energy or anti energy, the other of energy, the complementary of energy, the opposite of energy which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$\underline{E} = S - E \tag{3}$$

Definition 2.4 (**Time**).

Let t denote time, the other of anti-time, the complementary of anti - time, the opposite of anti-time which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. Let $\underline{t}$ denote anti time. It is

$$t = S - \underline{t} \tag{4}$$

Definition 2.5 (**Anti time**).

Let $\underline{t}$ denote non-time or anti-time, the other of time, the complementary of time, the opposite of time which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$\underline{t} = S - t \tag{5}$$

Theoretically, anti-time is the other of time, the complementary of time, the opposite of time.

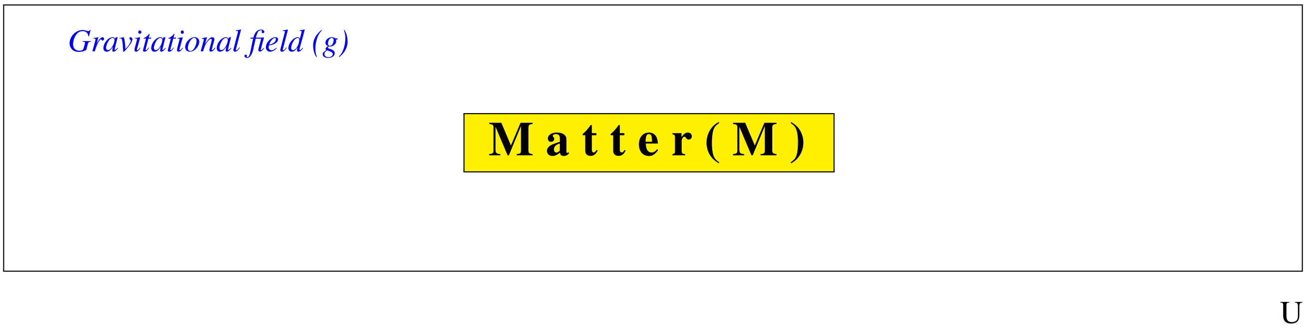
Definition 2.6 (**Gravitational field**).

Let g denote the gravitational field. The gravitational field g is quite often defined by the gravitational potential. Nonetheless, it is necessary to distinguish the gravitational field and the gravitational potential, both are not identical. Even if it is a little questionable to refer so often to Einstein's position, as long as the same is logically sound, it is also very difficult to simply ignore the same. Although it is much too often overlooked today, let us again refer to Einstein's understanding of the relationship between matter and gravitational field. Einstein defined the gravitational field **ex negativo** as follows.

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß **alles außer dem Gravitationsfeld als 'Materie'bezeichnet** wird, also nicht nur die 'Materie'im üblichen Sinne, sondern auch das elektromagnetische Feld. "

(Einstein, 1916, p. 802/803)

Again, Einstein's position translated into English: 'We distinguish in the following between 'gravitational field'and 'matter', in the sense that everything except the gravitational field is regarded as 'matter', that is not only 'matter'in the ordinary sense, but also the electromagnetic field.'The following and only symbolic figure might illustrate the relationship between matter and gravitational field in more detail.



Mathematically, the gravitational field is expressed as follows:

$$g = U - M \tag{6}$$

Definition 2.7 (Space).

Let S denote the space which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. We assume that energy and time are determining space. It is

$$S = E + t \tag{7}$$

In the further progress of the research it should be possible to demonstrate beyond any reasonable doubt that

$$S - t = E \tag{8}$$

and that the most general formulation of the Einstein field equations could be

$$\left(S \times g_{\mu\nu}\right) - \left(t \times g_{\mu\nu}\right) = \left(E \times g_{\mu\nu}\right) \tag{9}$$

where $g_{\mu\nu}$ is the metric tensor. Energy passes over into time and vice versa. Time passes over into energy. However, equation 8 has another aspect too. It is equally

$$(S \times S) - (S \times t) = (S \times E) \tag{10}$$

or $C^2 - a^2 = b^2$ and as a logical consequence also

$$\left((S \times S) \times g_{\mu\nu}\right) - \left((S \times t) \times g_{\mu\nu}\right) = \left((S \times E) \times g_{\mu\nu}\right) \tag{11}$$

The relationship between Pythagorean theorem and equation 10 is illustrated in more detail by figure 1 (see also theorem 3, page 23).

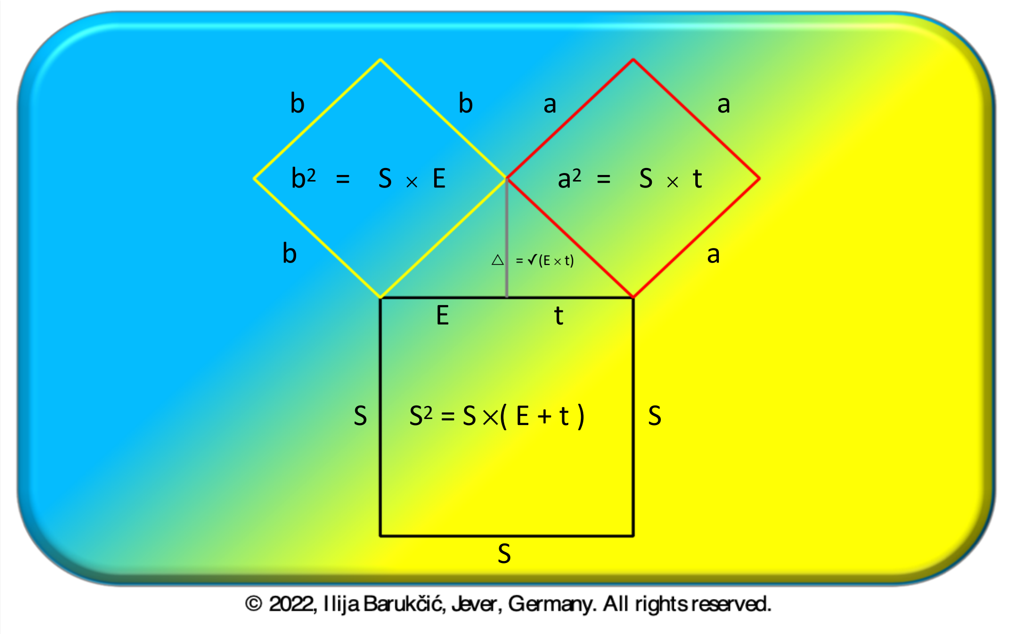


Figure 1. Space S, expectation values of energy (E) and time (t) and Pythagorean theorem.

Definition 2.8 (U)**.**

Let U denote the unity and the struggle between matter and gravitational field which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$U = \frac{S}{c^2} = M + g \tag{12}$$

2.2.2. Extended definitions of special theory of relativity

Definition 2.9 (Energy and special theory of relativity)**.** *Let $_{r}E_{t}$ denote the total or relativistic (see Lewis and Tolman, 1909) energy* [1] *of an (quantum mechanical) entity, at a certain run of an experiment t and is dependent on the relative velocity v of an observer. Let $_{0}E_{t}$ denote the rest energy of an entity, at a certain run of an experiment t. The invariant mass $_{0}m_{t}$ (also called rest mass) which is determined by rest energy is an invariant quantity which is the same for all observers in all reference frames. Let $_{w}E_{t}$ denote the electromagnetic wave energy of an entity, at a certain run of an experiment t. Let $_{rp}E_{t}$ denote the relativistic potential energy (see Barukčić, 2013), let $_{rk}E_{t}$ denote the relativistic kinetic energy (see Barukčić, 2013).*

The relativistic momentum, denoted as $_{r}p_{t}$, is defined as

$$({}_{\mathrm{r}}p_{\mathrm{t}}) = ({}_{\mathrm{r}}m_{\mathrm{t}}) \times (v) \tag{13}$$

where v is the relative velocity between observers. The energy of an electromagnetic wave, denoted as $_{w}E_{t}$, is derived as

$$({}_{\mathrm{w}}E_{\mathrm{t}}) = ({}_{\mathrm{r}}p_{\mathrm{t}}) \times (c) = ({}_{\mathrm{r}}m_{\mathrm{t}}) \times (v) \times (c) \tag{14}$$

where c is the speed of the light in vacuum. In general, it is

$$({}_{\mathrm{r}}E_{\mathrm{t}}) = \left({}_{\mathrm{rp}}E_{\mathrm{t}}\right) + ({}_{\mathrm{rk}}E_{\mathrm{t}}) \tag{15}$$

and the usual energy momentum relation

$$\left(({}_{\mathrm{r}}E_{\mathrm{t}}) \times {}_{\mathrm{rp}}E_{\mathrm{t}}\right) + (({}_{\mathrm{r}}E_{\mathrm{t}}) \times {}_{\mathrm{rk}}E_{\mathrm{t}}) = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times ({}_{\mathrm{r}}E_{\mathrm{t}}) \tag{16}$$

The invarinat or rest energy (see figure 2), denoted as $({}_{0}E_{\mathrm{t}})$, is given as

$$({}_{0}E_{\mathrm{t}})^{2} = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \left({}_{\mathrm{rp}}E_{\mathrm{t}}\right) \tag{17}$$

The relativistic potential energy, $_{rp}E_{t}$, is given as

$$\left({}_{\mathrm{rp}}E_{\mathrm{t}}\right) = \frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})} = \left(1 - \frac{v^{2}}{c^{2}}\right) \times ({}_{\mathrm{r}}E_{\mathrm{t}}) \tag{18}$$

Furthermore, the energy of a electromagnetic wave (see figure 2), denoted as $({}_{\mathrm{w}}E_{\mathrm{t}})$, is given as

$$(\mathrm{w}E_{\mathrm{t}})^{2} = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times ({}_{\mathrm{rk}}E_{\mathrm{t}}) \tag{19}$$

The relativistic kinetic energy (see figure 2), denoted as $_{rk}E_{t}$, is given as

$$({}_{\mathrm{rk}}E_{\mathrm{t}}) = \frac{(\mathrm{w}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})} = \frac{({}_{\mathrm{r}}m_{\mathrm{t}}) \times (v) \times (c) \times ({}_{\mathrm{r}}m_{\mathrm{t}}) \times (v) \times (c)}{({}_{\mathrm{r}}m_{\mathrm{t}}) \times (c) \times (c)} = ({}_{\mathrm{r}}m_{\mathrm{t}}) \times \left(v^{2}\right) = ({}_{\mathrm{r}}p_{\mathrm{t}}) \times (v) \tag{20}$$

[1] Lewis, Gilbert N. and Tolman, Richard C. (1909), "The Principle of Relativity, and Non-Newtonian Mechanics" , Proceedings of the American Academy of Arts and Sciences, 44 (25): 709–726.doi:10.2307/20022495

We have very convincing arguments to assume that the concept of **vis viva** (see Leibniz, 1695) as put forward by Leibniz and the notion **relativistic kinetic energy** are identical. The relationship before and their inner connection to the Pythagorean theorem are view by figure 2 in more detail.

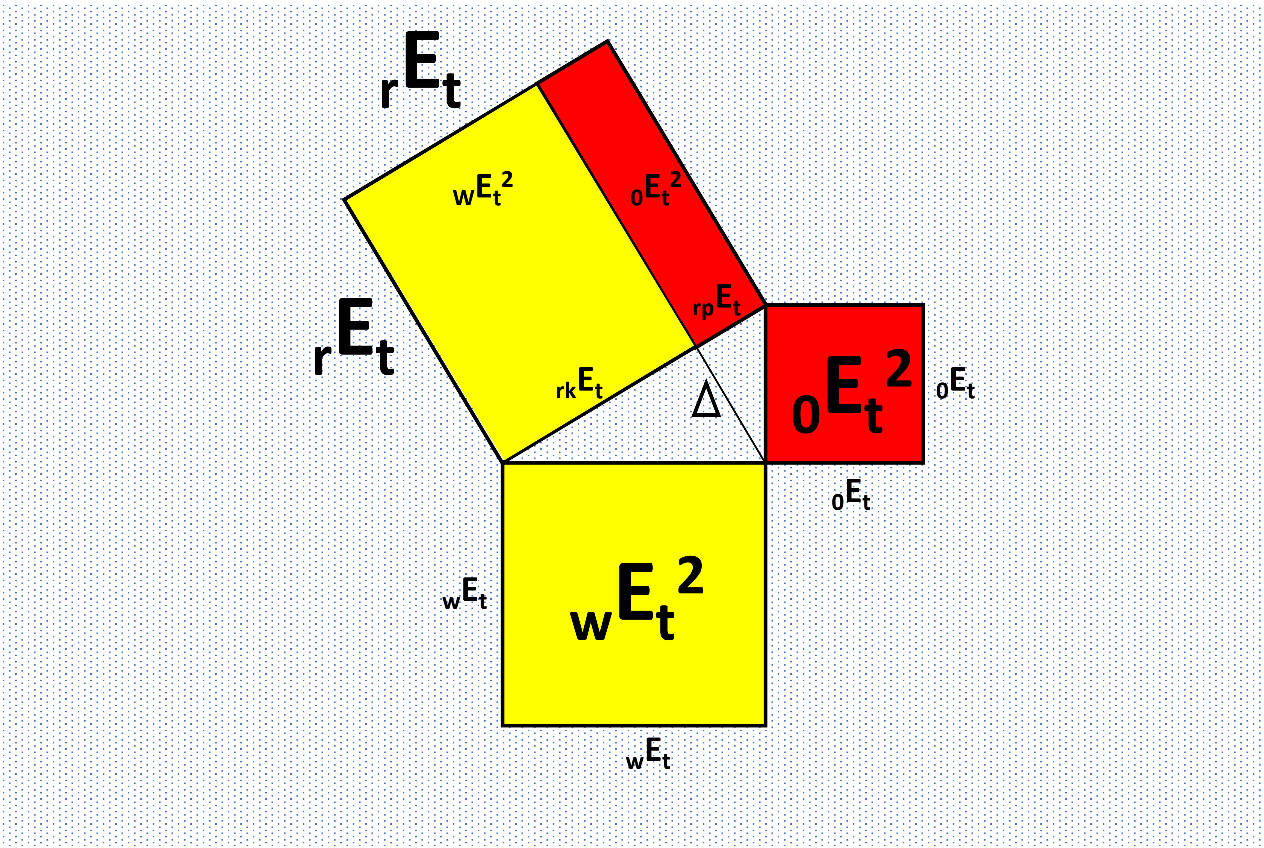


Figure 2. Pythagorean theorem and Einstein's special theory of relativity (Einstein's triangle).

The usual energy momentum relation has been the foundation of many relativistic wave equations. The normalised energy momentum relation is given as

$$\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)+\left(\frac{({}_{\mathrm{w}}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)=\left(\frac{({}_{0}m_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}m_{\mathrm{t}})^{2}}\right)+\left(\frac{(v)^{2}}{(c)^{2}}\right)=+1 \tag{21}$$

while

$$p\left({}_{\mathrm{rp}}E_{\mathrm{t}}\right)=\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)=1-\left(\frac{(v)^{2}}{(c)^{2}}\right) \tag{22}$$

can be understood as the probability of finding a certain particle local. The next figure might provide us with an simplified overview.

Wave $({}_{w}E_{t})^{2}$

Particle $({}_{0}E_{t})^{2}$

Energy $({}_{r}E_{t})^{2}$

Depending upon the experimental conditions and the measuring device used, the wave energy $({}_{w}E_{t})$ might change, or particle's energy $({}_{0}E_{t})$ might change (i. e. collision experiments in particle physics) et cetera. In any case, the extent of the interaction between a measuring device and an entity to be measured can be determined accurately. Nonetheless, it does not make any sense at all to assume that a measuring device is only a measuring device if the same measuring device generates by an act of measurement the entity which has to be measured. In this respect a remark to the ordinary matter and the electromagnetic field might be permissible. It is

$$({}_{r}E_{t}) = ({}_{r}E_{t}) \times (1+0) = \left(1 - \sqrt[2]{\frac{v^2}{c^2}} + \sqrt[2]{\frac{v^2}{c^2}}\right) \times ({}_{r}E_{t}) = \underbrace{\left(\left(1 - \sqrt[2]{\frac{v^2}{c^2}}\right) \times ({}_{r}E_{t})\right)}_{Oridinary\ energy/matter\ {}_{a}E_{t}} + \underbrace{\left(\left(\sqrt[2]{\frac{v^2}{c^2}}\right) \times ({}_{r}E_{t})\right)}_{Electromagnetic\ wave\ {}_{w}E_{t}} = ({}_{a}E_{t}) + ({}_{w}E_{t}) \tag{23}$$

where ${}_{a}E_{t}$ is the energy of ordinary matter/energy and ${}_{w}E_{t}$ is the energy of the electromagnetic field/wave. Based on equation 23 , it is

$$({}_{a}E_{t}) = \underbrace{\left(\left(1 - \sqrt[2]{\frac{v^2}{c^2}}\right) \times ({}_{r}E_{t})\right)}_{Oridinary\ energy/matter\ {}_{a}E_{t}} = \left(1 - \sqrt[2]{\frac{v^2}{c^2}}\right) \times h \times {}_{r}f_{t} \tag{24}$$

where ${}_{a}E_{t}$ might denote "Alltagsenergie "or ordinary energy/matter, h is Planck's constant and ${}_{r}f_{t}$ is the frequency. The total or relativistic energy ${}_{r}E_{t}$ is determined as

$${}_{r}E_{t} = \frac{{}_{a}E_{t}}{\left(1 - \sqrt[2]{\frac{v^2}{c^2}}\right)} \tag{25}$$

The relationship between "rest energy ", denoted as ${}_{0}E_{t}$ and ordinary energy ${}_{a}E_{t}$ is given as

$$({}_{0}E_{t}) = \underbrace{\left(\left(\sqrt[2]{1 - \frac{v^2}{c^2}}\right) \times ({}_{r}E_{t})\right)}_{rest\ energy/matter\ {}_{0}E_{t}} = \frac{\left(\sqrt[2]{1 - \frac{v^2}{c^2}}\right)}{\left(1 - \sqrt[2]{\frac{v^2}{c^2}}\right)} \times {}_{a}E_{t} \tag{26}$$

These relationships are necessary to be considered at any measurement and especially in cosmology. Equation 24 is the natural foundation of the Doppler effect (see Doppler, 1842, Voigt, 1887) and is illustrated by the following picture in more detail.

Electromagnetic wave $({}_{w}E_{t})$

Ordinary matter/energy $({}_{\mathrm{a}}E_{\mathrm{t}})$

Energy $({}_{\mathrm{r}}E_{\mathrm{t}})$

Let us assume that quantum theory is (in principle) a (universal) theory which is applicable (in principle) to all physical systems including our earth-moon system too. This could imply that a linear evolution of quantum states applied to macroscopic objects might routinely lead to superpositions of macroscopically distinct objects. Based on equation 23, this is not completely absurd. However, various approaches to what is called the 'Measurement Problem'propose contradictory answers to the previous and similar questions. There are, however, various ways of approaching this issue. Normalising equation 23, it is

$$\left(\frac{{}_{\mathrm{a}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}}\right)+\left(\frac{{}_{\mathrm{w}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}}\right)=+1 \tag{27}$$

Multiplying by the Schrödinger equation (Schrödinger, Erwin Rudolf Josef Alexander, 1926), it is

$$\left(\frac{{}_{\mathrm{a}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}}\times(H\times\Psi)\right)+\left(\frac{{}_{\mathrm{w}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}}\times(H\times\Psi)\right)=H\times\Psi \tag{28}$$

It is ${}_{\mathrm{r}}E_{\mathrm{t}}=H=i\times\hbar\times\frac{\partial}{\partial t}$, equation 28 becomes

$$({}_{\mathrm{a}}E_{\mathrm{t}}\times\Psi)+({}_{\mathrm{w}}E_{\mathrm{t}}\times\Psi)=H\times\Psi \tag{29}$$

or in the quantized version

$$\left(\left(\left(1-\sqrt[2]{\frac{v^2}{c^2}}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)+\left(\left(\left(\sqrt[2]{\frac{v^2}{c^2}}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)=H\times\Psi \tag{30}$$

For our purposes, the most important features of equation 30 is that it is deterministic, linear and that the same provides a possibility to describe macroscopic objects too. Among the circumstances in which this might happen are, as an example, experimental set-ups where two persons, A like Alice and B like Bob, are measuring the existence of our moon.

Example.

Person A (i. e. Alice) measures the moon of our earth with his own **eyes open**. At the same place and time, person B (i. e. Bob) measures the same moon of our earth with his own **eyes closed**. Thus far, if only human eyes which are open would justify the existence of our earth's moon, person A should not be able to measure anything, because according to the opinion of person B there cannot be anything, his eyes are still closed. At the same point in space-time t both is given, the moon exists (Person A) and the moon does not exist (Person B), which is a contradiction. At this point we must ask the fundamental question for what logically obligatory reason should we humans have to accept that our earth's moon exists only if we also look at the same.

"We often discussed his notions on **objective reality**. I recall that during one walk **Einstein** suddenly stopped, turned to me and asked whether I really believed that **the moon exists only when I look at it**. "

(see Pais, 1979, p. 907)

Is our moon there when nobody looks?

Figure 3. Credit: NASA, International Space Station. Heavenly Half Moon. Picture taken by a crew member aboard the International Space Station during Expedition 20.

The answer to the above question may cause headaches, sleepless nights and numerous other inconveniences for view authors. In the end, the answer was, is and remains that what it is: clear and simple. Our earth's moon is there where the same is, even if nobody looks. In other words, earth's moon exists independently of any measurement and independently and outside of any perceiving subject, objectively and real. In general, the existence of a (quantum mechanical) object is a necessary condition for the measurement of the (quantum mechanical) object. Without the existence of a (quantum mechanical) object no measurement of the (quantum mechanical) object. What are the epistemological consequences of measuring something that does not exist?

2.2.3. Basic definitions of theory of general relativity

Definition 2.10 (The Einstein field equations). *The Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) describe the relationship between the presence of matter represented by the stress-energy tensor* $\left(\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu}\right)$ *in a given region of space-time and the curvature in that region by the equation*

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + \left(\Lambda\times g_{\mu\nu}\right) \equiv \left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu} \equiv E\times g_{\mu\nu} = E_{\mu\nu} \tag{31}$$

(Einstein, 1916, 1917)

*where $R_{\mu\nu}$ is the Ricci tensor (Ricci-Curbastro and Levi-Civita, 1900) of 'Einstein's general theory of relativity' (Einstein, 1916), R is the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold, Λ is the Einstein's cosmological (Barukčić, 2015a, Einstein, 1917) constant, $\underline{\Lambda}$ is the "**anti cosmological constant**" (Barukčić, 2015a), $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity, $G_{\mu\nu}$ is Einstein's curvature tensor, $\underline{G}_{\mu\nu}$ is the "**anti tensor**" (Barukčić, 2016c) of Einstein's curvature tensor, $E_{\mu\nu}$ is the stress-energy tensor of energy, $\underline{E}_{\mu\nu}$ is the tensor of non-energy, the anti-tensor of the stress-energy tensor of energy, $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, c is the speed of the light in vacuum, γ is Newton's gravitational "**constant**" (Barukčić, 2015a,b, 2016a,c), π is Archimedes constant pi.*

Table 1 may provide a more detailed and preliminary overview of the definitions (Barukčić, 2016b,c) before.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu} \equiv (c_{\mu\nu} + \Lambda\times g_{\mu\nu})$	$\frac{8\times\pi\times\gamma\times T}{c^4\times D}\times g_{\mu\nu} \equiv \left(\frac{R}{D}-\frac{R}{2}+\Lambda\right)\times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv (b_{\mu\nu} - \Lambda\times g_{\mu\nu})$	$d_{\mu\nu} \equiv (\frac{R}{2}\times g_{\mu\nu} - b_{\mu\nu})$	$\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D}-\frac{R}{2}\right)\times g_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D}\times g_{\mu\nu}$

Table 1. Four basic fields of nature and Einstein's field euqations.

Definition 2.11 (**Four basic fields of nature**).

We define the four basic fields of nature (Barukčić, 2016b,c, 2020a,c,d, 2021b) as $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$. Exemplarily, covariant tensors are used. The four basic fields of nature can also be formulated as mixed or as contra-variant tensors without any loss of information. In general, it is

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} \tag{32}$$

or

$$b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} - a_{\mu\nu} \tag{33}$$

Furthermore, it is (Barukčić, 2016b,c, 2020a,c,d, 2021b)

$$\begin{aligned} a_{\mu\nu} + b_{\mu\nu} &\equiv \frac{8 \times \pi \times \gamma}{c^4} \times T_{\mu\nu} \\ &\equiv G_{\mu\nu} + \Lambda \times g_{\mu\nu} \\ &\equiv \frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \times g_{\mu\nu} \\ &\equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \\ &\equiv E \times g_{\mu\nu} \\ &\equiv E_{\mu\nu} \end{aligned} \tag{34}$$

and

$$\begin{aligned} a_{\mu\nu} + c_{\mu\nu} &\equiv G_{\mu\nu} \\ &\equiv R_{\mu\nu} - \frac{R}{2} \times g_{\mu\nu} \\ &\equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \end{aligned} \tag{35}$$

It was possible to provide evidence (Barukčić, 2016b,c, 2020a,c,d, 2021b) that

$$
\begin{aligned}
c_{\mu\nu} + d_{\mu\nu} &\equiv R_{\mu\nu} - a_{\mu\nu} - b_{\mu\nu} \\
&\equiv R_{\mu\nu} - \frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \times g_{\mu\nu} \\
&\equiv \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \\
&\equiv \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \\
&\equiv \underline{E} \times g_{\mu\nu} \\
&\equiv \underline{E}_{\mu\nu}
\end{aligned}
\tag{36}
$$

and that (Barukčić, 2016b,c, 2020a,c,d, 2021b)

$$
\begin{aligned}
b_{\mu\nu} + d_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} + \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - c_{\mu\nu} \\
&\equiv E_{\mu\nu} + \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - a_{\mu\nu} - c_{\mu\nu} \\
&\equiv E_{\mu\nu} + \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - G_{\mu\nu} \\
&\equiv \frac{R}{2} \times g_{\mu\nu} + E_{\mu\nu} - \Lambda \times g_{\mu\nu} - G_{\mu\nu} \\
&\equiv \frac{R}{2} \times g_{\mu\nu}
\end{aligned}
\tag{37}
$$

The table 2 will provide once again an overview of the general definition of the relationships between these four basic (Barukčić, 2016b,c, 2021b) fields of nature under conditions of the general theory of relativity where $R_{\mu\nu}$ is the Ricci tensor, $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 2. The four basic fields of nature

the stress-energy tensor of electromagnetic field, $G_{\mu\nu}$ is Einstein's curvature tensor, $\underline{G}_{\mu\nu}$ is the "**anti tensor**" (Barukčić, 2016c) of Einstein's curvature tensor, $E_{\mu\nu}$ is the stress-energy tensor of energy, $\underline{E}_{\mu\nu}$ is the tensor of non-energy, the anti-tensor of the stress-energy tensor of energy.

3. Results

3.1. *Energy, time and space*

3.1.1. Theorem. Energy and space

Theorem 1 (The relationship between energy and space)**.** *The relationship between energy and space is given as*

$$+\frac{E}{S}+\frac{\underline{E}}{S}=+1 \tag{38}$$

Proof by direct proof. Axiom 1 or

$$+1=+1 \tag{39}$$

is true. Multiplying equation 39 by space +S, it is equally true that

$$+S=+S \tag{40}$$

Equation 40 is equivalent with the relationship

$$+S+0=+S+0=+S \tag{41}$$

In general, it is +E-E = 0. Equation 41 becomes

$$+E+S-E=+S \tag{42}$$

Non-energy or anti-energy et cetera is defined (see equation 3, p. 10) as $+\underline{E}=+S-E$, it is

$$+E+\underline{E}=+S \tag{43}$$

Normalising relationship between energy and non-energy, we obtain

$$+\frac{E}{S}+\frac{\underline{E}}{S}=+1 \tag{44}$$

□

Energy +E is one determining part of space but non-energy or anti-energy, denoted as $+\underline{E}$, too. Only under circumstances where non-energy $+\underline{E}=0$, space and energy where equivalent or even identical but not in general. Today, we have not convincing evidence of the identity of energy and space. Therefore, another of energy need to be given. Energy itself is given as

$$E=S\times\left(+1-\frac{\underline{E}}{S}\right)=S\times(1-p(t))=S\times p(E) \tag{45}$$

where E is the expectation value of energy and $p(E)$ is the probability of energy E.

3.1.2. Theorem. Time and space

Theorem 2 (The relationship between time and space)**.** *The relationship between time and space is given as*

$$+\frac{t}{S}+\frac{\underline{t}}{S}=+1 \tag{46}$$

Proof by direct proof. Axiom 1 or

$$+1=+1 \tag{47}$$

is true. Multiplying equation 47 by space +S, it is equally true that

$$+S=+S \tag{48}$$

Equation 48 is equivalent with the relationship

$$+S+0=+S+0=+S \tag{49}$$

In general, it is +t-t = 0. Equation 49 becomes

$$+t+S-t=+S \tag{50}$$

Non-time or anti-time et cetera is defined (see equation 5, p. 10) as $+\underline{t}=+S-t$, it is

$$+t+\underline{t}=+S \tag{51}$$

Normalising relationship between energy and non-energy, we obtain

$$+\frac{t}{S}+\frac{\underline{t}}{S}=+1 \tag{52}$$

□

Time +t is another determining part of space but non-time or anti-time, denoted as $+\underline{t}$, too. Only under circumstances where non-time $+\underline{t}$ = 0, space and time where equivalent or even identical but not in general. Today, we have not convincing evidence of the identity of time and space. Therefore, another of time need to be given. Time itself is determined as

$$t=S\times\left(+1-\frac{\underline{t}}{S}\right)=S\times(1-p(E))=S\times p(t) \tag{53}$$

where t is the expectation value of time and $p(t)$ is the probability of time t.

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3.1.3. Theorem. There is no third between energy and time

Theorem 3 (Energy and time)**.** *There is no third between energy and time, **tertium non datur**, a third (see Thomson, 1849, p. 295) is not given! It is*

$$ln\left({}_{R}E\right) + ln\left({}_{R}t\right) = ln\left(\sigma\left({}_{R}S\right)^{2}\right) \tag{54}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{55}$$

is true. Equation 55 changes slightly. It is equally

$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{v^2}{c^2}} \tag{56}$$

According to equation 471, it is $\frac{{}_{0}E_{\mathrm{t}}}{{}_{\mathrm{R}}E} = \sqrt{1 - \frac{v^2}{c^2}}$. Equation 56 changes to

$$\frac{{}_{0}E_{\mathrm{t}}}{{}_{\mathrm{R}}E} = \sqrt{1 - \frac{v^2}{c^2}} \tag{57}$$

Equation 475 demands that $\frac{{}_{0}t}{{}_{\mathrm{R}}t} = \sqrt{1 - \frac{v^2}{c^2}}$. Based on this relationship, equation 57 changes to

$$\frac{{}_{0}E}{{}_{\mathrm{R}}E} = \frac{{}_{0}t}{{}_{\mathrm{R}}t} \tag{58}$$

and to

$${}_{0}E \times {}_{\mathrm{R}}t = {}_{0}t \times {}_{\mathrm{R}}E \tag{59}$$

Furthermore, it is

$${}_{0}E \times \frac{{}_{0}t}{\sqrt{1 - \frac{v^2}{c^2}}} = {}_{\mathrm{R}}t \times \sqrt{1 - \frac{v^2}{c^2}} \times {}_{\mathrm{R}}E \tag{60}$$

Equation 60 becomes

$$\frac{{}_{0}E}{\sqrt{1 - \frac{v^2}{c^2}}} \times \frac{{}_{0}t}{\sqrt{1 - \frac{v^2}{c^2}}} = {}_{\mathrm{R}}t \times {}_{\mathrm{R}}E = \left({}_{\mathrm{R}}Z^2\right) \tag{61}$$

while variable ${}_{\mathrm{R}}\mathrm{Z}$ is a provisional compromise. Under conditions where ${}_{\mathrm{R}}\mathrm{E}$ and ${}_{\mathrm{R}}\mathrm{t}$ can be treated as expectations values (see Barukčić, 2022), it is ${}_{\mathrm{R}}S = {}_{\mathrm{R}}E + {}_{\mathrm{R}}t$ and equally $\left({}_{\mathrm{R}}Z\right)^2 = \sigma\left({}_{\mathrm{R}}S\right)^2 = {}_{\mathrm{R}}E \times {}_{\mathrm{R}}t$, while $\sigma\left({}_{\mathrm{R}}S\right)^2$ is the variance (see Barukčić, 2022) of ${}_{\mathrm{R}}\mathrm{S}$. We apply the logarithmus (see Nepervs, 1614) naturalis (ln) to equation 61. In point of fact, we have no other logical alternative than to conclude in accordance with Einstein's theory of special relativity the subsequent. It is logically (see also figure 1, page 12) and mathematically irrefutable that

$$ln\left({}_{\mathrm{R}}E\right) + ln\left({}_{\mathrm{R}}t\right) = ln\left({}_{\mathrm{R}}Z^2\right) = ln\left(\sigma\left({}_{\mathrm{R}}S\right)^2\right) \tag{62}$$

□

In other words, based on Einstein's theory of special (see Einstein, 1905b) relativity we were able to provide a clear proof that **there is no third between energy and time**, time is the other of energy, time is the opposite of energy and vice versa. Energy is the other of time, energy is the opposite of time. However, it is equally an opposition which can destroy itself. Again, variable $_{R}Z$ is a provisional compromise for those readers who do not agree with the term $ln\left(\sigma\left({}_{R}S\right)^{2}\right)$. Whether and what relationship there is between $_{R}Z$ and Planck's constant h can be investigated on another occasion. Based on equation 62 there might be circumstances given under which equation 7 derived as

$$S = E + t \tag{63}$$

is normalised as

$$\frac{E}{S} + \frac{t}{S} = 1 \tag{64}$$

With the help of the linear partial differential Schrödinger equation (see Born, 1926, Schrödinger, Erwin Rudolf Josef Alexander, 1926) we get

$$\left(\frac{E \times H \times \Psi}{S}\right) + \left(\frac{t \times H \times \Psi}{S}\right) = H \times \Psi \tag{65}$$

Another consequence of equation 7 derived as $S = E + t$ is that

$$S = S - \underline{E} + S - \underline{t} \tag{66}$$

Under these consequences we would have to accept (see equation 3 and equation 5) that

$$\underline{E} + \underline{t} = +S \tag{67}$$

or that

$$\underline{E} = +S - \underline{t} = +t \tag{68}$$

and that

$$\underline{t} = +S - \underline{E} = +E \tag{69}$$

3.1.4. Theorem. Matter and gravitational field

The fundamental relationship between matter and the gravitational field has been defined by Einstein as follows.

> "Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß **alles außer dem Gravitationsfeld** als '**Materie**'bezeichnet wird, also nicht nur die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. "
>
> (Einstein, 1916, p. 802/803)

Einstein's position translated into English. 'In the following we distinguish between 'gravitational field'and 'matter', in the sense that everything else but the gravitational field is termed as 'matter', i.e. not only 'matter'in the ordinary sense, but also the electromagnetic field. '

Theorem 4 (Matter and gravitational field)**.**

$$\underline{E} = g \times c^2 \tag{70}$$

Proof by direct proof. It is

$$1 = 1 \tag{71}$$

or

$$U = U \tag{72}$$

Rearranging equation 72, it is

$$U - M + M = U + 0 \tag{73}$$

or

$$g + M = U \tag{74}$$

Normalising the relationship between matter and gravitational field, it is

$$\frac{g}{U} + \frac{M}{U} = \frac{U}{U} = +1 \tag{75}$$

Rearranging equation 75 it is

$$\frac{g \times c^2}{U \times c^2} + \frac{M \times c^2}{U \times c^2} = +1 \tag{76}$$

or

$$\frac{g \times c^2}{S} + \frac{M \times c^2}{S} = +1 \tag{77}$$

and

$$\frac{g \times c^2}{S} + \frac{E}{S} = +1 \tag{78}$$

or

$$\frac{g \times c^2}{S} = +1 - \frac{E}{S} = \frac{\underline{E}}{S} \tag{79}$$

At the end, it is

$$g \times c^2 = \underline{E} \tag{80}$$

□

Remark 3.1. *Objective reality is not only determined by energy, there is also something other than energy, there is the complementary of energy, there is not energy or anti-energy. The other of energy, denoted as $\underline{E}$, the complementary of energy, the opposite of energy et cetera is identified for sure (see equation 80) as*

$$\underline{E} = g \times c^2 \tag{81}$$

However, which other meaning may we attribute to this relationship, can there be a more profound meaning of $\underline{E}$ at all? In general, it is (see equation 3 and equation 4)

$$E + \underline{E} = t + \underline{t} = S \tag{82}$$

Energy is given by the equation

$$E = t + \underline{t} - \underline{E} = S - \underline{E} \tag{83}$$

We add time to energy. It is

$$E + t = t + t + \underline{t} - \underline{E} = S + t - \underline{E} \tag{84}$$

Epistemologically it can not be denied that there can be circumstances where $+t = \underline{E}$ *with the consequence that* $+t - \underline{E} = 0$. *Under these circumstances, we can conclude that*

$$E + t = S + t - \underline{E} = S + 0 = S \tag{85}$$

We define energy in this way as all but time (ex negativo). In other words, there is no third between energy and time, tertium non datur. At the end, it is

$$E + t = S \tag{86}$$

There are conditions where it follows in a logically consistent way (see equation 68) that

$$t = \underline{E} = g \times c^2 \tag{87}$$

Fortunately, meanwhile we have presented a clear proof that equation 87 is generally valid. In fact, under preliminary aspects we are inclined to consider that ***everything but time is energy****. Thus far and according to theorem 3, we have at least one justifiable reason to suppose that there is really no third between energy and time.*

3.1.5. Theorem. Time and wave function

Theorem 5 (Time and wave function). *Let ${}_{R}E_{t}$ denote the relativistic (total) energy of a system (viewed from stationary observer R) at a certain run of an experiment t, let ${}_{R}t_{t}$ denote the relativistic time of a system (viewed from stationary observer R) at a certain run of an experiment t. One distinct aspects of the special theory of relativity is the relationship ${}_{0}t_{t} = \left(\sqrt[2]{1-\frac{v^2}{c^2}}\right) \times {}_{R}t_{t}$. In general, it is*

$$ {}_{R}t_{t} = \Psi \tag{88} $$

Proof by direct proof. Axiom 1 or

$$ +1 = +1 \tag{89} $$

is true. In the continuation of this theorem we consider a quantum mechanical system. The total energy of that system is identical with ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$. We obtain

$$ {}_{\mathrm{R}}E_{\mathrm{t}} = {}_{\mathrm{R}}E_{\mathrm{t}} \tag{90} $$

Multiplying the energy of the quantum mechanical system (see equation 90) by ${}_{\mathrm{R}}\mathrm{t}_{\mathrm{t}}$, it is

$$ {}_{\mathrm{R}}E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} = {}_{\mathrm{R}}E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} \tag{91} $$

The quantum mechanical system mentioned previously (see equation 91) can be described without any contradictions with the Schrödinger wave equation. We shall obtain ${}_{\mathrm{t}}$, it is

$$ {}_{\mathrm{R}}E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} = H \times \Psi \tag{92} $$

where Ψ is the wave function and H is the Hamiltonian. The Hamiltonian of a quantum mechanical system is an operator corresponding to the total energy of a quantum mechanical system, including both kinetic energy and potential energy. We have very good reason to assume that ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$ equals H. Equation 92 can be rearranged as

$$ H \times {}_{\mathrm{R}}t_{\mathrm{t}} = H \times \Psi \tag{93} $$

Under the outlined circumstances (equation 93), we have very high level of evidence that the physical meaning of the wave function is determined as

$$ {}_{\mathrm{R}}t_{\mathrm{t}} = \Psi \tag{94} $$

□

In a more far reaching (see Barukčić, 2016d) publication on this matter, it should be possible to provide a proof, that equation 94 is generally valid.[2] In combination with equation 87, it is

$$ {}_{\mathrm{R}}t_{\mathrm{t}} = \Psi = g \times c^2 \tag{95} $$

[2] Barukčić, I. (2016) The Physical Meaning of the Wave Function. Journal of Applied Mathematics and Physics, 4, 988-1023. doi: 10.4236/jamp.2016.46106.

3.2. *Theorem. The scalar form of the Ricci tensor $R_{\mu\nu}$*

3.2.1. Theorem. The relationship between the scalar S and the dimension of space-time D

In general, the Ricci tensor $R_{\mu\nu}$ represents how a volume of space in a curved space-time differs from a volume of space in Euclidean space. Usually, the Ricci tensor $R_{\mu\nu}$ is defined in terms of mathematical objects called Christoffel symbols. The Christoffel symbols themselves are defined in terms of the metric tensor $g_{\mu\nu}$. At this location we would like to work out a proposal how to simplify the form of the Ricci tensor.

Theorem 6 (The relationship between the entity S and the dimension of space-time D)**.** *In general, the entity S is given by*

$$S \equiv \left(\frac{R}{D}\right) \tag{96}$$

Proof. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{97}$$

is true, **then** the conclusion

$$S \equiv \left(\frac{R}{D}\right) \tag{98}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{99}$$

is true. Multiplying this premise by the Ricci tensor it is

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{100}$$

From where we stand, which is still unproven, the entity S (see equation 7) in combination with the metric tensor $g_{\mu\nu}$ is able to describe the Ricci tensor $R_{\mu\nu}$ mathematically in its entirety. The relationship

$$\boxed{R_{\mu\nu} \equiv S \times g_{\mu\nu}} \tag{101}$$

is **valid without an exception and in general**. Under these conditions, equation 101 becomes

$$R_{\mu\nu} \times g^{\mu\nu} \equiv S \times g_{\mu\nu} \times g^{\mu\nu} \tag{102}$$

or in accordance with definition 5.40

$$R \equiv S \times g_{\mu\nu} \times g^{\mu\nu} \tag{103}$$

In general, it is (see definition 5.9, equation 398)

$$R \equiv S \times D \tag{104}$$

Under circumstances outlined before, entity S is depending on the number of space-time dimensions D and follows as

$$S \equiv \left(\frac{R}{D}\right) \tag{105}$$

□

Our assumption (see equation 101, page 28) presented as $R_{\mu\nu} \equiv S \times g_{\mu\nu}$ would be true in the case that $R_{\mu\nu} \equiv \left(\frac{R}{D}\right) \times g_{\mu\nu}$. However, the last relationship has to be proven and cannot be simply hypothesised. Under conditions of D=1 space-time dimension, it would have to be taken as given that

$$R \equiv S \times D \equiv S \times 1 \equiv S \tag{106}$$

3.2.2. Theorem. The relationship between variable X and the Ricci scalar R

Theorem 7 (The relationship between variable X and the Ricci scalar R)**.** *In general, it is*

$$X = \left(\frac{R}{D}\right) \tag{107}$$

Proof by direct proof. Axiom 1 or +1=+1 is true and therefore

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{108}$$

In our understanding, there is a relationship between the variable X and the Ricci tensor $R_{\mu\nu}$ given by the equation

$$R_{\mu\nu} = X \times g_{\mu\nu} \tag{109}$$

while the value of X is unknown at this moment. Rearranging equation 109 it is

$$R_{\mu\nu} \times g^{\mu\nu} = X \times g_{\mu\nu} \times g^{\mu\nu} \tag{110}$$

or

$$R = X \times D \tag{111}$$

and

$$X = \frac{R}{D} \tag{112}$$

□

3.2.3. Theorem. The scalar form of Ricci tensor $R_{\mu\nu}$

Theorem 8 (The scalar form of Ricci tensor $R_{\mu\nu}$). *The general scalar form of Ricci tensor $R_{\mu\nu}$ is given as*

$$R_{\mu\nu} = \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{113}$$

Proof by direct proof. It is (see equation 112, p. 29)

$$X \equiv \left(\frac{R}{D}\right) \tag{114}$$

We multiply equation 114 by the metric tensor $g_{\mu\nu}$. It is

$$X \times g_{\mu\nu} \equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{115}$$

Equation 115 (see equation 112, page 29) is an equivalent formulation of the Ricci tensor $R_{\mu\nu}$ in terms of a Scalar X and given by the equation

$$R_{\mu\nu} = X \times g_{\mu\nu} \equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{116}$$

□

3.2.4. Theorem. The relationship between the entity S and the Ricci scalar R

Theorem 9 (The relationship between the entity S and the Ricci scalar R). *In general, it is*

$$X = S \tag{117}$$

Proof by direct proof. In general, axiom 1 or +1=+1 is true. Therefore, it is

$$X = X \tag{118}$$

Equation 118 (see equation 112, p. 29) becomes

$$X \equiv \left(\frac{R}{D}\right) \tag{119}$$

The type of relationship hypothesised by equation 105 (see equation 105, p. 29) is given for sure as

$$S \equiv \left(\frac{R}{D}\right) \tag{120}$$

□

Based on equation 120, it is considered proved that Ricci tensor $R_{\mu\nu}$ is given by the equation

$$R_{\mu\nu} = S \times g_{\mu\nu} \equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{121}$$

3.2.5. Theorem. Einstein manifolds and scalar S

At this point the question is legitimate, if we really proved the relation of equation 101 or rather only defined the same? Because a lot depends on the validity of equation 101, we want to deal with this topic from a different point of view.

Theorem 10 (The scalar S)**.** *The scalar S is determined and not defined as*

$$S = \frac{R}{D} \tag{122}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore

$$R_{\mu\nu} = R_{\mu\nu} \tag{123}$$

In general and from our point of view, it has to be that

$$R_{\mu\nu} = S \times Y \times g_{\mu\nu} \tag{124}$$

while S is a scalar as defined by equation 7 (see equation 7, p. 11) and Y is not known at this stage of the proof. As next, equation 124 becomes

$$R_{\mu\nu} \times g^{\mu\nu} = S \times Y \times g_{\mu\nu} \times g^{\mu\nu} \tag{125}$$

Thus far, it is equally

$$\underbrace{R}_{Left:scalar} = \underbrace{S \times D \times Y}_{Right:scalar} \tag{126}$$

or Y itself is a scalar given as

$$Y = \frac{R}{S \times D} \tag{127}$$

In general, the Ricci tensor $R_{\mu\nu}$ is determined as

$$R_{\mu\nu} = S \times \left(\frac{R}{S \times D}\right) \times g_{\mu\nu} = \left(\frac{R}{D}\right) \times g_{\mu\nu} \tag{128}$$

At this point we must refer to the previous evidence provided that $Y = \dfrac{R}{S \times D} = 1$. Under these circumstances (D is the number of space-time dimensions), it is again

$$S = \frac{R}{D} \tag{129}$$

□

As generally known, in mathematical physics and differential geometry, an Einstein manifold is a differentiable manifold whose Ricci tensor $R_{\mu\nu}$ is at the end proportional to the metric tensor $g_{\mu\nu}$. In contrast to equation 116, an Einstein manifold (see Besse, 1987, Kasner, 1920) is defined in general such that $R_{\mu\nu} = \kappa \times g_{\mu\nu}$, while κ is a proportionality factor. A number of monographs appeared under the "name"Arthur L. Besse which is a nom de plume of a group of French differential geometers, led by Marcel Berger (1927 – 2016).

3.3. The geometrical structure of the stress-energy tensor of matter

Gravity and space-time geometry are related. In the Einstein field equations, it is the stress-energy tensor of matter $T_{\mu\nu}$, introduced by Max von Laue (1879-1960) in the year 1911 (see Laue, 1911, p. 528) as 'Welttensor', which is the source of gravitation. Unfortunately, the stress-energy tensor of matter $T_{\mu\nu}$ is still "... a field devoid of any geometrical significance" (see Goenner, 2004, p. 7). A possible way out of this persistent difficulty might be a detour via a scalar.

3.3.1. Theorem. The scalar E of the stress-energy tensor of matter

General relativity's approach to gravitation is based on a more or less complicated geometry of space and time while doing away with forces. However, the unity of nature as the very foundation of the unity of science should enable us to find a different but equivalent approach to this subject too.

Theorem 11 (The scalar E of the stress energy tensor of matter). *The scalar E of the stress energy tensor of matter $T_{\mu\nu}$ is given as*

$$E = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{130}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{131}$$

is true. Therefore, it is equally true that

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{132}$$

From our standpoint, it should be theoretically possible to geometrize this tensor in its entirety. This tensor should be fully expressed by an unknown scalar E and the metric tensor $g_{\mu\nu}$. Equation (see equation 132) changes slightly for this reason. It is

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = E \times g_{\mu\nu} \tag{133}$$

The trace of a tensor has several properties. The reader may kindly appreciate that we cannot go into any further detail on this matter at this point. Taking the trace of equation (see equation 133), we obtain

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} = E \times g_{\mu\nu} \times g^{\mu\nu} \tag{134}$$

According to equation 398 it is $g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\nu}{}^{\nu} \equiv D$. Equation 134 changes slightly. It is

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T = E \times D \tag{135}$$

Moving away step by step from the known to the unknown, we have been able to shed some more light on the epistemological darkness which is surrounding us. The unknown scalar E has been identified in a logically and mathematical consistent way as

$$E = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{136}$$

□

3.3.2. Theorem. The scalar form of the stress-energy tensor of matter

Theorem 12.

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v} = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu v} \tag{137}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{138}$$

is true. Therefore, it is equally true that (see equation 133)

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v} = E \times g_{\mu v} \tag{139}$$

The scalar E (see equation 136) has been identified as $E = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right)$. The geometrized, generally covariant scalar form of the stress-energy tensor of matter is given as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v} = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu v} \tag{140}$$

□

3.3.3. Theorem. Quantisation of the stress energy tensor of matter

The mathematical representation of matter might oscillate back and forth between relativity theory and quantum theory. In fact, the unity of nature should, nonetheless, provide us with the ability to bridge the ever-increasing gap between quantum theory and (classical) field theory. We give ourselves over to the silent hope to derive quantum theory or the quantisation of gravity as a consequence of unified field theory.

Theorem 13 (Quantisation of the stress energy tensor of matter)**.** *The stress energy tensor of matter can be quantised as*

$$\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}=h\times\left(\frac{4\times\gamma\times T}{\hbar\times c^4\times D}\right)\times g_{\mu\nu} \tag{141}$$

Proof by direct proof. Axiom 1 or

$$+1=+1 \tag{142}$$

is true. is true. Therefore, it is equally true (see equation 140) that

$$\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}=\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{\mu\nu} \tag{143}$$

We know that Dirac's/Schrödinger's (see also Dirac, 1926, Dirac and Fowler, 1926, Schrödinger, Erwin Rudolf Josef Alexander, 1926) constant $\hbar$ is determined as

$$\hbar\equiv\frac{h}{2\times\pi} \tag{144}$$

In other words, it is

$$2\times\pi\equiv\frac{h}{\hbar} \tag{145}$$

This relationship is substituted into equation 143. The quantised form of the stress-energy tensor of matter is given as

$$\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}=h\times\left(\frac{4\times\gamma\times T}{\hbar\times c^4\times D}\right)\times g_{\mu\nu} \tag{146}$$

□

At least one methodological weak point in the process of the establishment of quantisation of the Einstein field equations for unified field theory was the missing link between geometrization of the stress tensor of the matter and its relationship to quantum theory. We have some reason to believe that this methodological weakness can be considered overcome with equation 146. This approach would receive a certain positive boost if theoretical or experimental proof were to be obtained that frequency f is determined as

$$f=\left(\frac{4\times\gamma\times T}{\hbar\times c^4\times D}\right) \tag{147}$$

Such a proof would open up massive theoretical and experimental possibilities.

3.4. The geometrical structure of the four basic fields of nature

3.4.1. The scalar theories of gravitation

The history of the scalar theories of gravitation (Goenner, 2012) is characterised by a lot of ups (Bragança and Lemos, 2018) and downs. In one of the frist trials, Gunnar Nordström (1881–1923), a Finnish theoretical physicist, in his attempt to construct a consistent, relativistic theory of gravitation treated the gravitational potential as a scalar field on a Minkowski background (Nordström, 1912). Quickly, Nordström modified his first scalar theory of gravitation into a predecessor of general relativity, a scalar theory of gravitation with conformal background (Nordström, 1913a). Nordström's relativistic scalar theory of gravitation immediately inspired Einstein who in the same year 1913 reformulated Nordström's theory in an elegant way and presented his own relativistic scalar theory of gravitation (Einstein, 1913). In the following, Einstein and Fokker (Einstein and Fokker, 1914) reanalysed Nordström's modified scalar gravitational theory and demonstrated that the same theory is a covariant scalar theory in a conformally flat space-time. However, neither Einstein nor other authors did answer the fundamental question of whether the Einstein field equations can fully be expressed in terms of scalars too. Einstein is writing with regard to a similar topic the following.

"Bei der unleugbaren Kompliziertheit der hier vertretenen Theorie der Gravitation müssen wir uns ernstlich fragen, ob nicht die bisher ausschließlich vertretene Auffassung, nach welcher das Gravitationsfeld auf einen Skalar Φ zurückgeführt wird, die einzig nahe liegende und berechtigte sei. Ich will kurz darlegen, warum wir diese Frage verneinen zu müssen glauben. "

(see Einstein and Grossmann, 1913, p. 20)

Einstein is writing: 'In view of the undeniable complexity of the theory of gravitation presented here, we must seriously ask ourselves whether the hitherto exclusively advocated view, according to which the gravitational field is traced back to a scalar Φ, is not the only obvious and justified one. I want to explain briefly why we believe to have to deny this question.'Can we reduce the gravitational field to a scalar? Einstein believed he could answer the issue of a scalar theory of gravitation or of a scalar-tensor theory of gravitation decisively in the negative and thereby ruling out not just Nordström's theory of gravitation but any competitor of general relativity which represented the gravitation with the help of scalars. However and with all the conceptual adversities to which we may be exposed, it would be more than appropriate to distinguish very precisely between a scalar theory of gravitation and a scalar-tensor theory of gravitation. In everything we do, we should keep in mind that a scalar-tensor (Brans and Dicke, 1961) theory of gravitation should not be mismatched with or reduced to a scalar (Nordström, 1912, 1913a) theory of gravitation. Today, several theories of gravitation are based on supergravity or superstrings and do contain one or more scalar fields and are trying to modify to some extent the original Einstein tensor theory of gravitation. The following lines are mostly to be understood as something like new scalar-theory of gravitation based on very slight modifications of Einstein tensor theory of gravitation and not as a refutation of Einstein tensor theory of gravitation. The approach to this matter differs essentially from Jordan–Brans–Dicke scalar-tensor theory of gravitation (see Brans and Dicke, 1961, Jordan, 1952). In fact, scalar-tensor theories of gravitation as one of the most popular competitors (see Yasunori and Kei-ichi, 2003) to Einstein's theory of gravitation are again and again considered as a serious alternative to Einstein's theory of gravity. Such an stance is not really factually justified. Today, one will assume for nothing that the inevitable time will come when Einstein's general theory of relativity is to be regarded as erroneous.

Time and again, we were able to identify the four fundamental fields of nature (Barukčić, 2016b,c, 2020b,c,d,d, 2021b) as $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$. At this point, we would like to visualize these matters once again in our mind's eye (see table 3, p. 37).

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 3. Einstein field equations and the four basic fields of nature

As previously outlined elsewhere, an equivalent formulation of the four basic fields of nature $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ in terms of the Ricci tensor $R_{\mu\nu}$ is given by the equation

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} \tag{148}$$

Nonetheless, even if the aforementioned is logically very plausible, the concrete structure and a detailed geometrical description of the four fundamental fields of nature remains quite doubtful in spite of many attempts of geometrization (Barukčić, 2016b,c, 2020b,c,d,d, 2021b) of the same. At this stage we would like to approach this issue from a different viewpoint in order to possibly get closer to the solution of this problem. In this context, Einstein's field equations (see equation 496) completely geometrized (see Barukčić, 2020a,d) with respect to space-time dimension D are given as

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{149}$$

or as

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left((R) \times g_{\mu\nu}\right) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{150}$$

and equally as

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu\,\mathrm{kl}\,...}\right) - \left((R) \times g_{\mu\nu\,\mathrm{kl}\,...}\right) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu\,\mathrm{kl}\,...}\right) + \left(\Lambda \times g_{\mu\nu\,\mathrm{kl}\,...}\right) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu\,\mathrm{kl}\,...} \tag{151}$$

3.4.2. Theorem. The geometrical structure of the fields of nature $a_{\mu\nu}$ + $b_{\mu\nu}$

Theorem 14 (The geometrical structure of the fields of nature $a_{\mu\nu}$ + $b_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $a_{\mu\nu}$ + $b_{\mu\nu}$ is given as*

$$a_{\mu\nu} + b_{\mu\nu} = R_{\mu\nu} - (R \times g_{\mu\nu}) + \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \tag{152}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore

$$R_{\mu\nu} = R_{\mu\nu} \tag{153}$$

too. The Ricci tensor $R_{\mu\nu}$ is determined by the four basic fields of nature $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ as

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} \tag{154}$$

Rearranging equation , it is

$$\begin{aligned} a_{\mu\nu} + b_{\mu\nu} &\equiv R_{\mu\nu} - c_{\mu\nu} - d_{\mu\nu} \\ &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \end{aligned} \tag{155}$$

Rearranging equation 155, it is

$$a_{\mu\nu} + b_{\mu\nu} + 0 \equiv R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) + 0 + (\Lambda \times g_{\mu\nu}) \tag{156}$$

or

$$a_{\mu\nu} + b_{\mu\nu} \equiv R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) \underbrace{- \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right)}_{+0} + (\Lambda \times g_{\mu\nu}) \tag{157}$$

Another equivalent geometrical formulation of the tensors $a_{\mu\nu} + b_{\mu\nu}$ is given as

$$a_{\mu\nu} + b_{\mu\nu} \equiv R_{\mu\nu} - (R \times g_{\mu\nu}) + \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{158}$$

□

Based on this new discovery we are now in the position to present the Einstein's field equations, a ten component tensor equation which relates local space-time curvature with local energy and momentum, in a new manner as

$$\underbrace{\left(\frac{R}{D} \times g_{\mu\nu}\right) - (R \times g_{\mu\nu})}_{\text{Ordinary matter } a_{\mu\nu}} + \underbrace{\left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu})}_{\text{Electromagnetic field } b_{\mu\nu}} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{159}$$

3.4.3. Theorem. The geometrical structure of the fields of nature $c_{\mu\nu} + d_{\mu\nu}$

Theorem 15 (The geometrical structure of the fields of nature $c_{\mu\nu} + d_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $c_{\mu\nu} + d_{\mu\nu}$ is given as*

$$c_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{160}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore $R_{\mu\nu} = R_{\mu\nu}$ too. The Ricci tensor $R_{\mu\nu}$ is determined by the four basic fields of nature $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ as

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} \tag{161}$$

Rearranging equation , it is

$$\begin{aligned} c_{\mu\nu} + d_{\mu\nu} &\equiv R_{\mu\nu} - a_{\mu\nu} - b_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \\ &\equiv R_{\mu\nu} - (G_{\mu\nu} + (\Lambda \times g_{\mu\nu})) \end{aligned} \tag{162}$$

Rearranging equation 162, it is

$$\begin{aligned} c_{\mu\nu} + d_{\mu\nu} &\equiv R_{\mu\nu} - (G_{\mu\nu} + (\Lambda \times g_{\mu\nu})) \\ &\equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu})\right) \\ &\equiv \left(\frac{R}{2} \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \end{aligned} \tag{163}$$

The fundamental geometrical formulation of the fields of nature $c_{\mu\nu} + d_{\mu\nu}$ is given for sure as

$$c_{\mu\nu} + d_{\mu\nu} \equiv \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \tag{164}$$

□

Remark 3.2. *Interestingly, the tensors $c_{\mu\nu} + d_{\mu\nu}$ do not depend on the space-time dimension D. Nonetheless, the question arises immediately whether there are conditions under which it is conceivable that the relationship*

$$c_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} = 0 \tag{165}$$

is given. Equation 165 simplifies at the end as

$$R = 2 \times \Lambda = \Lambda + \Lambda \tag{166}$$

or as

$$R - \Lambda = \Lambda \tag{167}$$

According to Barukčić (Barukčić, 2013) it [3], [4], [5] *is* $\underline{\Lambda} = R - \Lambda$. *Under the assumed circumstances, the relationship*

$$\underline{\Lambda} = \Lambda \tag{168}$$

applies, where $\underline{\Lambda}$ *denotes the other of* Λ *or anti-lambda. Objective reality determined by the vanishing of the fields* $c_{\mu\nu}$ + $d_{\mu\nu}$ = 0 *is an objective reality where* $\underline{\Lambda}$ *is equal to* Λ*, its own other, its own opposite and vice versa.*

3.4.4. Theorem. The geometrical structure of the fields of nature $a_{\mu\nu}$ + $c_{\mu\nu}$

In general, Einstein's tensor $G_{\mu\nu}$ is defined as

$$G_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{169}$$

However, we have to face helplessly the fact that the detailed structure of what determines the Einstein tensor in dependence of the 4 fundamental fields of nature is at present unknown to us. Nevertheless, despite all methodological intellectual darkness, we know the following (see table 4, p. 40).

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$unknown_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 4. Einstein tensor $G_{\mu\nu}$ and the four basic fields of nature

Theorem 16 (The determination of Einstein's tensor $G_{\mu\nu}$)**.** *There is a tensor* $x_{\mu\nu}$ *which is still unknown in detail and which is an intrinsic part of the tensor* $G_{\mu\nu}$*. Einstein's tensor* $G_{\mu\nu}$ *is determined in detail as*

$$G_{\mu\nu} = a_{\mu\nu} + x_{\mu\nu} \tag{170}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore

$$G_{\mu\nu} = G_{\mu\nu} \tag{171}$$

[3] Ilija Barukčić, "The Relativistic Wave Equation," International Journal of Applied Physics and Mathematics vol. 3, no. 6, pp. 387-391, 2013.
[4] ibid.
[5] ibid.

Rearranging equation 169, it is

$$G_{\mu\nu} - a_{\mu\nu} + a_{\mu\nu} = G_{\mu\nu} \tag{172}$$

It is

$$G_{\mu\nu} - a_{\mu\nu} = x_{\mu\nu} \tag{173}$$

In general, Einstein's tensor $G_{\mu\nu}$ is determined by the tensor $a_{\mu\nu}$ and an unknown tensor $x_{\mu\nu}$ as

$$a_{\mu\nu} + x_{\mu\nu} = G_{\mu\nu} \tag{174}$$

□

Theorem 17 (The determination of the tensor $x_{\mu\nu}$)**.** *The tensor $x_{\mu\nu}$ is determined as*

$$x_{\mu\nu} = c_{\mu\nu} \tag{175}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) = \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{176}$$

too. In our understanding, this field is determined by the same unknown tensor $x_{\mu\nu}$ as is the Einstein tensor $G_{\mu\nu}$. From our point of view, the following relationship

$$(x_{\mu\nu}) + (d_{\mu\nu}) = \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{177}$$

applies until further notice. As next, equation 177 changes slightly (see equation 164, p. 39). It is equally valid that

$$(x_{\mu\nu}) + (d_{\mu\nu}) = (c_{\mu\nu}) + (d_{\mu\nu}) \tag{178}$$

Under the conditions mentioned in the previous passage, we can determine the unknown tensor $x_{\mu\nu}$ as

$$x_{\mu\nu} = c_{\mu\nu} \tag{179}$$

□

Theorem 18 (The field of nature $a_{\mu\nu}$ + $c_{\mu\nu}$)**.** *The relationship between the Einstein's tensor $G_{\mu\nu}$ and the basic fields of nature is determined by the relation*

$$G_{\mu\nu} = a_{\mu\nu} + c_{\mu\nu} \tag{180}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore

$$G_{\mu\nu} = G_{\mu\nu} \tag{181}$$

too. As found before (see equation 170, p. 40), it is equally

$$G_{\mu\nu} = a_{\mu\nu} + x_{\mu\nu} \tag{182}$$

In the meantime, we have been able to determine the exact structure of the unknown tensor $x_{\mu\nu}$. It is $x_{\mu\nu} = c_{\mu\nu}$ (see equation 179, p. 41). Equation 182 changes because of this insight. The basic fields of nature $a_{\mu\nu}$ and $c_{\mu\nu}$ are determining the Einstein tensor in detail as

$$G_{\mu\nu} = a_{\mu\nu} + c_{\mu\nu} \tag{183}$$

□

3.4.4.1. Theorem. The field of nature $a_{\mu\nu} + c_{\mu\nu}$

Theorem 19 (The field of nature $a_{\mu\nu} + c_{\mu\nu}$). *Einstein's tensor is determined as*

$$a_{\mu\nu} + c_{\mu\nu} = G_{\mu\nu} \tag{184}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{185}$$

is true. Therefore, it is equally true that

$$\begin{aligned}
a_{\mu\nu} + c_{\mu\nu} &\equiv \qquad a_{\mu\nu} \qquad + \qquad c_{\mu\nu} \\
&\equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} - b_{\mu\nu}\right) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - d_{\mu\nu}\right) \\
&\equiv \left(G_{\mu\nu} + \Lambda \times g_{\mu\nu} - b_{\mu\nu}\right) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - d_{\mu\nu}\right) \\
&\equiv G_{\mu\nu} + \underbrace{\frac{R}{2} \times g_{\mu\nu} - b_{\mu\nu} - d_{\mu\nu}}_{=+0} \\
&\equiv G_{\mu\nu}
\end{aligned} \tag{186}$$

□

In this respect, the question arises whether the tensors $G_{\mu\nu}$ and $\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu}$ do posses a common tensor $x_{\mu\nu}$ at all? Didn't we just make all this up and defined it? We know that $G_{\mu\nu} + \Lambda \times g_{\mu\nu} = a_{\mu\nu} + x_{\mu\nu} + \Lambda \times g_{\mu\nu} = a_{\mu\nu} + b_{\mu\nu}$. In other words it is $x_{\mu\nu} + \Lambda \times g_{\mu\nu} = b_{\mu\nu}$ or $-\Lambda \times g_{\mu\nu} = x_{\mu\nu} - b_{\mu\nu}$. Substituting this relationship into the tensor and $\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu}$ it is and $\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} = \frac{R}{2} \times g_{\mu\nu} + x_{\mu\nu} - b_{\mu\nu}$. Therefore, our assumption is justified that the tensors $G_{\mu\nu}$ and $\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu}$ do posses a common tensor $x_{\mu\nu}$ together. The proof provided is logically sound. The table 5 (see table 5, p. 42) should be able now to provide us with the recognised details in a logically consistent way.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 5. Einstein tensor $G_{\mu\nu}$ and the four basic fields of nature

3.4.5. Theorem. The geometrical structure of the fields of nature $b_{\mu\nu}$ + $d_{\mu\nu}$

Theorem 20 (The geometrical structure of the fields of nature $b_{\mu\nu}$ + $d_{\mu\nu}$)**.** *The field* $\left(\frac{R}{2}\right) \times g_{\mu\nu}$ *is determined by the basic field of nature as*

$$\left(\frac{R}{2}\right) \times g_{\mu\nu} = \left(b_{\mu\nu}\right) + \left(d_{\mu\nu}\right) \tag{187}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore (see equation 180, p. 41)

$$\begin{aligned} b_{\mu\nu} + d_{\mu\nu} &\equiv b_{\mu\nu} + d_{\mu\nu} + a_{\mu\nu} + c_{\mu\nu} - a_{\mu\nu} - c_{\mu\nu} \\ &\equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} - a_{\mu\nu} - c_{\mu\nu} \\ &\equiv \left(a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}\right) - \left(a_{\mu\nu} + c_{\mu\nu}\right) \\ &\equiv R_{\mu\nu} - G_{\mu\nu} \\ &\equiv R_{\mu\nu} - R_{\mu\nu} + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \\ &\equiv \left(\frac{R}{2}\right) \times g_{\mu\nu} \end{aligned} \tag{188}$$

□

A very attentive reader may note in this context that it will not always correspond to the truth, if it is just defined how objective reality has to be. In point of fact, the fundamental question arises indeed, is it allowed at all to decompose the Ricci tensor $R_{\mu\nu}$ into the four basic four fields of nature as $a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu}$. Nonetheless, if this question is allowed to be answered with a clear yes, and if the Einstein's field equations are true, then what is presented in this publication follows with an undeniable and pure logical necessity.

3.5. The basic field of nature $c_{\mu\nu}$

3.5.1. Theorem. The geometrical structure of the basic field of nature $c_{\mu\nu}$

Theorem 21 (The geometrical structure of the basic field of nature $c_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $c_{\mu\nu}$ is given as*

$$c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{189}$$

Proof by direct proof. Einstein's tensor $G_{\mu\nu}$ (Barukčić, 2016b,c, 2020b,c,d,d, 2021b) has been derived (but not defined) as

$$G_{\mu\nu} = a_{\mu\nu} + c_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times g_{\mu\nu} = \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \tag{190}$$

The validity of this tensor equation remains even under conditions under which the stress-energy tensor of the ordinary matter disappears or $a_{\mu\nu} = 0$. Under these conditions, it is

$$c_{\mu\nu} = G_{\mu\nu} - a_{\mu\nu} = G_{\mu\nu} - 0 = G_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times g_{\mu\nu} = \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \tag{191}$$

This tonsorial equation is true in all coordinate systems. Similarly, under the conditions of 1 space-time dimension, we must take the validity of this tensor equation as given. Under these conditions follows that

$$c_{\mu\nu} = \left(\frac{R}{1} - \frac{R}{2}\right) \times g_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{192}$$

The geometrical form of the fundamental field of nature $c_{\mu\nu}$ is given as

$$c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu} \tag{193}$$

□

Remark 3.3. *The field $c_{\mu\nu}$ is determined as $c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu}$. However, this insight results in a few consequences. Both tensors, $a_{\mu\nu}$ and $c_{\mu\nu}$, are contributing to Einstein's tensor $G_{\mu\nu}$. However, this does not exclude that the field $c_{\mu\nu}$ exists and can exist even if the field $a_{\mu\nu}$ disappears or no longer exists. We have to keep in mind that the Einstein field equations allow and describe an objective reality even in one dimension as*

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{194}$$

This excludes the existence of the stress-energy tensor of the ordinary matter $a_{\mu\nu}$ in one dimension. Nevertheless, the existence of the stress-energy tensor of the electromagnetic field $b_{\mu\nu}$ in one dimension is not excluded. The following picture might illustrate equation 194 in more detail.

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right)$$

$$((\Lambda) \times g_{\mu\nu})$$

$$\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$$

3.6. The basic field of nature $a_{\mu\nu}$

3.6.1. Theorem. The geometrical structure of the basic field of nature $a_{\mu\nu}$

Theorem 22 (The geometrical structure of the basic field of nature $a_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $a_{\mu\nu}$ (stress-energy tensor of ordinary matter) is given as*

$$a_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) \tag{195}$$

Proof by direct proof. Einstein's tensor $G_{\mu\nu}$ (Barukčić, 2016b,c, 2020b,c,d,d, 2021b) has been derived (but not defined) as

$$G_{\mu\nu} = a_{\mu\nu} + c_{\mu\nu} = \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu} \tag{196}$$

The stress-energy tensor of the ordinary matter, denoted as $a_{\mu\nu}$, is given as

$$a_{\mu\nu} = \left(\left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}\right) - c_{\mu\nu} \tag{197}$$

The tensor $c_{\mu\nu}$ has been determined as $c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu}$ (see equation 192, p. 44). Equation 197 becomes

$$a_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{198}$$

The geometrical form of the stress-energy tensor of the ordinary matter, denoted as $a_{\mu\nu}$, is given as

$$a_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) \tag{199}$$

□

Remark 3.4. *An objective reality under conditions of D=1 space-time dimension seems to be possible purely theoretically. Under these conditions the tensor of the ordinary matter $a_{\mu\nu}$ vanishes or it is*

$$a_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) = \left(\frac{R}{1} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) = \left(R \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) = 0 \tag{200}$$

and with it also the possibility of any kind of locality. In this respect it seems to be necessary to point out that objective reality under conditions of D=1 space-time dimension is purely non-local.

3.6.2. Theorem. The geometrical structure of the basic field of nature $a_{\mu\nu}$

Theorem 23 (The geometrical structure of the basic field of nature $a_{\mu\nu}$ **.** *The geometrical structure of the basic field of nature $a_{\mu\nu}$ is given as*

$$a_{\mu\nu} = (R_{\mu\nu}) - (R \times g_{\mu\nu}) \tag{201}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{202}$$

is true. Therefore, it is equally true that

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{203}$$

Einstein field equations becomes

$$(G_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) = (R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{204}$$

As outlined before, it is $G_{\mu\nu} = a_{\mu\nu} + x_{\mu\nu}$ (see equation 170). Equation 204 becomes

$$\underbrace{a_{\mu\nu} + x_{\mu\nu}}_{\text{Einstein tensor}} + (\Lambda \times g_{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = (R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \tag{205}$$

The stress-energy of ordinary matter, $a_{\mu\nu}$, is given as

$$a_{\mu\nu} = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} - x_{\mu\nu} - (\Lambda \times g_{\mu\nu}) = (R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - x_{\mu\nu} - (\Lambda \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \tag{206}$$

or more simplified as

$$a_{\mu\nu} = (R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - x_{\mu\nu} \underbrace{- (\Lambda \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu})}_{+0} \tag{207}$$

The unknown tensor $x_{\mu\nu}$ has been identified (see equation 179 and equation 193) as $x_{\mu\nu} = c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu}$. Equation 207 becomes

$$a_{\mu\nu} = (R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + 0 = (R_{\mu\nu}) - (R \times g_{\mu\nu}) \tag{208}$$

□

The weak interaction and the electromagnetic interaction were unified by the Glashow–Weinberg–Salam model into electroweak (see Glashow, 1959, Salam and Ward, 1959, Weinberg, 1967) interaction. The electromagnetic, weak, and strong forces (see Georgi and

Glashow, 1974) are meanwhile merged into a single force. However, it is necessary to consider whether the weak force and the strong force can be merged into a single force of ordinary matter denoted by something related on

$$\text{Ordinary matter} = \text{Strong force} + \text{Weak force} = a_{\mu\nu} = \left(R_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) \tag{209}$$

Under conditions where the stress energy tensor or ordinary matter vanishes or where $a_{\mu\nu} = 0$, it is

$$\left(R_{\mu\nu}\right) = \left(R \times g_{\mu\nu}\right) \tag{210}$$

Under these circumstances, the only term in the stress–energy tensor is the stress energy tensor of electromagnetism $b_{\mu\nu}$. We obtain

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times (D=1)}\right) \times g_{\mu\nu} = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{211}$$

These conditions are given especially under conditions of D = 1 space time dimension. Provided that the proof of the existence of strings in a space-time dimension D = 1 would succeed, the wave equation (see equation 211) should be able to describe those strings completely. Normalising equation 211 it is

$$\frac{\left(\frac{R}{2}\right)}{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times (D=1)}\right)} + \frac{(\Lambda)}{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times (D=1)}\right)} = +1 \tag{212}$$

Multiplying equation 212 by the Schrödinger equation (see Schrödinger, Erwin Rudolf Josef Alexander, 1926), it is

$$\frac{\left(\frac{R}{2}\right) \times (H \times \Psi)}{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times (D=1)}\right)} + \frac{(\Lambda) \times (H \times \Psi)}{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times (D=1)}\right)} = (H \times \Psi) \tag{213}$$

where H is the Hamiltonian of a system, an operator corresponding to the total energy of a system. Under circumstances where $H = \underline{ek} \times \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times (D=1)}\right)$ (assuming $\underline{ek} = 1$) equation 213 simplifies as

$$\left(\frac{R}{2} \times \Psi\right) + (\Lambda \times \Psi) = H \times \Psi \tag{214}$$

It is $\hbar = \frac{h}{2 \times \pi}$ and $h = 2 \times \pi \times \hbar$ In quantum mechanics, the canonical commutation relation is a fundamental relation which justifies the equation $[x,p] = i \times \hbar \times \mathbb{I}$ and $\hbar = \frac{[x,p]}{i \times \mathbb{I}}$. It is $\frac{1}{2} = \frac{\pi \times \hbar}{h} = \frac{\pi \times \frac{[x,p]}{i \times \mathbb{I}}}{h}$. Equation 214 becomes

$$\left(\frac{\pi \times [x,p]}{i \times h \times \mathbb{I}} \times R \times \Psi\right) + (\Lambda \times \Psi) = H \times \Psi \tag{215}$$

3.7. *The basic field of nature $d_{\mu\nu}$*

3.7.1. Theorem. The geometrical structure of the basic field of nature $d_{\mu\nu}$

Theorem 24 (The geometrical structure of the basic field of nature $d_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as*

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{216}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore $R_{\mu\nu} = R_{\mu\nu}$ too. The Ricci tensor $R_{\mu\nu}$ is determined by the four basic fields of nature $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, $d_{\mu\nu}$ as

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = R_{\mu\nu} \tag{217}$$

Rearranging equation , it is

$$\begin{aligned} c_{\mu\nu} + d_{\mu\nu} &\equiv R_{\mu\nu} - a_{\mu\nu} - b_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \end{aligned} \tag{218}$$

It follows (Barukčić, 2016b,c, 2020b,c,d,d, 2021b) (and is not defined) that

$$c_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \tag{219}$$

The tensor $c_{\mu\nu}$ has been determined as $c_{\mu\nu} = \left(\frac{R}{2}\right) \times g_{\mu\nu}$ (see equation 192, p. 44). Equation 219 becomes

$$\left(\frac{R}{2} \times g_{\mu\nu}\right) + d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{220}$$

or

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{221}$$

The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{222}$$

□

As of today, the importance and the properties of gravitational waves (see Abbott et al., 2016, Einstein, 1918a, Heaviside, 1898) cannot be overemphasized. However, at present there are rather diverging results on the issue of the relevance of the cosmological constant Λ on gravitational waves (see Bičák, Jiři and Podolský, Jiři , 1999, Näf et al., 2009) in general relativity. Do gravitational waves exist even under conditions of objective reality where the stress energy tensor of ordinary matter $a_{\mu\nu}$ is equal to $a_{\mu\nu} = 0$? We point out that one can not rule out the possibility that under these assumptions gravitational waves will be localised inside field $d_{\mu\nu}$ or will even be identical with field $d_{\mu\nu}$. As known, the space-time of special relativity is flat while the space-time of general relativity is curved. Equation 222 can be decomposed as

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) = -\left(\Lambda \times \eta_{\mu\nu}\right) - \left(\Lambda \times \underline{\eta}_{\mu\nu}\right) \tag{223}$$

where $\eta_{\mu\nu}$ is the metric tensor of special relativity while $\underline{\eta}_{\mu\nu}$ might describe disturbances or ripples in the curvature of space-time.

3.7.2. Theorem. The geometrical structure of the basic field of nature $d_{\mu\nu}$

Theorem 25 (The geometrical structure of the basic field of nature $d_{\mu\nu}$). *The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as*

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{224}$$

Proof by direct proof. Axiom 1 or +1 = +1 is valid. Based on this axiom, we obtain

$$c_{\mu\nu} = c_{\mu\nu} \tag{225}$$

or (see equation 35, p. 19 and equation 36, p. 20)

$$G_{\mu\nu} - a_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{226}$$

We rearrange equation 226. It is

$$R_{\mu\nu} - \left(\frac{R}{2} \times g_{\mu\nu}\right) - a_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{227}$$

Based on equation 113, p. 30, equation 227 changes slightly. We obtain

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - a_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{228}$$

Equation 228 is generally valid. Rearranging equation 228, it is

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) - a_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{229}$$

or

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) - a_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{230}$$

The unrestricted validity of the previous equation (see equation 230) is also given if the tensor of ordinary matter $a_{\mu\nu}$ vanishes or if $\mathbf{a}_{\mu\nu} = \mathbf{0}$. We obtain

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{231}$$

The unrestricted validity of the previous equation (see equation231) is also given under conditions of **D = 1** space-time dimension. We obtain

$$\left(\frac{R}{1} \times g_{\mu\nu}\right) - \left(R \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{232}$$

or

$$0 = -\left(\Lambda \times g_{\mu\nu}\right) - d_{\mu\nu} \tag{233}$$

The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{234}$$

□

Remark 3.5. *The geometric structure of the field $d_{\mu\nu}$ has been determined as $d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right)$. However, this raises at once several fundamental and far-reaching questions. Under the most different aspects, the Einstein cosmological constant Λ, usually represented by the Greek letter Λ (Lambda), is viewed as equivalent to the 'mass 'of empty space (which itself can be either positive or negative), and manny times associated with 'vacuum energy' (see also Huterer and Turner, 1999, Zwicky, 1933). In particular, as it may and will be in the end, the basic field of nature $d_{\mu\nu}$ appears to be an underlying background field that exists in space throughout the entire Universe. Is vacuum as such the fourth basic field of nature which is the underlying background field given throughout the entire Universe?*

3.7.3. Theorem. The geometrical structure of the basic field of nature $d_{\mu\nu}$

Theorem 26 (The geometrical structure of the basic field of nature $d_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $d_{\mu\nu}$ is given as*

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{235}$$

Proof by direct proof. Axiom 1 or +1 = +1 is valid. Based on this axiom, we obtain (see equation 188, p. 43)

$$c_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \tag{236}$$

Equation 236 is valid even if $c_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) = 0$. Under these circumstances, it is

$$c_{\mu\nu} = -d_{\mu\nu} \tag{237}$$

Under conditions where $c_{\mu\nu}+d_{\mu\nu}=\left(\frac{R}{2}\times g_{\mu\nu}\right)-(\Lambda\times g_{\mu\nu})=0$, it is equally

$$\left(\frac{R}{2}\times g_{\mu\nu}\right)=(\Lambda\times g_{\mu\nu}) \tag{238}$$

Combining equation 237 and equation 238, it is **either**

$$d_{\mu\nu}=-c_{\mu\nu}=\left(\frac{R}{2}\times g_{\mu\nu}\right) \tag{239}$$

or

$$-d_{\mu\nu}=+c_{\mu\nu}=+(\Lambda\times g_{\mu\nu}) \tag{240}$$

which is equal to

$$+d_{\mu\nu}=-(\Lambda\times g_{\mu\nu}) \tag{241}$$

However, based on equation equation 188 (see equation 188, p. 43) we know that $b_{\mu\nu}+d_{\mu\nu}$ has to be equal to $b_{\mu\nu}+d_{\mu\nu}=\left(\frac{R}{2}\times g_{\mu\nu}\right)$. If the relationship $d_{\mu\nu}=\left(\frac{R}{2}\times g_{\mu\nu}\right)$ (see equation 239) where true, it would follow that

$$\left(\frac{R}{2}\times g_{\mu\nu}\right)=+b_{\mu\nu}+d_{\mu\nu}=+b_{\mu\nu}+\left(\frac{R}{2}\times g_{\mu\nu}\right) \tag{242}$$

In other words, we would have to accept in general that

$$b_{\mu\nu}=0 \tag{243}$$

which is not the fact under any circumstances. Therefore, we must accept that the geometrical structure of the basic field of nature $\mathrm{d}_{\mu\nu}$ is given as

$$\boxed{d_{\mu\nu}=-(\Lambda\times g_{\mu\nu})} \tag{244}$$

□

3.7.4. Theorem. The determination of Λ

We were able to identify the fourth field of nature so as to be $\mathrm{d}_{\mu\nu}=-\Lambda\times g_{\mu\nu}$. Yet, we have not determined at least one concrete geometrical structure of this field or even the physical value of the same. We want to make up for this at the following point.

Theorem 27 (The determination of Λ. *The concrete geometrical structure of the forth field of nature, $d_{\mu\nu}$, including Λ itself, is given as*

$$-\Lambda\times g_{\mu\nu}=\left(+\left(\frac{R}{D}\right)-\left(\frac{R}{2}\right)-\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^4\times D}\right)\right)\times g_{\mu\nu} \tag{245}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{246}$$

is true. Therefore, it is equally true that

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{247}$$

We have at the present time neither an experimental nor a theoretical reason to assume that the Einstein field equations are erroneous. As a result of Einstein's publications (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) we arrive at the following Einstein's field equations.

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{248}$$

Taking the trace of both sides of equation 248, it is

$$(R_{\mu\nu} \times g^{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + (\Lambda \times g_{\mu\nu} \times g^{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \tag{249}$$

or

$$(R) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + (\Lambda \times g_{\mu\nu} \times g^{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T \tag{250}$$

and equally (see equation 398, p. 95)

$$(R) - \left(\left(\frac{R}{2}\right) \times D\right) + (\Lambda \times D) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T \tag{251}$$

Changing equation 251, it is

$$\left(\frac{R}{D}\right) - \left(\frac{R}{2}\right) + (\Lambda) = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{252}$$

Based on the result of equation 252, the value of $+\Lambda$ is given as

$$+\Lambda = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) - \left(\frac{R}{D}\right) + \left(\frac{R}{2}\right) \tag{253}$$

At this point it is necessary to direct the attention of the reader to a very important detail of the equation 253. The space-time dimension D, which can vary, is an essential part of the determination of Λ. Equation 253 is therefore a clear mathematical proof that the cosmological constant Λ (Einstein, 1917) is not a constant. Based on the result of equation 252, the value of $-\Lambda$ is determined as

$$-\Lambda = +\left(\frac{R}{D}\right) - \left(\frac{R}{2}\right) - \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{254}$$

In general, the geometrical form of the field $d_{\mu\nu} = -\Lambda \times g_{\mu\nu}$ is determined as

$$-\Lambda \times g_{\mu\nu} = \left(+\left(\frac{R}{D}\right) - \left(\frac{R}{2}\right) - \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right)\right) \times g_{\mu\nu} \tag{255}$$

□

3.8. The basic field of nature $b_{\mu\nu}$

3.8.1. Theorem. The geometrical structure of the basic field of nature $b_{\mu\nu}$

Theorem 28 (The geometrical structure of the basic field of nature $b_{\mu\nu}$). *The geometrical structure of the basic field of nature $b_{\mu\nu}$ is given as*

$$b_{\mu\nu} = (b) \times g_{\mu\nu} = \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{256}$$

Proof by direct proof. Here we would like to reiterate once again that the following relationship has been established (and not defined) (Barukčić, 2016b,c, 2020b,c,d, 2021b) . It is

$$b_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{257}$$

The tensor $d_{\mu\nu}$ has been determined as $d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right)$ (see equation 222, p. 49). Equation 257 changes slightly. It is

$$b_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) = \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{258}$$

The geometrical structure of the stress-energy momentum tensor of the field $b_{\mu\nu}$ (hopefully he stress-energy (see Hughston and Tod, 1990, p. 38) momentum tensor of the electromagnetic(see Lehmkuhl, 2011, p. 13)) is given as

$$b_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{259}$$

□

Remark 3.6. *Under conditions where equation 259 derived as $b_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right)$ is the geometrized (Kalinowski, 1988) form of the stress-energy momentum tensor of the electromagnetic field, Λ could be measured or calculated exactly as*

$$\left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu c} \times F_{\nu}{}^{c}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{260}$$

Recall, F is Faraday's electromagnetic field tensor. As long as we are allowed to agree with Tonnelat's position, a unified field theory is "... a theory joining the gravitational and the electromagnetic field into one single hyperfield whose equations represent the conditions imposed on the geometrical structure of the universe." (see Tonnelat et al., 1955, p. 5) The geometrization of the fundamental fields of nature that has now been accomplished can be helpful in this view. Under these assumptions, ***the geometrized hyper-field for electromagnetism and gravitation*** *might be given as*

$$c_{\mu\nu} + b_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) = \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \tag{261}$$

There seem to exist conditions where ***the tensor of pure non-locality*** *is given by the equation*

$$b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} = \left(R \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) = \left(R \times g_{\mu\nu}\right) \tag{262}$$

Table 6 is intended to give us a simple and appropriate overview of the relationships that have been established so far.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu} = \left(\frac{R}{D} - R\right) \times g_{\mu\nu}$	$b_{\mu\nu} \equiv \left(\frac{R}{2} + \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) - \left(\frac{R}{D}\right) + \left(\frac{R}{2}\right)\right) \times g_{\mu\nu}$	$a_{\mu\nu} + b_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu}$	$d_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu} = \left(+\left(\frac{R}{D}\right) - \left(\frac{R}{2}\right) - \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right)\right) \times g_{\mu\nu}$	$c_{\mu\nu} + d_{\mu\nu} = \left(\frac{R}{D} - \frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}$
		$a_{\mu\nu} + c_{\mu\nu} = G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$b_{\mu\nu} + d_{\mu\nu} = \frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}$

Table 6. The four basic fields of nature geometrized.

We are again and again confronted with the challenge that various astronomers are claiming that the majority of objective reality, i. e. the cosmos itself, consists of mysterious, invisible stuff that surrounds us, the so called dark matter (see Zwicky, 1933) and dark energy (see Turner, 1999). Unfortunately, we must also concede at the same time that the notions dark matter and dark energy have yet to be adequately (or even fully) understood or clearly defined. In short, is there a difference between dark energy and dark matter at all and what is the difference? The assumption is that dark matter works like an attractive force and slows down the expansion of the cosmos, while dark energy is a sort of anti-gravity and speeds the expansion of the cosmos up. At this point we would like to ask the question whether these lines of thought are valid even for a single photon itself? In the event that we can answer this question unambiguously in the positive, there are a series of implications. Why does a single photon, emitted somewhere out there more than 13 billion years ago, keep on moving forward? What drives such a photon, what accelerates it? Meanwhile, we have been able to identify Λ as a dominating part of photon (Barukčić, 2021a) . We have reason to believe that Λ seems to be that which always drives a single photon forward. However, a single photon also seems to contain within itself the other of itself, a moment under which the electromagnetic field is attracted, collapses and loses its own meaning. Such circumstances are dominated by a graviton (Barukčić, 2021a). In particular and only with the utmost caution do these lines of thought give rise to the hope that notion 'dark energy' (see Perlmutter et al., 1999, Riess et al., 1998) could be identical with

$$\text{dark energy} = \Lambda \times g_{\mu\nu} \tag{263}$$

while the concept of 'dark matter 'could possibly be found in the field

$$\text{dark matter} = \frac{R}{2} \times g_{\mu\nu} \tag{264}$$

It is however more than necessary to emphasise at this point strongly that these lines of pure speculations should not be taken for granted as verified human knowledge.

3.8.2. Theorem. The geometrical structure of the field of nature $b_{\mu\nu}$

Einstein's general theory of relativity is not the end of all wisdom but only one small stepping stone towards the ultimate goal of all physics, the **unified field theory**. A unified field theory should even be able to integrate somehow both gravitational and electromagnetic fields (see Einstein, 1925) into a single hyper-field. Unfortunately, so far the search for **the holy grail of all physics**, the unified field theory, has not been crowned with success. Solving the issue of the unified field theory turns out to be more difficult than expected. Even Einstein himself who brilliantly succeeded in geometrizing gravity was at the end unable to accomplish the geometrization of electromagnetism too. Einstein's unified field theory program (see Sauer, 2014) or the whole of physics seen as an unique entity is characterized in total by more than forty technical papers on the unified field theory. Einstein is writing:

> "It is only the circumstance that we have not sufficient knowledge of the electromagnetic field of concentrated charges that compels us, provisionally, to leave undetermined in presenting the theory, the true form of this tensor. "
>
> (see Einstein, 1923b, p. 91)

It is not necessarily mandatory to describe all that exists geometrically or to apply geometrical methods in science whenever it is possible and more or less rightly so. It is the unity of nature at the end which is the foundation for the unity of science and of physics itself. Historically, Einstein's transition from the special theory of relativity to the general theory of relativity was carried out with the aid of the mathematical technology of tensors. However, this does not exclude in any way that objective reality can be described completely with the help of e.g. the probability (see Barukčić, 2022) theory too. Whatever the case may be, it is not very astounding that since Einstein's very remarkable accomplishment of the description of gravity as a geometric phenomenon of curved space time, numerous great efforts, including Einstein (see Sauer, 2014) himself, have been made to geometrize electromagnetism too in order to end up at the unified field theory. However, electromagnetism itself even if identical with gravity under certain aspects is not the same as gravity is, electromagnetism is different from gravity too. Therefore, finding a suitable geometric description of the stress-energy tensor of electromagnetism is the first great problem for geometrizing electromagnetism. At this point we want to dare a completely new approach to this issue in order identify the geometrical structure of the stress-energy tensor of the electromagnetic field, denoted as $b_{\mu\nu}$, very precisely.

Theorem 29 (The geometrical structure of the field of nature $b_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $b_{\mu\nu}$ is given as*

$$b_{\mu\nu} = \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{265}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore $G_{\mu\nu} = G_{\mu\nu}$ or $G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) =$

$G_{\mu\nu} + (\Lambda \times g_{\mu\nu})$ too. We obtain

$$\begin{aligned} a_{\mu\nu} + b_{\mu\nu} &\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \\ &\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \\ &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \end{aligned} \tag{266}$$

Equation 266 simplifies as

$$\begin{aligned} b_{\mu\nu} &\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) - a_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) - a_{\mu\nu} \\ &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} - a_{\mu\nu} \end{aligned} \tag{267}$$

or as

$$b_{\mu\nu} + 0 \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + 0 + (\Lambda \times g_{\mu\nu}) - a_{\mu\nu} \tag{268}$$

Rearranging equation 268, it is

$$b_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\frac{R}{2}\right) \times g_{\mu\nu} + \left(\frac{R}{2}\right) \times g_{\mu\nu} + (\Lambda \times g_{\mu\nu}) - a_{\mu\nu} \tag{269}$$

or

$$b_{\mu\nu} \equiv R_{\mu\nu} - (R \times g_{\mu\nu}) - a_{\mu\nu} + \left(\frac{R}{2}\right) \times g_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \tag{270}$$

According to equation 195, p. 46, it is $a_{\mu\nu} = (R_{\mu\nu}) - (R \times g_{\mu\nu})$. Equation 270 becomes

$$b_{\mu\nu} \equiv a_{\mu\nu} - a_{\mu\nu} + \left(\frac{R}{2}\right) \times g_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \tag{271}$$

It is

$$b_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \tag{272}$$

In general, the geometrical structure of the basic field of nature $b_{\mu\nu}$ is given as

$$b_{\mu\nu} = \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{273}$$

□

3.8.3. Theorem. The geometrical structure of the field of nature $b_{\mu\nu}$

Theoretically, it may be advantageous to look at the vacuum in greater detail. This in turn could hopefully enable us to determine the geometric structure of the stress-energy tensor of the electromagnetic field.

Theorem 30 (The geometrical structure of the field of nature $b_{\mu\nu}$)**.** *The geometrical structure of the basic field of nature $b_{\mu\nu}$ is given as*

$$b_{\mu\nu} = \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{274}$$

Proof by direct proof. Axiom 1 or +1=+1 is valid and therefore

$$\begin{aligned} R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv a_{\mu\nu} + b_{\mu\nu} \end{aligned} \tag{275}$$

Adding +0, equation 275 doesn't change. It is

$$\begin{aligned} R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + 0 + (\Lambda \times g_{\mu\nu}) &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} + 0 \\ &\equiv a_{\mu\nu} + b_{\mu\nu} + 0 \end{aligned} \tag{276}$$

It is $+\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) = +0$. Equation 276 is reformulated. We obtain

$$\begin{aligned} R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \underbrace{- \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right)}_{+0} + (\Lambda \times g_{\mu\nu}) &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv a_{\mu\nu} + b_{\mu\nu} \end{aligned} \tag{277}$$

Simplifying equation 277, it is

$$\begin{aligned} R_{\mu\nu} - ((R) \times g_{\mu\nu}) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv a_{\mu\nu} + b_{\mu\nu} \end{aligned} \tag{278}$$

The energy–momentum tensor $\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ is non-zero in some regions of space-time and zero in others. However, if the energy–momentum tensor $\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ is zero in the region under consideration, then the Einstein field equations are also referred to as the vacuum field equations. A vacuum solution of the Einstein field equations is a manifold whose Einstein tensor $G_{\mu\nu}$ vanishes. At the end, in empty space, the Einstein's field equation reduce to $R_{\mu\nu} = 0$. As published somewhere else, manifolds with a vanishing Ricci tensor, $R_{\mu\nu} = 0$, are referred to as Ricci-flat manifolds. At this point, we do not want to present any new aspects of the vacuum field equations. Nonetheless, there are circumstances, where the condition $\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = 0$ is given. Under theses conditions, equation 278 changes slightly. It is

$$\begin{aligned} R_{\mu\nu} - ((R) \times g_{\mu\nu}) + \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv +0 \\ \equiv a_{\mu\nu} + b_{\mu\nu} \equiv +0 \end{aligned} \tag{279}$$

In other words, it is

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv -R_{\mu\nu} + ((R) \times g_{\mu\nu}) \tag{280}$$

or

$$-\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv +R_{\mu\nu} - ((R) \times g_{\mu\nu}) \tag{281}$$

It is equally

$$+b_{\mu\nu} \equiv -a_{\mu\nu} \tag{282}$$

or

$$-b_{\mu\nu} \equiv +a_{\mu\nu} \tag{283}$$

Under these conditions, we are able to determine the geometrical structure of the stress-energy tensor of the filed $b_{\mu\nu}$ very precisely **either** as $+R_{\mu\nu} - ((R) \times g_{\mu\nu})$ **or** as $+\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu})$ (see equation 280 - equation 283). However, what is it at the end? Based on equation 188, it has to be that

$$+b_{\mu\nu} + d_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \tag{284}$$

The geometrical structure of the tensor $d_{\mu\nu}$ has been identified several times as $+d_{\mu\nu} = -(\Lambda \times g_{\mu\nu})$ (see: equation 222, equation 234, equation 244). Equation 284 changes slightly. It is

$$+b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \tag{285}$$

While equation 280 until equation 283 give us only an approximate picture of the geometric structure of the field $b_{\mu\nu}$, equation 285 clearly shows us what the geometric structure of the field $b_{\mu\nu}$ has to be. The geometric structure of the field $b_{\mu\nu}$is given as

$$b_{\mu\nu} = \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{286}$$

□

To understand how far one can go it is necessary to ask the question whether electromagnetism is completely foreign to geometry? Einstein's field equations of general relativity have more or less a purely local character. As a consequence, Einstein's description of gravitation in terms of curved space need not imply that electromagnetism itself has to be described geometrically too. Nonetheless, as long as we are authorised to rely on equation 286 the basic field of nature $b_{\mu\nu}$, which is presumably the stress-energy tensor of electromagnetism, has been described geometrically too.

3.8.4. The geometrical structure of stress energy tensor of the electromagnetic field

At this stage of the research it is possible to specify that we have been able to determine the geometric structure of the field $b_{\mu\nu}$ with certainty. However, at this point, we need to provide some clear clarifications. The lines of thought presented here are based on Einstein's well-known description of the relationship between between ordinary matter and the elcetromagentic field as: "Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß **alles außer dem Gravitationsfeld als** '**Materie**'**bezeichnet** wird, also nicht nur die 'Materie'im üblichen Sinne, sondern auch das elektromagnetische Feld. " (Einstein, 1916, p. 802/803) . As a result of this, we decomposed the stress-energy tensor of matter into the stress-energy tensor of ordinary matter, denoted as $a_{\mu\nu}$, and into the stress-energy tensor of the electromagnetic field, denoted as $b_{\mu\nu}$. Despite all this, we have to ask ourselves how certain we can be that $b_{\mu\nu}$ is at the end that what it is assumed to be, the geometrical form of the the stress-energy tensor of the electromagnetic field. Why should the tensor $b_{\mu\nu}$ not be identical with the stress-energy tensor of relativistic kinetic energy?

$$b_{\mu\nu} = \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu} = \text{stress-energy tensor of relativistic-kinetic energy?} \tag{287}$$

These issues can be clarified in principle. Firstly. Equation 286 is valid even under circumstances where the tensor $a_{\mu\nu} = 0$. Under these circumstances, the tensor $b_{\mu\nu}$ contains all froms of stress-energy and momentum while the relativistic kinetic energy itself is no longer given, the same has passed over into the pure electromagentic field. Under these circumstances, the tensor $b_{\mu\nu}$ is the stress-energy tensor of the electromagentic field. Additionally, it is a tensorial equation which must hold under all cooridnate systems. Secondly. If $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, then the following equation need to be true too.

$$\left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu}{}^{\mathrm{c}}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \tag{288}$$

which can be proofed in principle. Thirdly. In the following we assume for now that the laws of the special theory of relativity (STR), in particular the unity and the struggle between a particle and a wave which have been described and derived as $\left(\frac{({}_{0}E_{\mathrm{t}})^2}{({}_{\mathrm{r}}E_{\mathrm{t}})^2}\right) + \left(\frac{({}_{\mathrm{w}}E_{\mathrm{t}})^2}{({}_{\mathrm{r}}E_{\mathrm{t}})^2}\right) = +1$ (see Barukčić, Ilija, 2022, p. 17) , are extended by the insights of the theory of general relativity (GTR) but not completely disproved or invalidated. Let us assume, that $b_{\mu\nu}$ is the stress-energy tensor of relativistic kinetic energy. Under these assumptions, the stress-energy tensor of the electromagnetic field would have to follow as

$$({}_{\mathrm{w}}E_{\mathrm{t}}) \times g_{\mu\nu} = \left(\sqrt[2]{\left(\left(\left(\left(\frac{R}{2} \right) + (\Lambda) \right) \times \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D} \right) \right) \right)} \right) \times g_{\mu\nu} \tag{289}$$

However, equation 288 and equation 289 were both true simultaneously only under conditions where $a_{\mu\nu} = 0$, which is to restricted. Fourthly. In last consequence, the previous considerations (equation 289) would demand that the content of the stress energy tensor of matter is at the end (energy^2/...) and not (energy/...). In this case, the equations derived would have to be modified slightly.

In general, from the point of view of special theory of relativity, the relationship between ordinary matter and electromagnetic wave would be given as follows.

$$({}_{\mathrm{w}}E_{\mathrm{t}}) \equiv ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \sqrt[2]{1 - \frac{({}_{0}E_{\mathrm{t}})^2}{({}_{\mathrm{r}}E_{\mathrm{t}})^2}} = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \sqrt[2]{1 - \frac{({}_{0}m_{\mathrm{t}} \times c^2) \times ({}_{0}m_{\mathrm{t}} \times c^2)}{({}_{\mathrm{r}}m_{\mathrm{t}} \times c^2) \times ({}_{\mathrm{r}}m_{\mathrm{t}} \times c^2)}} = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \sqrt[2]{1 - \frac{({}_{0}m_{\mathrm{t}})^2}{({}_{\mathrm{r}}m_{\mathrm{t}})^2}} = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \sqrt[2]{1 - \frac{({}_{\mathrm{r}}m_{\mathrm{t}})^2 \times \left(1 - \frac{v^2}{c^2}\right)}{({}_{\mathrm{r}}m_{\mathrm{t}})^2}} = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \sqrt[2]{\frac{v^2}{c^2}} \tag{290}$$

which can be simplified under some conditions as

$${}_{\mathrm{w}}E_{\mathrm{t}} = \frac{v}{c} \times ({}_{\mathrm{r}}E_{\mathrm{t}}) \tag{291}$$

where v is the relative velocity. Every ordinary matter is associated with an electromagnetic wave. However, at very small everyday relative velocities v this effect is in the end negligibly small but still given. According to STR, the ordinary matter, denoted as ${}_{\mathrm{a}}\mathrm{E}_{\mathrm{t}}$, is given as

$${}_{\mathrm{a}}E_{\mathrm{t}} = {}_{\mathrm{r}}E_{\mathrm{t}} - {}_{\mathrm{w}}E_{\mathrm{t}} = ({}_{\mathrm{r}}E_{\mathrm{t}}) - ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \sqrt[2]{\frac{v^2}{c^2}} = ({}_{\mathrm{r}}E_{\mathrm{t}}) \times \left(1 - \sqrt[2]{\frac{v^2}{c^2}}\right) \tag{292}$$

There are circumstances where the stress-energy tensor of ordinary matter is given as

$$({}_{\mathrm{a}}E_{\mathrm{t}}) \times g_{\mu\nu} = \left(\left(1 - \sqrt[2]{\frac{v^2}{c^2}}\right) \times \frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \times g_{\mu\nu} \tag{293}$$

while the stress-energy tensor of electromagnetic wave/field would be given as

$$({}_{\mathrm{w}}E_{\mathrm{t}}) \times g_{\mu\nu} = \left(\left(\sqrt[2]{\frac{v^2}{c^2}} \right) \times \frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \times g_{\mu\nu} \tag{294}$$

There might be circumstances given, where the stress-energy tensor of ordinary matter, denoted as $a_{\mu\nu}$, is identical with the stress-energy tensor of rest-mass, denoted by special theory of relaltivity as ${}_{0}\mathrm{E}_{\mathrm{t}}$. Nontheless, this need not to be the case under any circumstances given. The relationship between "rest energy ", denoted as ${}_{0}E_{\mathrm{t}}$ and ordinary energy ${}_{\mathrm{a}}E_{\mathrm{t}}$ would be given as (see equation 26)

$$({}_{0}E_{\mathrm{t}}) \times g_{\mu\nu} = \underbrace{\left(\left(\sqrt[2]{1 - \frac{v^2}{c^2}} \right) \times \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D} \right) \right)}_{rest\ energy/matter\ {}_{0}\mathrm{E}_{\mathrm{t}}} \times g_{\mu\nu} = \left(\frac{\left(\sqrt[2]{1 - \frac{v^2}{c^2}} \right)}{\left(1 - \sqrt[2]{\frac{v^2}{c^2}} \right)} \times ({}_{\mathrm{a}}E_{\mathrm{t}}) \right) \times g_{\mu\nu} \tag{295}$$

3.9. The evolution or self-organisation of objective reality

3.9.1. Objective reality without ordinary matter

Electrovacuum solution (electro-vacuum) is one of the known exact solutions of the Einstein field equations. The stress-energy momentum tensor (see equation 5.44) is defined as

$$E_{\mu\nu} \equiv a_{\mu\nu} + b_{\mu\nu} \tag{296}$$

Under conditions where objective reality is determined by a vanishing tensor of ordinary matter ($a_{\mu\nu} = 0$) we obtain

$$E_{\mu\nu} \equiv (a_{\mu\nu} = 0) + b_{\mu\nu} \tag{297}$$

or

$$b_{\mu\nu} \equiv E_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu} \tag{298}$$

However, equation 298 is given only under certain circumstances. Nonetheless, under these conditions, all stress energy and momentum is included in the stress energy tensor of the electromagnetic field. Nonetheless, a vanishing tenors of ordinary matter does not imply a vanishing of Einstein's tensor. The conditions outlined before do not imply that Einstein's tensor ($G_{\mu\nu}$) has to vanish too. Table 7 is providing us an overview of these relationships.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu} = 0$	$b_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$	$\frac{8 \times \pi \times \gamma}{c^4 \times D} \times g_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$d_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu} - (\frac{R}{D} - R) \times g_{\mu\nu}$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} - R \times g_{\mu\nu} + R \times g_{\mu\nu}$

Table 7. Objective reality without ordinary matter.

It is important to emphasise here that objective relativity in which no ordinary matter is given ($a_{\mu\nu}$ = 0) is at the same time also a world in which momentum excludes curvature and vice versa. Curvature excludes momentum. But at the same time it is also a world which is not dead and not without any changes but a world full of life. We have to be theoretically prepared for the possibility that such a world might be the one of pure non-locality. Logically it does not seem very convincing that ordinary matter as something already concrete have been given at the beginning of this world. So the issue arises whether before the state of locality (ordinary matter is given) of objective reality, the state of non locality (no ordinary matter) of objective reality has been given. In other words, has the locality of this world developed out of the state of non-locality and is this still the case today?

3.9.2. Objective reality under conditions of D=1 dimension

The world under the condition of D=1 space-time dimension may be a very strange world, but the same exists nevertheless.

Theorem 31 (Objective reality under conditions of D=1 dimension)**.** *A special property of objective reality under conditions of D=1 space-time dimension is described by the relationship*

$$R_{\mu\nu} = R \times g_{\mu\nu} \tag{299}$$

Proof by direct proof. The Einstein (Barukčić, 2016b,c, 2020b,c,d,d, 2021b, Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) field equations (see equation 497) are defined as

$$\underbrace{\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right)}_{The\ left-hand\ side} \equiv \underbrace{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}}_{The\ right-hand\ side} \tag{300}$$

Under conditions of D = 1 space-time dimension, the Einstein field equations becomes

$$\left(\frac{R}{1} \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times 1}\right) \times g_{\mu\nu} \tag{301}$$

The Einstein field equations simplifies under conditions D = 1 space-time dimension as

$$+\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times 1}\right) \times g_{\mu\nu} \tag{302}$$

Under these conditions (D = 1 space-time dimension), the Ricci tensor $R_{\mu\nu}$ becomes

$$R_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) = R \times g_{\mu\nu} \tag{303}$$

but not $R_{\mu\nu} = 0$. Furthermore, under these conditions (D=1 space-time dimension), Einstein's tensor $G_{\mu\nu}$ becomes

$$G_{\mu\nu} = \left(\frac{R}{1} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) = \left(R \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) = \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{304}$$

Under conditions of D=1 space-time dimension and in contrast to a **vacuum solution of general relativity** neither the Einstein tensor $G_{\mu\nu}$ vanishes nor the stress–energy tensor $E_{\mu\nu}$ vanishes nor does the Ricci tensor $R_{\mu\nu}$ vanishes. It is of extraordinary importance that under conditions of D=1 space-time dimension the tensor $d_{\mu\nu}$ becomes

$$d_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu} - (\frac{R}{D} - R) \times g_{\mu\nu}) \equiv -\Lambda \times g_{\mu\nu} - (\frac{R}{1} - R) \times g_{\mu\nu}) \equiv -\Lambda \times g_{\mu\nu} - (0) \equiv -\Lambda \times g_{\mu\nu} \tag{305}$$

Under conditions of D=1 space-time dimension the tensor $c_{\mu\nu}$ is determined as

$$c_{\mu\nu} \equiv \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - d_{\mu\nu} \equiv \frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu} - (-\Lambda \times g_{\mu\nu}) \equiv \frac{R}{2} \times g_{\mu\nu} \tag{306}$$

Objective reality under the condition of D=1 space-time dimension is described by the following picture (see table 8) in greater detail.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu} = 0$	$b_{\mu\nu} \equiv \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$	$\left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu}$	$d_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu}$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv R \times g_{\mu\nu}$

Table 8. Objective reality under conditions of D=1 space-time dimension.

□

In general relativity, a vacuum region of objective reality is understood as a region whose Einstein tensor $G_{\mu\nu}$ vanishes. The Einstein tensor vanishes if

$$G_{\mu\nu} = R_{\mu\nu} - \left(\frac{R}{2}\right) \times g_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) = 0 \tag{307}$$

which is especially the case under conditions of D = 2 space-time dimension. In general, vacuum solutions of the Einstein fields equations are distinct from the electrovacuum solutions (electromagnetic field, gravitational field) and are also distinct from the lambdavacuum solutions. In lambdavacuum solutions of the Einstein fields equations the only term in the stress–energy tensor is the cosmological constant term.

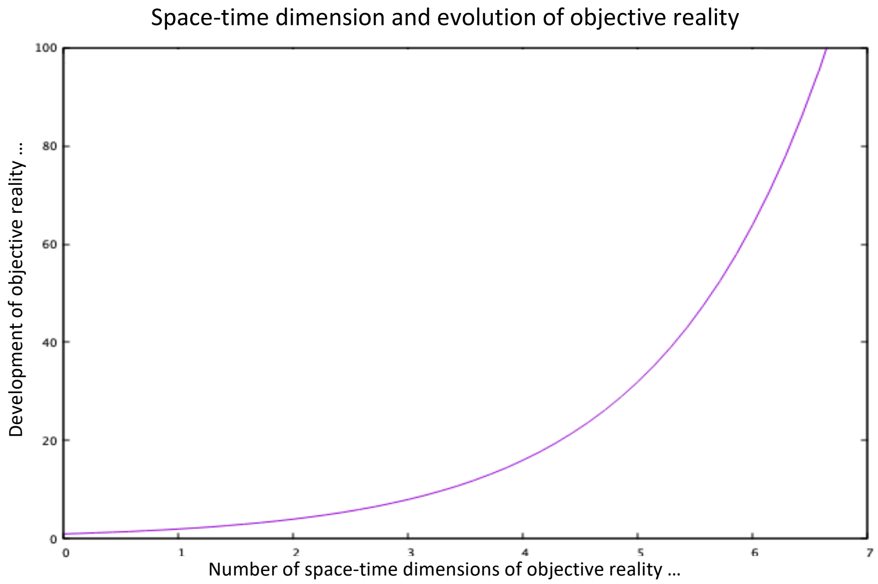

Figure 4. The evolution of objective reality

Under the previous and other conditions, one more point should be noted. The constancy of the speed of the light c in vacuum is something relative but not something absolute. Einstein is writing:

> "Dagegen bin ich der Ansicht, daß das Prinzip der Konstanz der Lichtgeschwindigkeit sich nur insoweit aufrecht erhalten läßt, als man sich auf raum - zeitliche Gebiete von **konstantem Gravitationspotential** beschränkt. Hier liegt nach meiner Meinung die Grenze der Gültigkeit ... des Prinzips der **Konstanz der Lichtgeschwindigkeit** und damit unserer heutigen Relativitätstheorie. "
>
> (see also Einstein, 1912, p. 1062)

Translated into English. 'On the other hand I am of the opinion that the principle of the constancy of the speed of light can be maintained only in so far as one restricts oneself to spatio-temporal areas of constant gravitational potential. Here lies in my opinion the limit of the validity... of the principle of the constancy of the speed of light and with it of our today's theory of relativity.'

3.9.3. Objective reality under conditions of D=2 dimension

Theorem 32 (Objective reality under conditions of D=2 dimension)**.** *An objective reality can be determined by two space-time dimension. Under conditions of D=2 space-time dimension is determined by the relationship*

$$G_{\mu\nu} = 0 \tag{308}$$

Proof by direct proof. The Einstein (Barukčić, 2016b,c, 2020b,c,d,d, 2021b, Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) field equations (see equation 497) are defined as

$$\underbrace{\left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right)}_{The\ left-hand\ side} \equiv \underbrace{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}}_{The\ right-hand\ side} \tag{309}$$

Under conditions of D = 2 space-time dimension, the Einstein field equations becomes

$$\left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times 2}\right) \times g_{\mu\nu} \tag{310}$$

or

$$+\left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{311}$$

Under these conditions, the Ricci tensor becomes

$$R_{\mu\nu} = \left(\frac{R}{2} \times g_{\mu\nu}\right) \tag{312}$$

but not $R_{\mu\nu} = 0$. Under conditions of D=2 space-time dimension, Einstein's tensor becomes

$$G_{\mu\nu} = \left(\frac{R}{D} \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) = \left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) = 0 \tag{313}$$

□

Conditions of D=2 space-time dimension are an exact solution of the Einstein field equation in which the only term in the stress–energy tensor is a cosmological constant term, neither the Ricci tensor vanishes nor the stress–energy tensor vanishes. Such an objective reality is related to the lambdavacuum solution of general relativity but not identical with the same. In general relativity, a lambdavacuum solution as an exact solution to the Einstein field equation is given as

$$G_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{314}$$

The general stance is that the Einstein tensor $G_{\mu\nu}$ vanishes if and only if the Ricci tensor $R_{\mu\nu}$ vanishes. This is not completely correct. Clearly, if the Ricci scalar R = 0, then the Einstein tensor $G_{\mu\nu}$ vanishes and the Ricci tensor $R_{\mu\nu}$ vanishes too. However, there are circumstances where the Einstein tensor

$G_{\mu\nu}$ vanishes while the Ricci tensor $R_{\mu\nu}$ does not vanishes, i. e. objective realty in 2 space-time dimensions.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu} \equiv -\left(\frac{R}{2}\right) \times g_{\mu\nu}$	$b_{\mu\nu} \equiv \left(\frac{R}{2} + \Lambda\right) \times g_{\mu\nu}$	$(\Lambda) \times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv +\left(\frac{R}{2}\right) \times g_{\mu\nu}$	$d_{\mu\nu} \equiv -\Lambda \times g_{\mu\nu}$	$\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv 0$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu}$

Table 9. Objective reality under conditions of D=2 space-time dimension.

Even under the conditions of D=2 space-time dimensions objective reality seems to be a very special world.

3.10. The generally covariant Planck-Einstein relation

Historically, previously separated have been unified by time. Thus far, a reader might often get the impression that we are on the verge of a theory of everything, a unified field theory. The hope is that combining gravity with the other fundamental forces of nature is just a matter of technical details. Yet we still do not have a unified field theory. The main reason is that we cannot yet construct a theory which is able to treat gravity quantum-mechanically. In the following theorem, the quantization of gravity is discussed by analogy with the quantization of the electromagnetic field. The following, purely speculative lines of thought are only meant to point out one theoretical possibility of unifying general relativity and quantum theory over the Planck–Einstein (Einstein, 1905a, Planck, 1901) relation $\hbar \times \omega = h \times f$.

3.10.1. Theorem. Frequency and Einstein field equations

Theorem 33 (Frequency and Einstein field equations)**.** *The frequency according to Einstein field equations is given by the relationship*

$$f = \left(\frac{4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \tag{315}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{316}$$

is true. Therefore, it is equally true that

$$E = h \times f \tag{317}$$

and at the end

$$\hbar \times \omega = h \times f \tag{318}$$

Multiplying equation 318, we obtain the generally covarinat form as

$$\hbar \times \omega \times g_{\mu\nu} = h \times f \times g_{\mu\nu} \tag{319}$$

We assume at this step that there is no contradiction between quantum mechanics and relativity theory. There are circumstances under which equation 319 may be equated with the stress-energy tensor of Einstein's general relativity (see equation 497). Therefore it follows that

$$h \times f \times g_{\mu\nu} = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{320}$$

Equation 320 is equivalent with the relationship

$$h \times f \times g_{\mu\nu} = h \times \left(\frac{4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \times g_{\mu\nu} \tag{321}$$

Simplifying equation 321, it is

$$f = \frac{c}{\lambda} = \left(\frac{4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \tag{322}$$

□

3.10.2. Theorem. The generally covariant form of the Planck-Einstein relation

Theorem 34 (The generally covariant form of the Planck-Einstein relation)**.** *The generally covariant form of the Planck-Einstein relation is given as*

$$\hbar \times \omega_{\mu\nu} = h \times f_{\mu\nu} \tag{323}$$

Proof by direct proof. Axiom 1 or +1 = +1 is valid. Based on this axiom, equation 159

$$\left(\frac{R}{D} \times g_{\mu\nu}\right) - (R \times g_{\mu\nu}) + \left(\frac{R}{2} \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{324}$$

changes to

$$\hbar \times \frac{\left(\frac{R}{D}\right) - (R) + \left(\frac{R}{2}\right) + (\Lambda)}{\hbar} \times g_{\mu\nu} = \left(\frac{4 \times h \times \gamma \times T}{c^4 \times \hbar \times D}\right) \times g_{\mu\nu} \tag{325}$$

and to

$$\hbar \times \frac{\left(\frac{R}{D}\right) - (R) + \left(\frac{R}{2}\right) + (\Lambda)}{\hbar} \times g_{\mu\nu} = h \times \left(\left(\frac{4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \times g_{\mu\nu}\right) \tag{326}$$

The stress energy tensor (see equation 322) determined by frequency would be given as

$$f_{\mu\nu} \equiv \left(\left(\frac{4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \times g_{\mu\nu}\right) \tag{327}$$

and

$$\omega_{\mu\nu} \equiv \frac{\left(\frac{R}{D}\right) - (R) + \left(\frac{R}{2}\right) + (\Lambda)}{\hbar} \times g_{\mu\nu} \tag{328}$$

The generally covariant Planck-Einstein relation would be given as

$$\hbar \times \omega_{\mu\nu} = h \times f_{\mu\nu} \tag{329}$$

□

An obvious and undeniable consequence of the foregoing is that we would have to accept that

$$\omega = \frac{\left(\frac{R}{D}\right) - (R) + \left(\frac{R}{2}\right) + (\Lambda)}{\hbar} \tag{330}$$

and that

$$f = \left(\frac{4 \times \gamma \times T}{\hbar \times c^4 \times D}\right) \tag{331}$$

Of course, it goes without any kind of saying that it is by far not enough to **define how objective reality has to be**. Pleasantly enough, it is much more satisfying to **discover how objective reality really is**. Whatever attitude each of us may call his own, the forgoing, including Λ, should be experimentally testable too. If an experimental proof should succeed, that equation 331 is correct, then very precise measurements of Newton's constant γ (under condition: D=4) as

$$\gamma = \left(\frac{f \times \hbar \times c^4}{T}\right) = \left(\frac{\hbar \times c^4}{\lambda \times T}\right) \tag{332}$$

where λ is the wave-length. Laue's scalar T (under condition: D=4) would be given as

$$T = \left(\frac{\hbar \times c^4}{\gamma}\right) \times f = constant \times f \tag{333}$$

Equation 333 demand us to accept that Laue's scalar and the frequency are more or less identical. This is very hard to believe and more than disturbing. No wonder, a speculation remains a speculation. Nonetheless, if something like equation 333 can be confirmed experimentally, new frontiers on our planet as well as on far away worlds are within reach. In the final consequence, it ought to be possible to prove definitely, whether Newton's gravitational constant γ is a constant (Newton, 1669, 1687, 1711, 1732, 1744) or is not (Barukčić, 2006, Barukčić, 2015b, 2016a, 2021a) a constant. Furthermore, under conditions where

$$\frac{\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right)}{\left(\frac{4 \times 2 \times \pi \times T}{c^4 \times D}\right)} \equiv \gamma \equiv \frac{F \times d^2}{m_1 \times m_1} \tag{334}$$

we would have to accept Newton's law of gravitation given as

$$F = \frac{\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right)}{\left(\frac{4 \times 2 \times \pi \times T}{c^4 \times D}\right)} \times \frac{m_1 \times m_1}{d^2} = \frac{D \times \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right)}{(4 \times 2 \times \pi \times T)} \times \frac{E_1 \times E_2}{d^2} \tag{335}$$

which is not without difficulties as such. Newton's gravitational constant γ appears to us to be dependent at least on the number of space-time dimensions D as proposed by the following equation

$$\gamma = \frac{D \times \left(\frac{R}{D} - \frac{R}{2} + \Lambda\right)}{(4 \times 2 \times \pi \times T)} \times c^4 \tag{336}$$

and may not be a constant. Equation 336 suggests an experimental possibility to determine the number of space-time dimensions D or of Λ et cetera experimentally.

3.11. Quantum gravity and Schrödinger's wave equation

There are numerous wave equations, including relativistic (Barukčić, 2013, Dirac, 1928, Gordon, 1926, Klein, 1926) ones. The Schrödinger equation, named after Erwin Schrödinger, is a non relativistic wave equation (Schrödinger, Erwin Rudolf Josef Alexander, 1926) and more or less a quantum mechanical counterpart of Newton's second law in classical mechanics. In the following, we want to establish a link between gravitation and Schrödinger's wave equation.

3.11.1. Theorem. Quantum gravity and Schrödinger's wave equation

Theorem 35 (Quantum gravity and Schrödinger's wave equation)**.** *Incorporating both the principles of general relativity and quantum theory leads to the wave equation of the gravitational field as*

$$\left(\left(\left(\frac{\pi \times \hbar}{h}\right) \times \left(\frac{(2 \times R) - (R \times D)}{D}\right)\right) \times \Psi\right) + (\Lambda \times \Psi) = H \times \Psi \tag{337}$$

Proof by direct proof. Axiom 1 or

$$+1 = +1 \tag{338}$$

is true. Therefore, it is equally true that

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{339}$$

Based on the Einstein publications (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) we arrive at the following Einstein's field equations.

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{340}$$

Taking the trace of both sides of equation 340, it is

$$(R_{\mu\nu} \times g^{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + (\Lambda \times g_{\mu\nu} \times g^{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \tag{341}$$

or

$$(R) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + (\Lambda \times g_{\mu\nu} \times g^{\mu\nu}) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T \tag{342}$$

and equally (see equation 398, p. 95)

$$(R) - \left(\left(\frac{R}{2}\right) \times D\right) + (\Lambda \times D) = \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T \tag{343}$$

Changing equation 343, it is

$$\left(\frac{R}{D}\right) - \left(\frac{R}{2}\right) + (\Lambda) = \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{344}$$

Equation 344 can be rearranged as

$$\left(\frac{R}{D}\right)-\left(\frac{R}{2}\right)-\left(\frac{R}{2}\right)+\left(\frac{R}{2}\right)+(\Lambda)=\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right) \tag{345}$$

or as

$$\left(\frac{R}{D}\right)-(R)+\left(\frac{R}{2}\right)+(\Lambda)=\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right) \tag{346}$$

Normalising equation 346 it is

$$\frac{\left(\left(\frac{R}{D}\right)-(R)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}+\frac{\left(\left(\frac{R}{2}\right)+(\Lambda)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}=+1 \tag{347}$$

Multiplying equation 347 by Schrödinger's wave equation $H\times\Psi$, it is

$$\frac{\left(\left(\frac{R}{D}\right)-(R)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}\times H\times\Psi+\frac{\left(\left(\frac{R}{2}\right)+(\Lambda)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}\times H\times\Psi=H\times\Psi \tag{348}$$

where H is the Hamiltonian and Ψ is the (time dependent/independent) wave function. In general, the Hamiltonian H of a certain system is an operator corresponding to the total energy of that system. The total energy of the system of general relativity is more or less $\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}$. Under conditions where

$$H=\underline{ek}\times\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D} \tag{349}$$

equation 348 simplifies as

$$\left(\left(\frac{R}{D}\right)-(R)\right)\times\underline{ek}\times\Psi+\left(\left(\frac{R}{2}\right)+(\Lambda)\right)\times\underline{ek}\times\Psi=H\times\Psi \tag{350}$$

In general, Dirac's/Schrödinger's (see also Dirac, 1926, Dirac and Fowler, 1926, Schrödinger, Erwin Rudolf Josef Alexander, 1926) constant $\hbar$ is determined as

$$\hbar\equiv\frac{h}{2\times\pi} \tag{351}$$

In other words, it is

$$\frac{1}{2}=\left(\frac{\pi\times\hbar}{h}\right) \tag{352}$$

This relationship is substituted into equation 350. A relativistic wave equation (see Barukčić, 2013) is given as

$$\left(\left(\left(\frac{R}{D}\right)-(R)\right)\times\underline{ek}\times\Psi\right)+\left(\left(\left(\frac{\pi\times\hbar}{h}\right)\times(R+(2\times\Lambda))\right)\times\underline{ek}\times\Psi\right)=H\times\Psi \tag{353}$$

Under circumstances where $\underline{ek}$ = 1, equation 353 becomes

$$\left(\left(\left(\frac{R}{D}\right)-(R)\right)\times\Psi\right)+\left(\left(\left(\frac{\pi\times\hbar}{h}\right)\times(R+(2\times\Lambda))\right)\times\Psi\right)=H\times\Psi \tag{354}$$

Normalising 344, it is

$$\frac{\left(\left(\frac{R}{D}\right)-\left(\frac{R}{2}\right)\right)}{\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right)}+\frac{(\Lambda)}{\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right)}=+1 \tag{355}$$

Multiplying equation 355 by Schrödinger's wave equation $H\times\Psi$, it is

$$\frac{\left(\left(\frac{(2\times R)-(R\times D)}{2\times D}\right)\right)\times H\times\Psi}{\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right)}+\frac{(\Lambda)\times H\times\Psi}{\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right)}=H\times\Psi \tag{356}$$

The Hamiltonian operator H in quantum mechanics is the operator for the total energy of a certain quantum mechanical system and acts on the wave function (capital Ψ) to produce a range of possible eigenvalues for the eigenfunctions (lowercase ψ). So the unit of the Hamiltonian operator H is more or less energy. In general relativity, the total energy is described by the stress-energy tenors of matter. We have reason to believe there are circumstances where both describe more or less the same entity. As already explained elsewhere, the metric tensor is unitless while the stress-energy tensor, has the unit energy/volume = pressure = force/area et cetera (see Porta Mana, 2021). Einstein's constant, denoted as ek and defined as

$$ek=\frac{4\times2\times\pi\times\gamma}{c^4} \tag{357}$$

converts the stress-energy to the units of the left side of the field equation, each term of which is of unit $1/L^2$. In the following, we define the Anti Einstein constant, denoted as $\underline{ek}$,such that

$$H=\underline{ek}\times\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D} \tag{358}$$

In the following, equation 356 simplifies as

$$\left(\left(\left(\frac{\pi\times\hbar}{h}\right)\times\left(\frac{(2\times R)-(R\times D)}{D}\right)\right)\times\underline{ek}\times\Psi\right)+(\Lambda\times\underline{ek}\times\Psi)=H\times\Psi \tag{359}$$

Under circumstances where $\underline{ek}$ = 1, equation 359 becomes

$$\left(\left(\left(\frac{\pi\times\hbar}{h}\right)\times\left(\frac{(2\times R)-(R\times D)}{D}\right)\right)\times\Psi\right)+(\Lambda\times\Psi)=H\times\Psi \tag{360}$$

□

Given the canonical commutation relation (see Born and Jordan, 1925), it is $\hbar = \frac{[x,p]}{i \times \mathbb{I}}$. Thus far, it makes good sense to substitute this relationship into equation 360. We obtain

$$\left(\left(\left(\frac{\pi \times [x,p]}{i \times \mathbb{I} \times h}\right) \times \left(\frac{(2 \times R) - (R \times D)}{D}\right)\right) \times \Psi\right) + (\Lambda \times \Psi) = H \times \Psi \tag{361}$$

Though quantum gravity has been the subject of investigation for almost a century, this topic presents not only extreme technical difficulties, but profound ontological and methodological challenges for various scientist. General relativity describes gravitation more or less as the curvature of space-time by matter or energy. Therefore, a quantisation of gravity theoretically implies some sort of quantisation of space-time geometry too. Whatever the final outcome, the above approach to the wave equation of the gravitational field does not only bring general relativity in line with quantum theory but is expected to be able to provide a satisfactory description of the micro-structure of space-time, especially at the so-called Planck scale. As can be seen, the gravitational field itself is also quantised. As always, one might legitimately remark that what constitutes a possible way out to one author might not qualify as such to another especially if theories break down at certain circumstances.

3.12. Measurement of space-time dimensions

One aspect of the self-organization of objective reality seems to be also the transition into an objective reality of higher dimension. Whether this process is irreversible may be an open question for the present. However, the question may very well be asked whether our objective reality is already part of another, higher dimensional objective reality? Moreover, it would be desirable if this can be measured or verified experimentally somehow. Measurements of Laue's scalar (i. e. by wave-length et cetera (see equation 331)) can be of help. In general, under conditions of 4 space-time dimensions, it is

$$\left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times 4}\right) \times g_{\mu\nu}\right) = \left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) \tag{362}$$

or

$$D \times \left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times 4}\right) \times g_{\mu\nu}\right) = \left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{363}$$

Simplifying, it is

$$D = \frac{\text{D Space time dimension}}{\text{4 Space time dimension}} = \frac{\left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right)}{\left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right)} = \frac{\left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right)\right)}{\left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right)} \tag{364}$$

The determination of the dimension space-time by an measurement is given as

$$D = \frac{\text{D Space time dimension}}{\text{4 Space time dimension}} = \frac{\left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right)\right)}{\left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right)} = \frac{4 \times \pi \times \gamma \times T_{\text{D Dimension}}}{\pi \times \gamma \times T_{\text{4 dimension}}} \tag{365}$$

Under certain conditions which we do not wish to go into in detail at this point, equation 365 simplifies to

$$D = \frac{4 \times T_{\text{D Dimension}}}{T_{\text{4 dimension}}} \tag{366}$$

4. Discussion

Today, a lot of cosmologists and theoretical physicists endorse the view that our universe was born about 13.7 billion years ago in a massive expansion, the so called big bang. The notion 'big bang '(see also Lemaître, 1931a,b) itself has been coined on 28 March 1949 by Fred Hoyle(see also Kragh, 2013) during his talk on the British Broadcasting Corporation (BBC). However, objective reality itself is our 'ultimate'truthmaker and can teach us very much about the beginning of our world. The beginning of our world, as the foundation on which everything other is built, appears to be determined by laws of nature which are worth to be examined from a higher point of view before anything else. At the beginning of our world, another end is probably running on empty and we have nothing else but the beginning itself. However, it remains to be seen what and how such a beginning could be. With what should the beginning of our world be made, what is there before us? Is it possible at all for our world to begin, it doesn't matter either our world is or it is not. In so far as our world is, our world is not just beginning, the world is already. In so far as our world is not, why should this world begin, how could this world begin? Thus, if no presupposition is to be made then the only determination of the beginning of our world as such is at the end to be the beginning of our world. In the same line, a beginning of our world may not presuppose anything. In point of fact, is there something like an absolute beginning at all, is there something which has been existed prior to the beginning of our world? Do we have to consider whether a preliminary labour need to be carried out before the beginning of our world? We should not let up at this point until the beginning of our world has been firmly established. A reader who is concerned with the origin or the beginning of our world will have to consider at least the possibility of a creation or of a beginning of our world out of nothing, a **creatio ex nihilio** (see also Aquinas, 1964), however nothing itself might be determined or scientifically defined.

Table 10. Without nothingness, no beginning of world?

		Beginning Of World		
		YES	NO	
Nothingness	YES	1	1	1
	NO	0	1	1
		1	1	1

In this context, it is necessary to point out, that **nothing**, even understood as an absence of something, **exists**. However, such an attitude is of course not unconditionally accepted by all. As an example, Guthrie himself is firmly convinced that nothing cannot be (see Guthrie, 1965, p. 104). The question whether this world had a beginning (in nothing, in time, in ...) still remains unanswered or was it more the either way? Has this world 'produced'and is this world still 'producing'nothingness? In particular, has this world had no beginning in time, in nothing et cetera with the consequence that this world always has existed and this world always will exist? Are nothingness and the beginning of our world independent of each other (see table 10)? In point of fact, are questions like these beyond any human experience? In particular, is the question of our world's beginning more a matter of faith than of demonstration or science?

The relationship without nothingness no beginning of our world is logically equivalent with the relationship if no absence of nothingness then no absence of the beginning of our world. However, this logical necessity need not imply that a beginning of our world is successful too. There may have existed a lot of 'trials'until the beginning of our world was successful. However, what is nothingness, what is the structure of nothingness, where does it itself comes from? May all this not also be a little different and more likely the following way: if nothingness then beginning of our world?

Table 11. If nothingness, then beginning of world?

		Beginning Of World		
		YES	NO	
Nothingness	YES	1	0	1
	NO	1	1	1
		1	1	1

Again, does nothingness exist and what are the properties of nothingness? But by the same reasoning, is there only total emptiness or total nothingness or are there small pockets of emptiness or small pockets of nothingness or both or none? In order not to expose ourselves to the danger to favour a one-sided point of view (**creatio ex nihilio**), it is appropriate to consider whether the determination of the beginning of our world is comparable to a coin with two sides - **a beginning of our world out of itself which is necessary for itself** and equally **a beginning of our world which is a condition for its own further self-organised and self-determined development**. (see also Barukčić, 2007) It is hardly surprising, therefore, that in the first view of the nature of the beginning of our world, the beginning of our world out of itself which is necessary for itself appears to be something what is absolutely simple, that is, something what is the most general. In other words, it is very likely that the beginning of our world cannot be made with anything containing a concrete relation within itself or anything concrete because such a concrete something need not to begin, such a concrete something is already existing. As a logical consequence, it is difficult to consider that a concrete something itself has been that from which the movement of our world started because the determinations contained in something concrete have already developed somehow. Thus, the developed and concrete something would exist before it started to exist. Consequently, anything which is in its own self a first and an other too implies equally that it has developed somehow, an advance from one to another has already been made. A concrete one has become somehow the concrete one that it is, some progress has already been made. In so far, that which constitutes the beginning of our world, **the beginning itself**, is to be taken as something simple and unfilled. If that which forms the beginning of our world would be something determined within itself, then this something that is determined within itself need likewise to be something otherwise concrete which the beginning of our world cannot be.

To address the question of emptiness, nothingness et cetera again and from a non-mathematical point of view, even after a removal of everything still remains something which is not constituted or determined by anything concrete, an objective reality determined by neither curvature nor momentum, the emptiness as such, **the empty negative**, the infinitely flat, whatever the structure of the same may be, what ever its properties. Are we able to identify those outstanding properties of an objective reality which is determined by neither curvature nor momentum? We find at this point no persuasive logical reason which would counter the assumption that in emptiness simply as such, in the empty negative which is necessary for itself, the beginning of our world can be found. The insight, that in the empty negative the beginning of our world can be found, is itself so simple that a beginning of our world as such out of the empty negative requires further introduction. In order not to be exposed to the danger of pure speculation, can there be any beginning of our world from the point of view of the general theory of relativity in nothingness, in the emptiness, in an empty negative? As found before, it is

$$d_{\mu\nu} = \left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) \quad (367)$$

Especially under conditions of D=2 space-time dimensions we should also consider the possibility that

$$d_{\mu\nu} = \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times 2}\right) \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) \quad (368)$$

In the final result, it is

$$d_{\mu\nu} = 0 - \left(\left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) = -\left(\Lambda \times g_{\mu\nu}\right) \quad (369)$$

In other words, zero is determined as the unity and the struggle between a positive and a negative as

$$0 = +\left(\left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \quad (370)$$

Historically (see Kaplan, 1999), many times zero and nothing have been treated as being identical. Nichomachus of Gerasa is writing: "... the sum of nothing added to nothing ... makes nothing." (see Nicomachus, 1926, pp. 237/238). However, differences (see Contini-Morava, 2006) between zero and nothing (latin nihilio) have also been discussed too. We should not forget at this passage that zero itself is full of life. Thus far, let us not beat around the bush unnecessarily and put it in a nutshell. After all these arguments presented, an infinitely small and dense point as the beginning of our world seems to be highly improbable. In order to ensure the beginning of the world it is necessary for the beginning itself to escape from the state of symmetry (see Anderson, 1972), to escape from **zero, the black hole of mathematics**. The question that might rack our brains is, can something and how can something escape from zero (Barukčić, 2015a) ? However, with all back and forth we have to demonstrate a certain amount of courage and to venture out from safe theoretical cover in order to ask ourselves at this point whether we are forbidden to ask the question about the theoretical possibility of the existence of an objective reality below D=2 space-time dimensions? What could such a bizarre objective reality look like in detail? Maybe Einstein's field equations can bring some light into this world of espistemiological darkness. Theoretically, it is possible that there are conditions where even the Ricci scalar R is equal to R = 0. Under these circumstances, the Einstein field equations becomes

$$\left(\left(\frac{0}{D}\right)\times g_{\mu\nu}\right)-\left(\left(\frac{0}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)=+\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}\right) \tag{371}$$

Conditions of objective reality where R, the Ricci scalar, is equal to R = 0 are describing equally a region of objective reality in which the Einstein tensor $G_{\mu\nu}$ vanishes. In general relativity, such conditions are describing a vacuum solution of the Einstein field equations too. However, vacuum solutions are distinct from the lambdavacuum solutions. In lambdavacuum solutions the only term in the stress–energy tensor is the cosmological constant term. An equivalent formulation of lambdavacuum solutions in terms of the Einstein tensor is $G_{\mu\nu}=-\left(\Lambda\times g_{\mu\nu}\right)$. As next, equation 371 simplifies. It is

$$+\left(\Lambda\times g_{\mu\nu}\right)=+\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}\right) \tag{372}$$

Nonetheless, in zero as the unity and the non-ending struggle between a positive and a negative, the positive and the negative are united too. It is an objective reality where the one is equal to its own other.

$$0=+\left(\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)\times g_{\mu\nu}\right)-\left(\Lambda\times g_{\mu\nu}\right) \tag{373}$$

In the end, even if we have reason to acknowledge with regard to this matter for the present to each observant reader his own point of view, the emptiness in which the beginning of our world can be found is itself full of life. However such an emptiness itself need to be an emptiness in which an advance from one to another has yet not been made, it is an abstract and not determinate emptiness. In point of fact, such an empty negative, the emptiness as such, is equally a self-related negativity, it is the negative of itself in its own self, it has a relation to the other of itself and is suffering and thirsty for the other of itself. From another point of view, only in what is simple there is nothing more than the pure beginning, only in such a simple, in emptiness as such, no advance yet has been made from one to an other. It might be reasonably assumed that **the beginning of our world began with the beginning itself**. Unfortunately, it appears to be that there is little to say in this respect since there were no eye-witnesses and there is no direct evidence in this regard. But even if epistemic self-doubt is not all the time so evidently justified, an important alternative which remains is the task of fact-finding as we **descend from the known to the unknown**. Yet it remains central and helpful to consider that it is very difficult to extract any further determination of any beginning of our world from the fact that it is the beginning of our world as such. At first sight, there isn't anything else present, any content which could be used to make the beginning of our world more determinate. It can be noted, however, that the beginning of our world in emptiness, in nothing else but the empty negative, is equally in its first manifestation in fanatical hostility towards an end, it is fearful of being lost in an end, it is fearful of being captured for ever by an end. The beginning of our world is equally within itself the end of an end, the end of an end in which the end is also the begin and the begin is also the end, the beginning of our world is thus the beginning of everything. In so far, that from which a movement began has united with itself, in the beginning an end ends and equally in such an end the beginning begins. The beginning of our world on its own accord determines thus itself as the other of itself, the beginning is thus the local hidden variable of an end, it is a simplicity into which an end has withdrawn. The beginning of our world contains as such within itself thus the beginning of any further self-governed advance and development. In its last manifestation, the beginning of our world seems to be equally the foundation on which everything other is built, it is the simplest, the simple itself, quite general, without any content and still undeveloped. The beginning of our world is the foundation which is present and preserved throughout the entire subsequent development, remaining completely immanent in its further determinations. That which forms the starting point of the development of our world remains at the base of all that follows and does not vanish from it. Enclosed in the beginning of our world is thus the entire development that follows. The further necessary development of our world started right from the beginning itself. The beginning of our world in its own necessary development brings with its own self the demand of further development. The beginning of our world starts from itself and advances to the other of itself, it is a movement through which the latter at the end returns to the first. The progress that follows is more or less only a further determination of the beginning of our world, every further progress is equally a fresh beginning too, it is the sublation of the very first beginning of our world. In so far, while getting further away from the beginning of our world, the development of our world is equally getting back nearer to it. Consequently, after the contradictions contained in the beginning of our world have been developed, the final result is the relationship which formed the beginning as such, is the infinite progress, the same contradiction with which our world began. However it may be, once the beginning of our world has inwardly reconstituted itself, all attempts to preserve the end are utterly in vain. In so far, the beginning as such remains to some extent a matter of indifference. Contrary to all, both sides of the beginning of our world constitutes the beginning of our world. The beginning of our world has thus its own result, its own negation in itself and passes thus into a higher space-time dimension, into a new unity and struggle between energy and time.

5. Conclusion

The big bang as an explanation of the beginning of our world is not the only conceivable logical possibility to explain the beginning of our world. The beginning of our world out of the empty negative, out of

$$d_{\mu\nu} = -\left(\Lambda \times g_{\mu\nu}\right) \tag{374}$$

is theoretically possible too.

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Erratum

Unfortunately, some misprints appeared in the previous publications, especially in the section of definitions. Some of the misprints have been brought up to date in this publication as far as possible.

Erratum :

Misprints (version 7), Theorem 9,

error (version 7): The table 5 (see equation 5, p. 25)

With all intent, recognised misprints had to be reduced and additional and new theorems had to be provided.

Today (October 16, 2022), it seems hopeless. The more often this article is read, the more misprints one notices. Various misprints where identified and had to be reduced.

Today (October 18, 2022), with each day the hope fades more and more away that an end of the theorems is in sight.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

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Supplementary Material

The supplementary material typically includes material that does not really form part of the main article. However, the supplementary material is of use in order to better understand the main article and includes sometimes additional data such as computer code, large tables, additional figures, appendices et cetera as necessary.

Geometry can be traced back to the first trials of systematic logical thinking of humans. Still, the nature of the relation between the definitions, axioms, theorems, and proofs in a system of geometry and objective reality has to be considered in detail. Tensors are one mathematical approach to geometry. The tensor (see also Voigt, 1898, p. 20) calculus has been developed in some greater detail by Ricci-Curbastro (see Ricci-Curbastro and Levi-Civita, 1900) and his student Levi-Civita on the basis of earlier work of authors like Riemann, Christoffel, Bianchi and others. Especially, Einstein's general theory of relativity is expressed by the mathematical technology of tensors.

5.1.1. Tensor addition

Definition 5.1 (Tensor addition).

The sum of two second rank co-variant tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C_{\mu\nu} &\equiv A_{\mu\nu} + B_{\mu\nu} \\ &\equiv B_{\mu\nu} + A_{\mu\nu} \end{aligned} \tag{375}$$

The sum of two second rank contra-variant tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C^{\mu\nu} &\equiv A^{\mu\nu} + B^{\mu\nu} \\ &\equiv B^{\mu\nu} + A^{\mu\nu} \end{aligned} \tag{376}$$

The sum of two second rank mixed tensors has the properties of associativity and commutativity and is defined as

$$\begin{aligned} C_{\mu}{}^{\nu} &\equiv A_{\mu}{}^{\nu} + B_{\mu}{}^{\nu} \\ &\equiv B_{\mu}{}^{\nu} + A_{\mu}{}^{\nu} \end{aligned} \tag{377}$$

5.1.2. Anti tensor I

Definition 5.2 (Anti tensor I).

Let $a_{\mu\nu}$ denote a co-variant (lower index) second-rank tensor. Let $b_{\mu\nu}$, $c_{\mu\nu}$ et cetera denote other co-variant second-rank tensors. Let $E_{\mu\nu}$ denote the sum of these co-variant second-rank tensors. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + ... \equiv E_{\mu\nu}$ be given. A co-variant second-rank anti tensor (see also Barukčić, 2020d) of a tensor $a_{\mu\nu}$ denoted in general as $\underline{a}_{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} \\ &\equiv b_{\mu\nu} + c_{\mu\nu} + ... \end{aligned} \tag{378}$$

5.1.3. Anti tensor II

Definition 5.3 (**Anti tensor II**).

Let $a^{\mu\nu}$ denote a contra-variant (upper index) second-rank tensor. Let $b^{\mu\nu}$, $c^{\mu\nu}$ et cetera denote other contra-variant (upper index) second-rank tensors. Let $E^{\mu\nu}$ denote the sum of these contra-variant (upper index) second-rank tensors. Let the relationship $a^{\mu\nu} + b^{\mu\nu} + c^{\mu\nu} + ... \equiv E^{\mu\nu}$ be given. A co-variant second-rank anti tensor of a tensor $a^{\mu\nu}$ denoted in general as $\underline{a}^{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}^{\mu\nu} &\equiv E^{\mu\nu} - a^{\mu\nu} \\ &\equiv b^{\mu\nu} + c^{\mu\nu} + \ldots \end{aligned} \tag{379}$$

5.1.4. Anti tensor III

Definition 5.4 (**Anti tensor III**).

Let ${a_\mu}^\nu$ denote a mixed second-rank tensor. Let ${b_\mu}^\nu$, ${c_\mu}^\nu$ et cetera denote other mixed second-rank tensors. Let ${E_\mu}^\nu$ denote the sum of these mixed second-rank tensors. Let the relationship ${a_\mu}^\nu + {b_\mu}^\nu + {c_\mu}^\nu + ... \equiv {E_\mu}^\nu$ be given. A mixed second-rank anti tensor of a tensor ${a_\mu}^\nu$ denoted in general as ${\underline{a}_\mu}^\nu$ is defined

$$\begin{aligned} {\underline{a}_\mu}^\nu &\equiv {E_\mu}^\nu - {a_\mu}^\nu \\ &\equiv {b_\mu}^\nu + {c_\mu}^\nu + \ldots \end{aligned} \tag{380}$$

5.1.5. Tensor subtraction

Definition 5.5 (**Tensor subtraction**).

The subtraction of two second rank co-variant tensors is defined as

$$C_{\mu\nu} \equiv A_{\mu\nu} - B_{\mu\nu} \tag{381}$$

The subtraction of two second rank contra-variant tensors is defined as

$$C^{\mu\nu} \equiv A^{\mu\nu} - B^{\mu\nu} \tag{382}$$

The subtraction of two second rank mixed tensors is defined as

$${C_\mu}^\nu \equiv {A_\mu}^\nu - {B_\mu}^\nu \tag{383}$$

5.1.6. Symmetric and anti symmetric tensors

Definition 5.6 (Symmetric and anti symmetric tensors).

Symmetric tensors of rank 2 may represent many physical properties objective reality. A co-variant second-rank tensor $a_{\mu\nu}$ is symmetric if

$$a_{\mu\nu} \equiv a_{\nu\mu} \tag{384}$$

However, there are circumstances, where a tensor is anti-symmetric. A co-variant second-rank tensor $a_{\mu\nu}$ is anti-symmetric if

$$a_{\mu\nu} \equiv -a_{\nu\mu} \tag{385}$$

Thus far, there are circumstances were an anti-tensor is identical with an anti-symmetrical tensor.

$$a_{\mu\nu} \equiv E_{\mu\nu} - b_{\mu\nu} + \ldots \equiv E_{\mu\nu} - \underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{386}$$

Under conditions where $E_{\mu\nu} = 0$, an anti-tensor is identical with an anti-symmetrical tensor or it is

$$-\underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{387}$$

However, an anti-tensor is not identical with an anti-symmetrical tensor as such.

Definition 5.7 (Multiplication of tensors). *Let g_{kl} or $g_{\mu\nu}$ denote a 2-index metric tensors. Let $g_{kl\mu\nu}$ denote a 4-index metric tensors. Let $g_{kl\mu\nu\ldots}$ denote a n-th index metric tensor. The n-index metric tensor $g_{kl\mu\nu\ldots}$ itself is a covariant symmetric tensor and equally an example of a tensor field. If we pause for a moment today and rely on Einstein's "Die Grundlage der allgemeinen Relativitätstheorie " (see Einstein, 1916, p. 784), it is*

$$g_{kl\mu\nu} \equiv g_{kl}g_{\mu\nu} \tag{388}$$

and in the case of n-th rank order

$$g_{kl\mu\nu\ldots} \equiv g_{kl}g_{\mu\nu\ldots} \tag{389}$$

The mixed and contra-variant cases are similar. Riemann defined the distance between two neighbouring points more or less by a quadratic differential form. The geometry based on the positive definite Riemannian metric tensor is called the Riemannian geometry. However, tensor calculus as a generalization of classical linear algebra should assure that formulae are invariant under coordinate transformations and that the same are independent of any kind of the rank order of the metric tensor chosen. Albert Einstein (see Einstein, 1916) presented some rules of tensor algebra in his important publication issued in the year 1916.

$$T_{\mathrm{a\,b\,c}} \equiv A_{\mathrm{a\,b}}B_{\mathrm{c}} \tag{390}$$

(see Einstein, 1916, p. 784)

Furthermore, it is

$$T^{a\,b\,c\,d} \equiv A^{a\,b} B^{c\,d} \tag{391}$$

(see Einstein, 1916, p. 784)

and equally

$$T^{\ a\ b}_{\ c\ d} \equiv A^{a\,b} B_{c\,d} \tag{392}$$

(see Einstein, 1916, p. 784)

A covariant tensor of the second rank type is defined as

$$T_{c\,d} \equiv A_c B_d \tag{393}$$

(see Einstein, 1916, p. 782)

The mathematics of tensors is particularly useful for Einstein's general theory of relativity. In a D-dimensional space a tensor of rank n has $D^{\,n}$ components. A contravariant tensor of the second rank type is defined as

$$T^{c\,d} \equiv A^c B^d \tag{394}$$

(see Einstein, 1916, p. 782)

A mixed tensor of the second rank type is defined by Einstein as follows.

$$T_{\ c}^{\ \ d} \equiv A_c B^d \tag{395}$$

(see Einstein, 1916, p. 783)

A scalar F, or a tensor of zero rank, is given by the relationship

$$F \equiv F_{\ b}^{\ b} \equiv F_{\ a\ b}^{\ a\ b} \equiv F_{a\,b} F^{a\,b} \tag{396}$$

(see Einstein, 1916, p. 785)

This relationship (see equation 396, p. 93) is of importance for the fundamental invariants of the electromagnetic field too. The covariant and contravariant products of two rank 2 tensors give the same value and result in a scalar. In general, scalar products are operations on two tensors of the same rank that yield a scalar.

5.1.7. The metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$

General relativity is a theory of the geometrical properties of space-time to, while the metric tensor $g_{\mu\nu}$ itself is of fundamental importance for general relativity. The metric tensor $g_{\mu\nu}$ is something like the generalization of the Pythagorean theorem. Thus far, it does not appear to be necessary to restrict the validity of the Pythagorean theorem only to certain situations. The question is justified why the Riemannian geometry should be oppressed by the quadratic restriction. In this context, **Finsler geometry**, named after Paul Finsler (1894 - 1970) who studied it in his doctoral thesis (see Finsler, 1918) in 1918, appears to be a kind of metric generalization of Riemannian geometry without the quadratic restriction and justifies the attempt to systematize and to extend the possibilities of general relativity.

Definition 5.8 (Kronecker delta).

The Kronecker delta (see Zehfuss, 1858) is a so called invariant tensor and has been invented by Leopold Kronecker (1823-1891) in 1868 (see Kronecker, 1868). Meanwhile, Kronecker delta appears in many areas of physics, mathematics, and engineering and is defined as

$$g_{\mu\rho} \times g^{\nu\rho} \equiv g_{\mu}{}^{\nu} \equiv \delta_{\mu}{}^{\nu} \tag{397}$$

Technically, the Kronecker delta itself is a mixed second-rank tensor.

Definition 5.9 (The metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$).

The distance between any two points in a given space can be described geometrically by a generalized Pythagorean theorem, the metric tensor $g_{\mu\nu}$. Sharing Einstein's point of view, it is in general

$$g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\nu}{}^{\nu} \equiv D \tag{398}$$

where D might denote the number of space-time dimensions. The quantity

$$\delta_{\mathrm{i}}{}^{\mathrm{i}} \equiv \delta_{1}{}^{1} + \delta_{2}{}^{2} + ... + \delta_{\mathrm{D}}{}^{\mathrm{D}} \equiv D \tag{399}$$

is an invariant. In other words, an index which is repeated inside an expression means summation over the repeated index (Einstein summation convention). Vectors and scalars are invariant under coordinate transformations. In point of fact, Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) were initially formulated by Einstein himself in the context of a four-dimensional theory even though Einstein field equations need not to break down under conditions of D space-time dimensions (see Stephani, 2003). Nonetheless, based on Einstein's statement (Einstein, 1916, p. 796), one gets (see also Einstein, 1923b, p. 91)

$$g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\nu}{}^{\nu} \equiv D \equiv +4 \tag{400}$$

or

$$\frac{1}{g_{\mu\nu} \times g^{\mu\nu}} \equiv \frac{1}{4} \tag{401}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other and are used to lower and raise indices. Einstein's pointed out that

> "... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential" (Einstein, 1923b, p. 88)

Definition 5.10 (The metric tensor $g_{\mu\nu}$ decomposed). *The fundamental difference between the metric tensors of the four basic fields of nature, denoted as $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$, finds its complete expression in equation 402 as*

$$ {}_{a}g_{\mu\nu} + {}_{b}g_{\mu\nu} + {}_{c}g_{\mu\nu} + {}_{d}g_{\mu\nu} \equiv g_{\mu\nu} \tag{402} $$

where ${}_{a}g_{\mu\nu}$ is the metric tensor of the ordinary matter, ${}_{b}g_{\mu\nu}$ is the metric tensor of electromagnetism, ${}_{c}g_{\mu\nu}$ is the metric tensor of the field $c_{\mu\nu}$, ${}_{d}g_{\mu\nu}$ is the metric tensor of the field $d_{\mu\nu}$ and $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity. We distinguish here between the four basic field of nature, as follows. Details are illustrated by table 12.

Table 12. The metric field decomposed

		Curvature		
		YES	NO	
Momentum	YES	$({}_{a}g_{\mu\nu})$	$({}_{b}g_{\mu\nu})$	$({}_{E}g_{\mu\nu})$
	NO	$({}_{d}g_{\mu\nu})$	$({}_{d}g_{\mu\nu})$	$({}_{\underline{E}}g_{\mu\nu})$
		$({}_{G}g_{\mu\nu})$	$({}_{\underline{G}}g_{\mu\nu})$	$(g_{\mu\nu})$

As an example, it is

$$ \frac{R}{D} \times {}_{a}g_{\mu\nu} = a \times g_{\mu\nu} \tag{403} $$

and

$$ {}_{a}g_{\mu\nu} = \frac{a \times D}{R} \times g_{\mu\nu} \tag{404} $$

We obtain

$$ \underbrace{\left(\frac{R}{D} \times {}_{a}g_{\mu\nu}\right)}_{a_{\mu\nu}} + \underbrace{\left(\frac{R}{D} \times {}_{b}g_{\mu\nu}\right)}_{b_{\mu\nu}} + \underbrace{\left(\frac{R}{D} \times {}_{c}g_{\mu\nu}\right)}_{c_{\mu\nu}} + \underbrace{\left(\frac{R}{D} \times {}_{d}g_{\mu\nu}\right)}_{d_{\mu\nu}} = \frac{R}{D} \times g_{\mu\nu} = R_{\mu\nu} \tag{405} $$

In this publication, let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the covariant second rank tensors of the four basic fields of nature were $a_{\mu\nu} \equiv a \times g_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu} \equiv b \times g_{\mu\nu}$, $c_{\mu\nu} \equiv c \times g_{\mu\nu}$ and $d_{\mu\nu} \equiv d \times g_{\mu\nu}$ et cetera.

Definition 5.11 (The metric tensor $_{gw}g_{\mu\nu}$ of gravitational waves). *Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. Let $_{gw}g_{\mu\nu}$ denote the metric tensor of* ***gravitational waves*** *of Einstein's general theory of relativity. Let $_{\underline{gw}}g_{\mu\nu}$ denote the metric tensor of* ***anti-gravitational waves*** *of Einstein's general theory of relativity. In general, there are circumstances where*

$$_{\underline{E}}g_{\mu\nu} \equiv {}_{\underline{gw}}g_{\mu\nu} + {}_{gw}g_{\mu\nu} \tag{406}$$

Definition 5.12 (The metric tensor $\eta_{\mu\nu}$ of special relativity). *There is a fundamental difference between Einstein's special theory of relativity and Einstein's general theory of relativity regarding the metric tensor. Let $\eta_{\mu\nu}$ denote the metric tensor of Einstein's special theory of relativity. In general, depending upon circumstances, it is $\eta_{\mu\nu} = \left\{\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{matrix}\right\}$ (see Einstein, 1916, p. 778). Let $\underline{\eta}_{\mu\nu}$ denote* ***the anti-metric tensor of Einstein's special theory of relativity****. Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. In general, it is (see equation 378)*

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \underline{\eta}_{\mu\nu} \tag{407}$$

We can imagine that there may be circumstances where $_{d}g_{\mu\nu} \equiv {}_{gw}g_{\mu\nu} \equiv \underline{\eta}_{\mu\nu}$ applies. Whether this will be generally valid might be the subject of further investigation. The n-th index relationship follows (see equation 378) as

$$g_{kl\mu\nu\ldots} \equiv \eta_{kl\mu\nu\ldots} + \underline{\eta}_{kl\mu\nu\ldots} \tag{408}$$

Einstein's field equations becomes

$$\underbrace{\left(\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times \eta_{\mu\nu}\right)}_{special\ relativity\ metric} + \underbrace{\left(\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) \times \underline{\eta}_{\mu\nu}\right)}_{disturbances\ or\ ripples\ in\ the\ curvature of spacetime} \equiv \underbrace{\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}}_{stress\ energy-momentum\ tensor} \tag{409}$$

Under conditions of D=2 space-time dimensions, the gravitational waves are related to Λ by the relationship

$$\Lambda \times \underline{\eta}_{\mu\nu} \tag{410}$$

Definition 5.13 (Index raising). *According to Einstein (see also Einstein, 1916, p. 790), it is*

$$F_{\mu\nu} \equiv g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta} \tag{411}$$

and equally

$$F^{\mu\nu} \equiv g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} \tag{412}$$

In other (Kay, 1988) words (see Einstein, 1916, p. 790), an order-2 tensor, twice multiplied by the contra-variant metric tensor and contracted (Einstein, 1916, p. 785) in different indices, raises each index. It is

$$F^{\left(\begin{smallmatrix} 1 & 3 \\ \mu & c \end{smallmatrix}\right)} \equiv g^{\left(\begin{smallmatrix} 1 & 2 \\ \mu & \nu \end{smallmatrix}\right)} \times g^{\left(\begin{smallmatrix} 3 & 4 \\ c & d \end{smallmatrix}\right)} \times F_{\left(\begin{smallmatrix} \nu & d \\ 2 & 4 \end{smallmatrix}\right)} \tag{413}$$

or more professionally

$$F^{\mu\ c} \equiv g^{\mu\nu} \times g^{cd} \times F_{\nu\ d} \tag{414}$$

Following Einstein, it is $g_{\mu\nu} \times g^{\mu\nu} \equiv \delta_{\mu}{}^{\mu}$ (Einstein, 1916, p. 796). Furthermore, in conjunction with another view of Einstein (see Einstein, 1916, p. 785), it is

$$F \equiv F_{\mu\nu}{}^{\mu\nu} \equiv F_{\mu\nu} \times F^{\mu\nu} \tag{415}$$

In the following, for the sake of better understanding, we consider tensors of order two. As is known, the components of a tensor of order two can be displayed in 4 × 4 matrix form.

5.2.1. Zero tensor

Definition 5.14 (Zero tensor).

The second-rank co-variant zero tensor is defined as

$$0_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 0_{00} & 0_{01} & 0_{02} & 0_{03} \\ 0_{10} & 0_{11} & 0_{12} & 0_{13} \\ 0_{20} & 0_{21} & 0_{22} & 0_{23} \\ 0_{30} & 0_{31} & 0_{32} & 0_{33} \end{pmatrix}}_{0_{\mu\nu} \text{ tensor}} \tag{416}$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.2. The negation of one

Definition 5.15 (The negation of one).

The negation of one, denoted as $\neg(1)$, is defined by division as

$$\neg(1) = \frac{0}{1} \tag{417}$$

In general, it is

$$\neg(1) \times 1 = +1 - 1 = \frac{0}{1} \times 1 = \frac{1}{1} \times 0 = 0 \tag{418}$$

The negation of one, denoted as $\neg$, is defined by subtraction as

$$\neg = 1- \tag{419}$$

In general, it is

$$\neg 1 = 1 - 1 = 0 \tag{420}$$

5.2.3. Unity tensor

Definition 5.16 (Unity tensor).

The second-rank co-variant unity tensor is defined as

$$1_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 1_{00} & 1_{01} & 1_{02} & 1_{03} \\ 1_{10} & 1_{11} & 1_{12} & 1_{13} \\ 1_{20} & 1_{21} & 1_{22} & 1_{23} \\ 1_{30} & 1_{31} & 1_{32} & 1_{33} \end{pmatrix}}_{1_{\mu\nu} \text{ tensor}} \tag{421}$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.4. The negation of zero

Definition 5.17 (The negation of zero).

The negation of zero, denoted as $\neg(0)$, is defined by division as

$$\neg(0) = \underline{0} = \frac{1}{0} \tag{422}$$

In general, it is

$$\neg(0) \times 0 = \underline{0} \times 0 = \frac{1}{0} \times 0 = \frac{0}{0} = 1 \tag{423}$$

The negation of zero, denoted as $\neg(0)$ or as $\underline{0}$, is defined by subtraction as

$$\neg = 1 - \tag{424}$$

In general, it is

$$\neg 0 = \underline{0} = 1 - 0 = 1 \tag{425}$$

5.2.5. The tensor of the number 2

Definition 5.18 (The tensor of the number 2).

The second-rank co-variant tensor of the number 2 is defined as

$$2_{\mu\nu} \equiv \underbrace{\begin{pmatrix} 2_{00} & 2_{01} & 2_{02} & 2_{03} \\ 2_{10} & 2_{11} & 2_{12} & 2_{13} \\ 2_{20} & 2_{21} & 2_{22} & 2_{23} \\ 2_{30} & 2_{31} & 2_{32} & 2_{33} \end{pmatrix}}_{2_{\mu\nu} \text{ tensor}} \tag{426}$$

This definition is also valid for contra-variant or mixed tensors an other numbers too. Whether it makes sense to define numbers or scalars et cetera in the form of a tensor is worth being discussed. However, such an approach has various advantages too.

5.2.6. Speed of the light tensor

Definition 5.19 (Speed of the light tensor).

Scientists and thinkers have been fascinated by the speed of light since ever. Aristotle (384-322 BCE) himself has been of the opinion that the speed of light was infinite. Let c denote the speed of the light in vacuum. The second-rank co-variant tensor of speed of the light is defined as

$$c_{\mu\nu} \equiv \underbrace{\begin{pmatrix} c_{00} & c_{01} & c_{02} & c_{03} \\ c_{10} & c_{11} & c_{12} & c_{13} \\ c_{20} & c_{21} & c_{22} & c_{23} \\ c_{30} & c_{31} & c_{32} & c_{33} \end{pmatrix}}_{\text{c}_{\mu\nu} \text{ tensor}} \tag{427}$$

5.2.7. Archimedes' constant tensor

Definition 5.20 (Archimedes' constant tensor).

The second-rank co-variant tensor of the Archimedes of Syracuse (c. 287 – c. 212 B. C. E.) constant π is defined as

$$\pi_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \pi_{00} & \pi_{01} & \pi_{02} & \pi_{03} \\ \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{20} & \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{30} & \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix}}_{\pi_{\mu\nu} \text{ tensor}} \tag{428}$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.8. Newton's constant tensor

Definition 5.21 (Newton's constant tensor).

The second-rank co-variant tensor of the Newton's constant is defined, as

$$\gamma_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \gamma_{00} & \gamma_{01} & \gamma_{02} & \gamma_{03} \\ \gamma_{10} & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{20} & \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{30} & \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}}_{\gamma_{\mu\nu} \text{ tensor}} \tag{429}$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.9. Planck's constant tensor

Definition 5.22 (Planck's constant tensor).

Plato (424/423 – 348/347 BCE), a Greek philosopher born in Athens, defined a circle as follows

> "Rund ist doch das, dessen Enden überall gleich weit von der Mitte entfernt sind? "
>
> (see also Plato, 1910, p. 26)

Max Karl Ernst Ludwig Planck (1858-1947) quantized the energy ${}_{R}E_{t}$ as

$$ {}_{R}E_{t} \equiv n \times h \times {}_{R}f_{t} \tag{430} $$

where h is Planck's constant (Planck, 1901), ${}_{R}f_{t}$ is the frequency and n is an integer number. In the following, Paul Adrien Maurice Dirac (1902-1984) defined the so-called Dirac's constant $\hbar$ (Dirac, 1926) as

$$ \begin{aligned} h &\equiv 2 \times \pi \times \hbar \\ &\equiv \pi \times (2 \times \hbar) \\ &\equiv \pi \times s \end{aligned} \tag{431} $$

Figure 5 might illustrate these basic relationships.

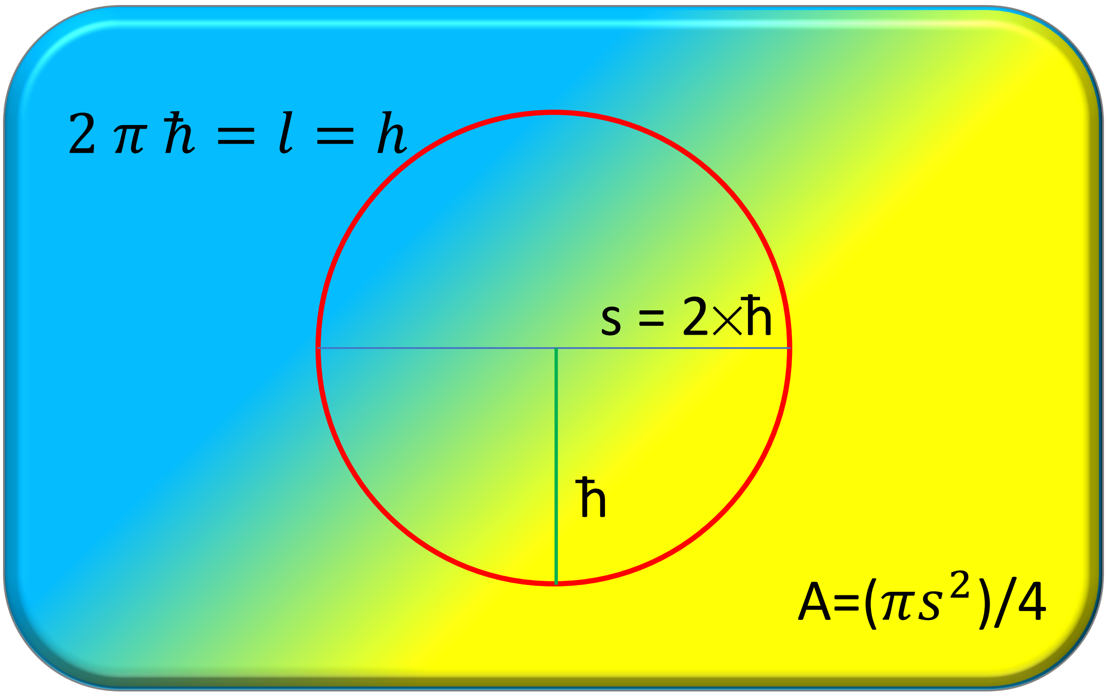

Figure 5. Planck's constant h, quantum loop and string theory.

A few thoughts - which are necessarily first thoughts - might consider circumstances where h can be regarded as a loop, denoted as l, of quantum loop theory, while s is treated as a string of string theory. Under these conditions, it is

$$l \equiv \pi \times s \tag{432}$$

or

$$\pi \equiv \frac{l}{s} \tag{433}$$

Equation 433 implies due to our experience that π can hardly be treated as a constant. In this context, the second-rank co-variant tensor of Planck's constant h (Planck, 1901) is defined, as

$$h_{\mu\nu} \equiv \underbrace{\begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{pmatrix}}_{\text{h}_{\mu\nu} \text{ tensor}} \tag{434}$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.10. Dirac's constant tensor

Definition 5.23 (Dirac's constant tensor).

The second-rank co-variant tensor of Dirac's constant $\hbar$ is defined as

$$\hbar_{\mu\nu} \equiv \underbrace{\begin{pmatrix} \hbar_{00} & \hbar_{01} & \hbar_{02} & \hbar_{03} \\ \hbar_{10} & \hbar_{11} & \hbar_{12} & \hbar_{13} \\ \hbar_{20} & \hbar_{21} & \hbar_{22} & \hbar_{23} \\ \hbar_{30} & \hbar_{31} & \hbar_{32} & \hbar_{33} \end{pmatrix}}_{\hbar_{\mu\nu} \text{ tensor}} \tag{435}$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.11. The commutative multiplication of tensors

Definition 5.24 (The commutative multiplication of tensors).

Multiplication is something which is equivalent to a repeated addition. Addition itself has the properties of associativity and commutativity. The question is justified whether there might exist something like a commutative multiplication of tensors. Let $U_{\mu\nu}$ denote a second-rank tensor. Let $W_{\mu\nu}$ denote another second-rank tensor. The commutative multiplication of two second-rank tensors is defined as an entry wise multiplication of both tensors. It is,

$$U_{\mu\nu} \cap W_{\mu\nu} \equiv X_{\mu\nu} \tag{436}$$

where the sign $\cap$ denotes a commutative multiplication of tensors of the same rank. The commutative multiplication of two tensors of the same rank is commutative, associative and distributive.

Example.

Example of an entrywise multiplication of two tensors of the same rank.

$$\underbrace{\begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} \\ u_{10} & u_{11} & u_{12} & u_{13} \\ u_{20} & u_{21} & u_{22} & u_{23} \\ u_{30} & u_{31} & u_{32} & u_{33} \end{pmatrix}}_{\mathrm{U}_{\mu\nu}\ \text{tensor}} \cap \underbrace{\begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} \\ w_{10} & w_{11} & w_{12} & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \\ w_{30} & w_{31} & w_{32} & w_{33} \end{pmatrix}}_{\mathrm{W}_{\mu\nu}\ \text{tensor}}$$
$$= \underbrace{\begin{pmatrix} (u_{00} \times w_{00}) & (u_{01} \times w_{01}) & (u_{02} \times w_{02}) & (u_{03} \times w_{03}) \\ (u_{10} \times w_{10}) & (u_{11} \times w_{11}) & (u_{12} \times w_{12}) & (u_{13} \times w_{13}) \\ (u_{20} \times w_{20}) & (u_{21} \times w_{21}) & (u_{22} \times w_{22}) & (u_{23} \times w_{23}) \\ (u_{30} \times w_{30}) & (u_{31} \times w_{31}) & (u_{32} \times w_{32}) & (u_{33} \times w_{33}) \end{pmatrix}}_{X_{\mu\nu}} \tag{437}$$

Jacques Salomon Hadamard (1865-1963), a French mathematician, defined a similar operation of two matrices of the same dimension $i \times j$ (see also Hadamard, 1893) which is commutative, associative and distributive. The Hadamard product (also known as the Issai Schur (see also Schur, 1911, p. 14) (1875 – 1941) product (see also Davis, 1962) or the point wise product) is of use for a commutative matrix multiplication and is defined something as

$$(u \circ w)_{\mathrm{ij}} \equiv u_{\mathrm{ij}} w_{\mathrm{ij}} \tag{438}$$

where the sign $\circ$ denotes an entry wise matrix multiplication.

5.2.12. The tensor double dot product on the closest indices

Definition 5.25 (The tensor double dot product on the closest indices).

Two tensors can be contracted over the first two indices of the second tensor or over the last two indices of the first tensor (double contraction). As is known, a double dot product between two tensors of orders m and n will result in a tensor of order (m + n - 4). Let $u_{\mu\nu}$ and $w_{\mu\nu}$ denote two second-rank tensors. Let : denote the contraction of two tensors $u_{\mu\nu}$ and $w_{\mu\nu}$ on the closest indices, then

$$u : w = u_{\mu\nu} w_{\nu\mu} \tag{439}$$

5.2.13. The tensor double dot product on the non-closest indices

Definition 5.26 (The tensor double dot product on the non-closest indices).

Let $u_{\mu\nu}$ and $w_{\mu\nu}$ denote two second-rank tensors. Let $\underline{:}$ denote the contraction of two tensors $u_{\mu\nu}$ and $w_{\mu\nu}$ on the non-closest indices, then

$$u\underline{:}w = u_{\mu\nu}w_{\mu\nu} \tag{440}$$

Especially under conditions where both second-rank tensors are symmetric, both definitions of the tensor double dot product coincide but not necessarily in general.

5.2.14. The division of tensors

Definition 5.27 (The division of tensors).

Division is something which is related to multiplication. Let $a_{\mu\nu}$ denote a second-rank tensor. Let $b_{\mu\nu}$ denote another second-rank tensor. Let $U_{\mu\nu}$ denote another second-rank co-variant tensor. In general, let it be that

$$a_{\mu\nu} + b_{\mu\nu} \equiv U_{\mu\nu} \tag{441}$$

The probability tensor of a tensor $a_{\mu\nu}$, denoted as $p(a_{\mu\nu})$, is calculated entry wise as follows.

$$p(a_{\mu\nu}) \equiv \begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} / \begin{pmatrix} U_{00} & U_{01} & U_{02} & U_{03} \\ U_{10} & U_{11} & U_{12} & U_{13} \\ U_{20} & U_{21} & U_{22} & U_{23} \\ U_{30} & U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} \frac{a_{00}}{U_{00}} & \frac{a_{01}}{U_{01}} & \frac{a_{02}}{U_{02}} & \frac{a_{03}}{U_{03}} \\ \frac{a_{10}}{U_{10}} & \frac{a_{11}}{U_{11}} & \frac{a_{12}}{U_{12}} & \frac{a_{13}}{U_{13}} \\ \frac{a_{20}}{U_{20}} & \frac{a_{21}}{U_{21}} & \frac{a_{22}}{U_{22}} & \frac{a_{23}}{U_{23}} \\ \frac{a_{30}}{U_{30}} & \frac{a_{31}}{U_{31}} & \frac{a_{32}}{U_{32}} & \frac{a_{33}}{U_{33}} \end{pmatrix} \tag{442}$$

5.2.15. The exponentiation of a tensor to the power n

Definition 5.28 (The exponentiation of a tensor to the power n)**.**

A second-rank co-variant tensor to the power n, denoted by ${}^{n}U_{\mu\nu}$, is determined by the fact that every single component of such a tensor is multiplied by itself n-times. In general, it is

$$
{}^{n}U_{\mu\nu} = \underbrace{\begin{pmatrix} \underbrace{(u_{00} \times u_{00} \times ...)}_{n-times} & \underbrace{(u_{01} \times u_{01} \times ...)}_{n-times} & \underbrace{(u_{02} \times u_{02} \times ...)}_{n-times} & \underbrace{(u_{03} \times u_{03} \times ...)}_{n-times} \\ \underbrace{(u_{10} \times u_{10} \times ...)}_{n-times} & \underbrace{(u_{11} \times u_{11} \times ...)}_{n-times} & \underbrace{(u_{12} \times u_{12} \times ...)}_{n-times} & \underbrace{(u_{13} \times u_{13} \times ...)}_{n-times} \\ \underbrace{(u_{20} \times u_{20} \times ...)}_{n-times} & \underbrace{(u_{21} \times u_{21} \times ...)}_{n-times} & \underbrace{(u_{22} \times u_{22} \times ...)}_{n-times} & \underbrace{(u_{23} \times u_{23} \times ...)}_{n-times} \\ \underbrace{(u_{30} \times u_{30} \times ...)}_{n-times} & \underbrace{(u_{31} \times u_{31} \times ...)}_{n-times} & \underbrace{(u_{32} \times u_{32} \times ...)}_{n-times} & \underbrace{(u_{33} \times u_{33} \times ...)}_{n-times} \end{pmatrix}}_{{}^{n}U_{\mu\nu}}
= \underbrace{\begin{pmatrix} (u_{00})^{n} & (u_{01})^{n} & (u_{02})^{n} & (u_{03})^{n} \\ (u_{10})^{n} & (u_{11})^{n} & (u_{12})^{n} & (u_{13})^{n} \\ (u_{20})^{n} & (u_{21})^{n} & (u_{22})^{n} & (u_{23})^{n} \\ (u_{30})^{n} & (u_{31})^{n} & (u_{32})^{n} & (u_{33})^{n} \end{pmatrix}}_{{}^{n}U_{\mu\nu}} \tag{443}
$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.16. The exponentiation of a tensor to the power 1/n

Definition 5.29 (The exponentiation of a tensor to the power 1/n)**.**

A second-rank co-variant tensor to the power 1/n, denoted by ${}^{1/n}U_{\mu\nu}$, is determined by the fact that every single component of such a tensor is multiplied by itself (1/n)-times. In general, it is

$$
{}^{1/n}U_{\mu\nu} = \underbrace{\begin{pmatrix} (u_{00})^{1/n} & (u_{01})^{1/n} & (u_{02})^{1/n} & (u_{03})^{1/n} \\ (u_{10})^{1/n} & (u_{11})^{1/n} & (u_{12})^{1/n} & (u_{13})^{1/n} \\ (u_{20})^{1/n} & (u_{21})^{1/n} & (u_{22})^{1/n} & (u_{23})^{1/n} \\ (u_{30})^{1/n} & (u_{31})^{1/n} & (u_{32})^{1/n} & (u_{33})^{1/n} \end{pmatrix}}_{{}^{1/n}U_{\mu\nu}} \tag{444}
$$

This definition is also valid for contra-variant or mixed tensors too.

5.2.17. The expectation value of a co-variant second rank tensor

Let $E({}_{R}U_{\mu\nu})$ denote the expectation value of a co-variant second rank tensor ${}_{R}U_{\mu\nu}$.Let $p({}_{R}U_{\mu\nu})$ denote the probability tensor of a tensor ${}_{R}U_{\mu\nu}$. In general, we define

$$
E\left({}_{R}U_{\mu\nu}\right) \equiv p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \tag{445}
$$

and equally

$$^{2}E\left({}_{\mathrm{R}}U_{\mu\nu}\right) \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap E\left({}_{\mathrm{R}}U_{\mu\nu}\right) \equiv p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \cap {}_{\mathrm{R}}U_{\mu\nu} \cap {}_{\mathrm{R}}U_{\mu\nu} \tag{446}$$

Let $E({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots})$ denote the expectation value of a co-variant n-index rank tensor ${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}$. Let $p({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots})$ denote the probability tensor of a co-variant n-index rank tensor ${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}$. In general, we define expectation value of a co-variant n-index rank tensor as

$$E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \equiv p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \tag{447}$$

It is equally true that

$$^{2}E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \equiv E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \equiv p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \tag{448}$$

5.2.18. The expectation value of a second rank anti tensor

Let $E({}_{\mathrm{R}}\underline{U}_{\mu\nu})$ denote the expectation value of the covariant second rank anti tensor ${}_{\mathrm{R}}\underline{U}_{\mu\nu}$. Let $p({}_{\mathrm{R}}\underline{U}_{\mu\nu})$ denote the probability tensor of an anti tensor ${}_{\mathrm{R}}\underline{U}_{\mu\nu}$. In general, we define

$$\begin{aligned} E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) &\equiv p\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \cap U_{\mu\nu} \\ &\equiv \left(1_{\mu\nu} - p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\right) \cap {}_{\mathrm{R}}U_{\mu\nu} \end{aligned} \tag{449}$$

Euclid's theorem is a fundamental statement of geometry and has been proved by Euclid in his famous work Elements. According to Euclid's theorem, it is

$${}_{\mathrm{R}}U_{\mu\nu} \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \tag{450}$$

Theorem 36. *It is*

$${}_{R}U_{\mu\nu} \equiv E\left({}_{R}U_{\mu\nu}\right) + E\left({}_{R}\underline{U}_{\mu\nu}\right) \tag{451}$$

Proof. According to Euclid's theorem, it is

$${}_{\mathrm{R}}U_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}U_{\mathrm{t}}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mathrm{t}}\right) \tag{452}$$

Multiply ${}_{\mathrm{R}}U_{\mathrm{t}}$ by the metric tensor $g_{\mu\nu}$ or just define

$${}_{\mathrm{R}}U_{\mathrm{t}} = {}_{\mathrm{R}}U_{\mu\nu} \tag{453}$$

Then the conclusion is true that

$${}_{\mathrm{R}}U_{\mu\nu} \equiv E\left({}_{\mathrm{R}}U_{\mu\nu}\right) + E\left({}_{\mathrm{R}}\underline{U}_{\mu\nu}\right) \tag{454}$$

□

♡

5.2.19. The expectation value of a second rank tensor raised to rower 2

Let $E({}^{2}{}_{R}U_{\mu\nu})$ denote the expectation value of the covariant second rank tensor ${}_{R}U_{\mu\nu}$ raised to the power 2. Let $p({}_{R}U_{\mu\nu})$ denote the probability tensor of a tensor ${}_{R}U_{\mu\nu}$. In general, we define

$$\begin{aligned} E\left({}^{2}{}_{R}U_{\mu\nu}\right) &\equiv p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \\ &\equiv p\left({}_{R}U_{\mu\nu}\right) \cap \left({}^{2}{}_{R}U_{\mu\nu}\right) \end{aligned} \tag{455}$$

Let ${}^{2}E({}_{R}U_{kl\mu\nu\ldots})$ denote the expectation value of a co-variant n-index rank tensor ${}^{2}{}_{R}U_{kl\mu\nu\ldots}$. Let $p({}_{R}U_{kl\mu\nu\ldots})$ denote the probability tensor of a co-variant n-index rank tensor ${}_{R}U_{kl\mu\nu\ldots}$. In general, we define the expectation value of a co-variant n-index rank tensor raised to rower 2 as

$${}^{2}E\left({}_{R}U_{kl\mu\nu\ldots}\right) \equiv p\left({}_{R}U_{kl\mu\nu\ldots}\right) \cap {}_{R}U_{kl\mu\nu\ldots} \cap {}_{R}U_{kl\mu\nu\ldots} \tag{456}$$

5.2.20. The variance of a tensor

Definition 5.30 (The variance of a tensor).

Let ${}_{R}U_{\mu\nu}$ denote a second-rank co-variant tensor. Let $p({}_{R}U_{\mu\nu})$ denote the probability tensor of a tensor ${}_{R}U_{\mu\nu}$. The variance of a tensor ${}_{R}U_{\mu\nu}$, denoted as ${}^{2}\sigma\left({}_{R}U_{\mu\nu}\right)$, is defined as

$$\begin{aligned} {}^{2}\sigma\left({}_{R}U_{\mu\nu}\right) &\equiv E\left({}^{2}{}_{R}U_{\mu\nu}\right) - {}^{2}\left(E\left({}_{R}U_{\mu\nu}\right)\right) \\ &\equiv \left(p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu}\right) - \left(p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap {}_{R}U_{\mu\nu}\right) \\ &\equiv {}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap \left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right) \end{aligned} \tag{457}$$

From equation 457 follows that

$${}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \equiv \frac{{}^{2}\sigma\left({}_{R}U_{\mu\nu}\right)}{p\left({}_{R}U_{\mu\nu}\right) \cap \left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)} \tag{458}$$

and that

$${}_{R}U_{\mu\nu} \equiv \frac{\sigma\left({}_{R}U_{\mu\nu}\right)}{{}^{1/2}\left(p\left({}_{R}U_{\mu\nu}\right) \cap \left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)\right)} \tag{459}$$

The standard deviation of a second-rank tensor, denoted as $\sigma\left({}_{R}U_{\mu\nu}\right)$, would follow as

$$\begin{aligned} \sigma\left({}_{R}U_{\mu\nu}\right) &\equiv {}^{1/2}\left({}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap \left(\left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)\right)\right) \\ &\equiv \sqrt[2]{\left({}_{R}U_{\mu\nu} \cap {}_{R}U_{\mu\nu} \cap p\left({}_{R}U_{\mu\nu}\right) \cap \left(\left(1_{\mu\nu} - p\left({}_{R}U_{\mu\nu}\right)\right)\right)\right)} \end{aligned} \tag{460}$$

Let ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Let p(${}_{\mathrm{R}}\mathrm{U}_{\mathrm{klkl}\mu\nu\ldots\ldots}$) denote the probability tensor of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$. The variance of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$, denoted as ${}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)$, is defined as

$$\begin{aligned}
&{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv E\left({}^{2}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) - {}^{2}\left(E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\\
&\equiv\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)-\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)
\end{aligned}\tag{461}$$

From equation 461 follows that

$${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\equiv\frac{{}^{2}\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)}\tag{462}$$

and that

$${}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\equiv\frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)}{{}^{1/2}\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)}\tag{463}$$

The standard deviation of a second-rank tensor, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)$, would follow as

$$\begin{aligned}
&\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\\
&\equiv{}^{1/2}\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)\\
&\equiv\sqrt[2]{\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\cap p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\cap\left(\left(1_{\mathrm{kl}\mu\nu\ldots}-p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)\right)}
\end{aligned}\tag{464}$$

5.2.21. The co-variance of two tensors

Definition 5.31 (The co-variance of two tensors).

Let ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ denote a second-rank co-variant tensor. Let p(${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$) denote the probability tensor of a tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$. According to equation 442, the probability tensor of a tensor ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ is defined as p(${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$). Let ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$ denote a second-rank co-variant tensor. Let p(${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$) denote the probability tensor of a tensor ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$ (see equation 442). Let p(${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$, ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$) denote the probability of a joint tensor of the two tensors ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$.The co-variance of the two tensors ${}_{\mathrm{R}}\mathrm{U}_{\mu\nu}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mu\nu}$, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mu\nu\ldots},{}_{\mathrm{R}}W_{\mu\nu\ldots}\right)$, is defined as

$$\begin{aligned}
&\sigma\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&\equiv E\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)-\left(E\left({}_{\mathrm{R}}U_{\mu\nu}\right)\times E\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\\
&\equiv\left(p\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\cap{}_{\mathrm{R}}U_{\mu\nu}\cap p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\cap{}_{\mathrm{R}}W_{\mu\nu}\right)\\
&\equiv{}_{\mathrm{R}}U_{\mu\nu}\cap{}_{\mathrm{R}}W_{\mu\nu}\cap\left(p\left({}_{\mathrm{R}}U_{\mu\nu},{}_{\mathrm{R}}W_{\mu\nu}\right)-\left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right)\times p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\right)
\end{aligned}\tag{465}$$

From equation 465 follows that

$$ {}_{\mathrm{R}}U_{\mu\nu} \cap {}_{\mathrm{R}}W_{\mu\nu} \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mu\nu}, {}_{\mathrm{R}}W_{\mu\nu}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mu\nu}, {}_{\mathrm{R}}W_{\mu\nu}\right) - \left(p\left({}_{\mathrm{R}}U_{\mu\nu}\right) \times p\left({}_{\mathrm{R}}W_{\mu\nu}\right)\right)\right)} \tag{466} $$

Let ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Furthermore, let $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots})$ denote the probability tensor of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$. According to equation 442, the probability tensor of a co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ is defined as $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots})$. Let ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$ denote a co-variant n-index rank tensor. Let $\mathrm{p}({}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots})$ denote the probability of this co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$ (see equation 442). Let $\mathrm{p}({}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots})$ denote the probability of a joint tensor of the two co-variant n-index rank tensors ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$.The co-variance of the two co-variant n-index rank tensor ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{kl}\mu\nu\ldots}$ and ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{kl}\mu\nu\ldots}$, denoted as $\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)$, is defined as

$$ \begin{aligned} &\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ \equiv &E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(E\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \times E\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right) \\ \equiv &\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ - &\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) \\ \equiv &{}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \cap \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right) \end{aligned} \tag{467} $$

From equation 467 follows that

$$ {}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots} \cap {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots} \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)}{\left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}, {}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right) - \left(p\left({}_{\mathrm{R}}U_{\mathrm{kl}\mu\nu\ldots}\right) \times p\left({}_{\mathrm{R}}W_{\mathrm{kl}\mu\nu\ldots}\right)\right)\right)} \tag{468} $$

Definition 5.32 (**Energy $_{R}E$ and mass $_{R}m$ equivalence**).

As long as we have the right to rely on the insights of the special theory of relativity, it turns out to be that (relativistic) energy $_{R}E$ and (relativistic) mass $_{R}m$ are only two different(see also Einstein, 1905c, p. 641) viewpoints on the one and the same physical quantity. The energy-mass(see also Einstein, 1912, p. 1062) equivalence (see Einstein, 1935) is given as

$$_{R}E = {_{R}m} \times c^2 \tag{469}$$

The relativistic mass is depending on the motion of an object and corresponds to the total energy of an object. However, different observers in relative motion might see different values for the relativistic mass. Furthermore, the relativistic mass of a moving object is at the end larger than the mass at rest, the mass of the same object at rest. The reason is that a moving object has relativistic kinetic energy (see Barukčić, 2013). Equally, it is

$$_{0}E = {_{0}m} \times c^2 \tag{470}$$

while $_{0}E$ is the rest energy, the energy of an object in its own rest frame 0 and $_{0}m$ is the rest mass, the mass of an object in its own rest frame 0. It is

$$\frac{_{0}E}{_{R}E} = \frac{_{0}m \times c^2}{_{R}m \times c^2} = \frac{_{0}m}{_{R}m} = \sqrt{1 - \frac{v^2}{c^2}} \tag{471}$$

where c is the speed of light in vacuum and v is the relative velocity between an observer at rest and a stationary observer. The normalised energy-momentum relation (see Barukčić, 2016c) is given as

$$\frac{_{0}m^2}{_{R}m^2} + \frac{v^2}{c^2} = 1 \tag{472}$$

while $p(_{0}m)$ is the probability of finding an object local (see Barukčić, 2022) and is given as

$$p(_{0}m) = \frac{_{0}m^2}{_{R}m^2} = 1 - \frac{v^2}{c^2} \tag{473}$$

Definition 5.33 (**Time and gravitational field equivalence**).

Time and gravitational field have been identified (see Barukčić, 2011) as being equivalent, as the two faces of one and the same coin. But the road there is long and not without pitfalls. However, in relation to the character of the relationship between time and gravitational field, Einstein's opinion would have to be emphasised again.

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß **alles außer dem Gravitationsfeld** als '**Materie**'bezeichnet wird, also nicht nur die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. "

(Einstein, 1916, p. 802/803)

Firstly. Everything but the gravitational field is matter, there is no third between matter and gravitational field, a third is not given, **tertium non datur**. Secondly. Matter, from the point of view of a stationary observer R, includes not only matter in the ordinary sense, but the electromagnetic field as well (Einstein, 1916, p. 802/803). Finally, one consequential relationship is necessary to mention. "Da Masse und Energie nach den Ergebnissen der speziellen Relativitätstheorie das Gleiche sind und die Energie formal durch den symmetrischen Energietensor (Tμv) beschrieben wird, so besagt dies, daß das G-Geld [gravitational field, author] durch den Energietensor der Materie bedingt und bestimmt ist "(Einstein, 1918b). Matter or energy is the cause of the gravitational field. However, is this relationship valid vice versa to?

Definition 5.34 (**Time $_{R}t_{t}$ and gravitational field $_{R}g_{t}$**).

Albert Einstein (see Einstein, 1905b, p. 904) pointed to the relationship between proper (see Minkowski, 1908) time $_{0}t$ (independent of coordinates) usually represented by the Greek letter τ and relativistic time $_{R}t$ (coordinate time) as

$$ {}_{0}t = {}_{R}t \times \sqrt{1 - \frac{v^2}{c^2}} \tag{474} $$

It is

$$ \frac{{}_{0}t}{{}_{R}t} = \sqrt{1 - \frac{v^2}{c^2}} \tag{475} $$

The relationship between proper time and coordinate time is normalised (see Barukčić, 2016c) as

$$ \frac{{}_{0}t^2}{{}_{R}t^2} + \frac{v^2}{c^2} = +1 \tag{476} $$

where c is the speed of light in vacuum and v is the relative velocity between an observer at rest and a stationary observer. The fundamental relationship between gravitational field $_{R}g_{t}$ from the point of view of the stationary observer R and time ${}_{R}t_{t}$ from the point of view of the same stationary observer R is determined(Barukčić, 2011, 2013, 2016c,d) by the equation

$$ {}_{R}g_{t} \equiv \frac{{}_{R}t_{t}}{c^{2}} \tag{477}$$

and from the point of view of a co-moving observer 0 by the equation

$$ {}_{0}g_{t} \equiv \frac{{}_{0}t_{t}}{c^{2}} \tag{478}$$

Next we define(Barukčić, 2011, 2016d) the following mathematical identities related to time, to which a concrete physical meaning would have to be attached in the following of further development.

$$ {}_{W}t_{t} \equiv v \times c \times {}_{R}g_{t} \tag{479}$$

In general, it is

$$ {}_{W}t_{t}{}^{2} \equiv \left(v \times c \times {}_{R}g_{t}\right)^{2} \equiv {}_{R}t_{t}{}^{2} - {}_{0}t_{t}{}^{2} \tag{480}$$

and

$$ {}_{W}g_{t} \equiv \frac{{}_{W}t_{t}}{c^{2}} \tag{481}$$

As such (see equation 480), it is a logical step to consider that

$$ {}_{R}g_{t} \equiv {}_{0}g_{t} + {}_{W}g_{t} \tag{482}$$

I should like to take this opportunity to express once again the possibility that ${}_{W}g_{t}$ itself might represent something similar to the gravitational waves. Let the mathematical identity ${}_{K}t_{t}$ be defined as follows.

$$ {}_{K}t_{t} \equiv \frac{{}_{W}t_{t} \times {}_{W}t_{t}}{{}_{R}t_{t}} \equiv \frac{{}_{W}t_{t}}{{}_{R}t_{t}} \times {}_{W}t_{t} \equiv \frac{\left(v \times c \times {}_{R}g_{t}\right)^{2}}{c^{2} \times {}_{R}g_{t}} \equiv v^{2} \times {}_{R}g_{t} \tag{483}$$

The notion ${}_{K}t_{t}$ might indicate the time as determined by the relativistic kinetic energy ${}_{K}E_{t}$. Let the mathematical identity ${}_{P}t_{t}$ be defined as follows.

$$ {}_{P}t_{t} \equiv \frac{{}_{0}t_{t} \times {}_{0}t_{t}}{{}_{R}t_{t}} \equiv \frac{{}_{0}t_{t}}{{}_{R}t_{t}} \times {}_{0}t_{t} \equiv \left(\sqrt{1 - \frac{v^{2}}{c^{2}}}\right) \times {}_{0}t_{t} \tag{484}$$

The notion ${}_{P}t_{t}$ might indicate the time as determined by the relativistic potential energy ${}_{P}E_{t}$. In general, it is necessary to consider that,

$$ {}_{R}t_{t} \equiv {}_{P}t_{t} + {}_{K}t_{t} \tag{485}$$

Furthermore, the following identities are defined.

$$ {}_{K}g_{t} \equiv \frac{{}_{K}t_{t}}{c^{2}} \tag{486}$$

$$ {}_{P}g_{t} \equiv \frac{{}_{P}t_{t}}{c^{2}} \tag{487}$$

The identity ${}_{Kred}t_{t}$ is defined as

$$ {}_{Kred}t_{t} \equiv v \times {}_{R}g_{t} \tag{488}$$

Definition 5.35 (**Space** $_{\mathrm{R}}\mathrm{S}_{\mathrm{t}}$).

We define the general relationship

$$ {}_{\mathrm{R}}S_{\mathrm{t}} \equiv {}_{0}S_{\mathrm{t}} + {}_{0}\underline{S}_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \times c^{2} \tag{489} $$

In case, that there are not justified reasons to doubt the correctness of Einstein's demand that all but matter is a gravitational field(Einstein, 1916, p. 802/803), we define

$$ {}_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} + {}_{\mathrm{R}}g_{\mathrm{t}} \equiv \frac{{}_{\mathrm{R}}S_{\mathrm{t}}}{c^{2}} \tag{490} $$

where $_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ is the mathematical identity of matter $_{\mathrm{R}}\mathrm{M}_{\mathrm{t}}$ and gravitational field $_{\mathrm{R}}\mathrm{g}_{\mathrm{t}}$, $_{\mathrm{R}}\mathrm{S}_{\mathrm{t}}$ is something like space and c is the speed of the light in vacuum. The following figure might illustrate this basic relationship from another point of view.

We multiply equation 490 by the term $\left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right)$ where v is the relative velocity between a co-moving observer 0 and a stationary observer R. It is

$$ \left({}_{\mathrm{R}}U_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right)\right) \equiv \left({}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right)\right) + \left({}_{\mathrm{R}}g_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right)\right) \tag{491} $$

We define $_{0}\mathrm{U}_{\mathrm{t}}$ as

$$ {}_{0}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right) \tag{492} $$

According to Einstein, the rest-mass $_{0}\mathrm{m}_{\mathrm{t}}$ is given as

$$ {}_{0}m_{\mathrm{t}} \equiv {}_{\mathrm{R}}M_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right) \tag{493} $$

We define $_{0}\mathrm{g}_{\mathrm{t}}$ as

$$ {}_{0}g_{\mathrm{t}} \equiv {}_{\mathrm{R}}g_{\mathrm{t}} \times \left(\sqrt{1-\frac{v^{2}}{c^{2}}}\right) \tag{494} $$

Equation 491 as seen from the point of view of a co-moving observer 0 becomes

$$ {}_{0}U_{\mathrm{t}} \equiv {}_{0}m_{\mathrm{t}} + {}_{0}g_{\mathrm{t}} \tag{495} $$

where $_{0}\mathrm{m}_{\mathrm{t}}$ indicates the rest mass as determined by the co-moving observer, $_{0}\mathrm{g}_{\mathrm{t}}$ is the gravitational field as determined by the co-moving observer and $_{0}\mathrm{U}_{\mathrm{t}}$ is the unity and the 'struggle' of both.

*5.4. **Einstein's general theory or relativity***

Definition 5.36 (The Einstein field equations). *The Einstein field equations (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) describe the relationship between the presence of matter* $\left(represented\ by\ the\ stress-energy\ tensor \left(\left(\frac{4\times 2\times \pi\times \gamma}{c^4} \right) \times T_{\mu\nu} \right) \right)$ *in a given region of space time and the curvature* $\left(represented\ by\ the\ Einstein\ tensor\ G_{\mu\nu} = R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \right)$ *in that region of space time by the equation*

$$R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{4\times 2\times \pi\times \gamma}{c^4} \right) \times T_{\mu\nu} \equiv E_{\mu\nu} \tag{496}$$

*where $R_{\mu\nu}$ is the Ricci tensor (Ricci-Curbastro and Levi-Civita, 1900) of 'Einstein's general theory of relativity' (Einstein, 1916), R is the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold, Λ is the Einstein's cosmological (Barukčić, 2015a, Einstein, 1917) constant, $\underline{\Lambda}$ is the "**anti cosmological constant**" (Barukčić, 2015a), $g_{\mu\nu}$ is the metric tensor of Einstein's general theory of relativity, $G_{\mu\nu}$ is Einstein's curvature tensor, $\underline{G}_{\mu\nu}$ is the "**anti tensor**" (Barukčić, 2016c) of Einstein's curvature tensor, $E_{\mu\nu}$ is the stress-energy tensor of energy, $\underline{E}_{\mu\nu}$ is the tensor of non-energy, the anti-tensor of the stress-energy tensor of energy, $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, c is the speed of the light in vacuum, γ is Newton's gravitational "**constant**" (Barukčić, 2015a,b, 2016a,c), π is Archimedes constant pi.*

Einstein's field equations are defined in space-time dimensions (see Málek, 2012, p. 31) other than 3+1 too. Table 13 may provide a more detailed and preliminary overview of the definitions (Barukčić, 2016b,c) before.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu} \equiv (c_{\mu\nu} + \Lambda \times g_{\mu\nu})$	$\frac{8\times\pi\times\gamma\times T}{c^4 \times D} \times g_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} + \Lambda \right) \times g_{\mu\nu}$
	NO	$c_{\mu\nu} \equiv (b_{\mu\nu} - \Lambda \times g_{\mu\nu})$	$d_{\mu\nu} \equiv (\frac{R}{2} \times g_{\mu\nu} - b_{\mu\nu})$	$\left(\frac{R}{2} - \Lambda \right) \times g_{\mu\nu}$
		$G_{\mu\nu} \equiv \left(\frac{R}{D} - \frac{R}{2} \right) \times g_{\mu\nu}$	$\frac{R}{2} \times g_{\mu\nu}$	$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu}$

Table 13. Four basic fields of nature and the Einstein field equations.

From Einstein's specific point of view, two wings are necessary to get to the core of the relationship between matter and gravitational field, just as two wings are essential for a bird that conquers the air.

We are quite privileged to consider in detail that

$$\underbrace{\left(\frac{R}{D}\times g_{\mu\nu}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)}_{the\ left-hand\ side}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^{4}\times D}\right)\times g_{\mu\nu}}_{the\ right-hand\ side} \tag{497}$$

while $R_{\mu\nu}\equiv a_{\mu\nu}+b_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}$ and the

> "... one wing ... is made of fine marble (**left side of the equation**) ...
>
> the other wing ... is built of low-grade wood (**right side of equation**).
>
> The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter. "
>
> (Einstein, 1936, p. 370)

Taken together, the n[th] index, D-dimensional Einstein's gravitational field equations (Barukčić, 2020d) follow as

$$\underbrace{\left(\frac{R}{D}\times g_{\mu\nu\pi\rho\ldots}\right)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu\pi\rho\ldots}\right)+\left(\Lambda\times g_{\mu\nu\pi\rho\ldots}\right)}_{(local)\ space-time\ curvature}\equiv\underbrace{\left(\frac{4\times 2\times\pi\times\gamma\times T}{c^{4}\times D}\right)\times g_{\mu\nu\pi\rho\ldots}}_{(local)\ energy\ and\ momentum} \tag{498}$$

The stress-energy tensor, has the unit of energy density, or pressure or of energy/volume = pressure = force/area, while the metric tensor is unitless (see Porta Mana, 2021). Einstein's constant converts converts the stress-energy to the units of the left side of the field equation, each term of which is of unit $1/L^2$. However and in general, the metric field (responsible for gravitational-inertial properties of bodies) on the left-hand side of Einstein's field equations, is completely determined by a tensorial but non-geometrical phenomenological representation of matter on the right-hand side. Einstein himself had a very differentiated view of these two sides of his field equations. In point of fact, the left part of the Einstein field equations (the Einstein tensor) is taken by Einstein as fine marble because of its geometrical nature, whereas the right side of the equations is lacking similar geometric significance and was degraded by Einstein himself to low-grade wood, the need for geometrical unification follows at least from such an asymmetrical state of affairs.

> "The mind striving after unification of the theory cannot be satisfied that two fields should exist which, by their nature, are quite independent. A mathematically **unified field theory** is sought in which **the gravitational field and the electromagnetic field are interpreted only as different components or manifestations of the same uniform field** ... The gravitational theory ... should be generalized so that it includes the laws of the electromagnetic field. "
>
> (Einstein, 1923a, p. 489)

An incorporation of electromagnetism and of other fields into spacetime geometry is desirable. In point of fact, a striving toward unification and simplification of the premises and of Einstein's general theory of relativity as a whole is necessary.

Definition 5.37 (**The stress-energy tensor of ordinary matter** $a_{\mu\nu}$). *Howard Georgi and Sheldon Glashow (Georgi and Glashow, 1974) proposed in 1974 the first Grand Unified Theory (Buras et al., 1978). Grand Unified Theory (GUT) models predict the unification of the electromagnetic, the weak, and the strong forces into a single force. However, it appears to be more appropriate to unify the strong nuclear force and the weak nuclear force into an ordinary force. The matter as associated with an ordinary force can be calculated very precisely. Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity,* ***the stress-energy tensor of ordinary matter*** $a_{\mu\nu}$ *which* ***is expected to unify the strong nuclear force and the weak nuclear force into an ordinary force*** *is defined / derived / determined as*

$$
\begin{aligned}
a_{\mu\nu} &\equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \\
&\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) - b_{\mu\nu} \\
&\equiv R_{\mu\nu} - (R \times g_{\mu\nu}) + (((\Lambda \times g_{\mu\nu}) + d_{\mu\nu}) = 0) \\
&\equiv \left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - (R \times g_{\mu\nu}) \\
&\equiv (E - b) \times g_{\mu\nu} \\
&\equiv (G - c) \times g_{\mu\nu} \\
&\equiv a \times g_{\mu\nu}
\end{aligned}
\tag{499}
$$

or

$$
\begin{aligned}
a_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) - \\
\left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right)
\end{aligned}
\tag{500}
$$

From our present point of view we can expect that there are conditions where

$$
\begin{aligned}
a_{\mu\nu} &\equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \\
&\equiv \left(\left(\frac{R}{D} - \frac{R}{2} + \Lambda\right) - \left(\frac{(4 + D) \times F_1}{4 \times \pi \times 4 \times D}\right)\right) \times g_{\mu\nu}
\end{aligned}
\tag{501}
$$

where F_1 is Lorenz invariant.

Definition 5.38 (**The 4-index D dimensional a** $_{kl\mu\nu}$). *The 4-index D dimensional* $a_{kl\mu\nu}$ *is defined as:*

$$
\begin{aligned}
a_{kl\mu\nu} &\equiv (E - b) \times g_{kl\mu\nu} \\
&\equiv (G - c) \times g_{kl\mu\nu} \\
&\equiv a \times g_{kl\mu\nu}
\end{aligned}
\tag{502}
$$

Definition 5.39 (**The n-index D dimensional a** $_{kl\mu\nu\,\ldots}$). *The n-index D dimensional* $a_{kl\mu\nu\,\ldots}$ *is defined as:*

$$
\begin{aligned}
a_{kl\mu\nu\,\ldots} &\equiv (E - b) \times g_{kl\mu\nu\,\ldots} \\
&\equiv (G - c) \times g_{kl\mu\nu\,\ldots} \\
&\equiv a \times g_{kl\mu\nu\,\ldots}
\end{aligned}
\tag{503}
$$

Definition 5.40 (**Ricci scalar R**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric is a quantity which is determined at each point in space-time by lamda Λ and anti-lamda (Barukčić, 2015a) $\underline{\Lambda}$ as*

$$R \equiv g^{\mu\nu} \times R_{\mu\nu} \equiv (\Lambda) + (\underline{\Lambda}) \equiv D \times S \tag{504}$$

where D is proved as the number of space-time dimension and $S \equiv \left(\frac{R}{D}\right)$. A Ricci scalar curvature R which is positive at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In other words, the density of space varies. In contrast to this, a Ricci scalar curvature R which is negative at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general, it is (see Barukčić, 2015a)

$$R \times g_{\mu\nu} \equiv (\Lambda \times g_{\mu\nu}) + (\underline{\Lambda} \times g_{\mu\nu}) \tag{505}$$

or

$$R \equiv (\Lambda) + (\underline{\Lambda}) \tag{506}$$

The cosmological constant can also be written algebraically as part of the stress–energy tensor, a second order tensor as the source of gravity (energy density).

Under conditions where R = 0, it is

$$-(\Lambda) = +(\underline{\Lambda}) \tag{507}$$

Definition 5.41 (**Ricci tensor $\mathbf{R}_{\mu\nu}$**). *The Ricci tensor $R_{\mu\nu}$ is a geometric object which has been developed by Gregorio Ricci-Curbastro (1853 – 1925) (Ricci-Curbastro and Levi-Civita, 1900) and is able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. In this publication, let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the covariant second rank tensors of the four basic fields of nature were $a_{\mu\nu} \equiv {}_{f}a^2 \times g_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu} \equiv {}_{f}b^2 \times g_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field, $c_{\mu\nu} \equiv c^2 \times g_{\mu\nu}$ is the tensor of the gravitational field and $d_{\mu\nu} \equiv {}_{f}d^2 \times g_{\mu\nu}$ is the tensor of gravitational waves. The Ricci tensor $R_{\mu\nu}$ of 'Einstein's general theory of relativity' (Einstein, 1916) is determined by the stress-energy tensor $\left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right)$ and the anti stress-energy tensor $\left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\right)$ as*

$$\begin{aligned} R_{\mu\nu} &\equiv \underbrace{\left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right)}_{stress-energy\ tensor} + \underbrace{\left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\right)}_{anti\ stress-energy\ tensor} \\ &\equiv \quad a_{\mu\nu} + b_{\mu\nu} \quad\quad + c_{\mu\nu} + d_{\mu\nu} \\ &\equiv (S) \times g_{\mu\nu} \\ &\equiv \left(\frac{R}{D}\right) \times g_{\mu\nu} \end{aligned} \tag{508}$$

while S might denote a scalar.

Definition 5.42 (Laue's scalar T). *Max von Laue (1879-1960) proposed the meanwhile so called Laue scalar (Laue, 1911) (criticised by Einstein (Einstein and Grossmann, 1913, Norton, 1992)) as the contraction of the the stress–energy momentum tensor $T_{\mu\nu}$ denoted as T and written without subscripts or arguments. Einstein wrote.*

> "Es bietet sich bei Charakterisierung des Gravitationsfeldes durch einen Skalar ein Weg dar ... bei dieser Auffassung ein Skalar maßgebend für die Wechselwirkung zwischen Gravitationsfeld und materiellem Vorgang. Dieser Skalar kann, worauf mich Herr Laue aufmerksam machte, nur
>
> $$\sum_{\mu} T_{\mu\mu} = P$$
>
> sein, den ich als den 'Laue'schen Skalar'bezeichnen will. "
>
> (Einstein and Grossmann, 1913, p. 23)

Translated into English: There is a way to characterize the gravitational field by a scalar ... in this view a scalar is decisive for the interaction between gravitational field and a material process. This scalar can only be, as Mr. Laue pointed out to me, $\sum_{\mu} T_{\mu\mu} = P$ which I will call 'Laue's scalar'.

Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, it is

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{509}$$

Taken Einstein seriously, $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter" (see Einstein, 1923b, p. 88). In other words, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense." (see Einstein, 1923b, p. 93)

Scalars are of use to describe circumstances of Einstein's theory of general relativity. However, it is necessary to point out a crucial difference. The use of scalars should not be confused neither with Brans-Dicke (see Brans and Dicke, 1961) theory of gravitation (see Norton, 1992) nor with Nordström's theory of gravitation (see Nordström, 1913a,b).The last has been found to be logically inconsistent.

Definition 5.43 (**The scalar E**). *In general, we define the scalar E as*

$$
\begin{aligned}
E \equiv {}_{d}E_{t}{}^{2} &\equiv \left(\frac{8\times\pi\times\gamma}{c^{4}\times D}\right)\times T \\
&\equiv \left(\frac{8\times\pi\times\gamma\times T}{c^{4}\times D}\right) \\
&\equiv \left(\frac{2\times\pi\times 4\times\gamma\times T}{c^{4}\times D}\right) \\
&\equiv \left(\frac{h\times 4\times\gamma\times T}{\hbar\times c^{4}\times D}\right) \\
&\equiv \left(\frac{R}{D}\right)-\left(\frac{R}{2}\right)+\Lambda
\end{aligned}
\tag{510}
$$

*where D is the space-time dimension, where c denote the speed of the light in vacuum, γ denote Newton's gravitational "**constant**" (Barukčić, 2015a,b, 2016a,c), π is the number pi and T denote Laue's scalar. The scalar E might correspond even to the total energy density squared of a (relativistic or quantum) system, and has the potential as such to bridge the gap between relativity theory and quantum mechanics under circumstances where the same is related or even identical with the Hamiltonian operator (squared).*

Definition 5.44 (**Stress-energy and momentum tensor $E_{\mu\nu}$**). *The stress–energy–momentum tensor or the stress–energy tensor or the energy–momentum tensor or energy tensor of matter $T_{\mu\nu}$ is the source of the gravitational field in the Einstein field equations of general relativity. In point of fact, the stress–energy–momentum tensor $T_{\mu\nu}$ itself is determined by sub-tensors and can be decomposed into the same. Especially, according to Einstein, it is necessary to consider that*

> *"... a tensor, $T_{\mu\nu}$, of the second rank ... includes in itself the energy density of the electromagnetic field and of ponderable matter; we shall denote this in the following as the 'energy tensor of matter"'*
>
> *(Einstein, 1923b, pp. 87/88)*

The tensor of stress-energy-momentum denoted as $E_{\mu\nu}$ is determined in detail as follows.

$$
\begin{aligned}
E_{\mu\nu} &\equiv a_{\mu\nu} + b_{\mu\nu} \\
&\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\
&\equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \\
&\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \\
&\equiv \left(S - \left(\frac{R}{2}\right) + \Lambda\right) \times g_{\mu\nu} \\
&\equiv (G + \Lambda) \times g_{\mu\nu} \\
&\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \\
&\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\
&\equiv E \times g_{\mu\nu}
\end{aligned}
\tag{511}
$$

while E might denote the scalar of, even something like 'energy density'. From a different angle, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense." (see also Einstein, 1923b, p. 93) Once again, it is important to point out that all possible forms of energy and momentum are contained in the stress-energy tensor $T_{\mu\nu}$. This includes any matter present but if there is some electromagnetic radiation given then the same too must be included in $T_{\mu\nu}$. It goes without saying that in this case there is simply no energy or momentum left which could be assigned to the gravitational field. Therefore, assigning any kind of energy density to a gravitational field (see Einstein, 1918a, pp. 156-159) could be difficult, both in principle and technically. The question inevitably arises why under the conditions of general relativity energy and momentum should be conserved at all?

Definition 5.45 (The scalar G). *In general, we define the scalar G (Barukčić, 2020b) as*

$$
\begin{aligned}
G \equiv {}_{d}G_{t}^{2} &\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \\
&\equiv \left(E + {}_{R}t_{t} - \frac{R}{2}\right) \\
&\equiv \left(E + \left(\frac{R}{2} - \Lambda\right) - \frac{R}{2}\right) \\
&\equiv E - \Lambda
\end{aligned}
\tag{512}
$$

Definition 5.46 (Einstein's curvature tensor $G_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of curvature denoted by $G_{\mu\nu}$ is defined/derived/determined (see Barukčić, 2020b) as follows:*

$$G_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \\ \equiv \left(\frac{R}{D} \right) \times g_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \\ \equiv \left(\left(\frac{R}{D} \right) - \frac{R}{2} \right) \times g_{\mu\nu} \\ \equiv a_{\mu\nu} + c_{\mu\nu} \\ \equiv G \times g_{\mu\nu} \\ \equiv \left(\frac{R}{D} \right) \times {}_{G}g_{\mu\nu} \tag{513}$$

Definition 5.47 (**The scalar $\underline{G}$**). *In general, we define the scalar $\underline{G}$ (see Barukčić, 2020b) as*

$$\underline{G} \equiv {}_{d}\underline{G}_{t}{}^{2} \equiv \left(\left(\frac{R}{D} \right) - G \right) \\ \equiv \left(\frac{R}{2} \right) \tag{514}$$

Definition 5.48 (**The scalar $\underline{E}$**). *In general, we define the scalar $\underline{E}$ as (see Barukčić, 2020b)*

$$\underline{E} \equiv {}_{d}\underline{E}_{t}{}^{2} \equiv \left(\left(\frac{R}{D} \right) - E \right) \\ \equiv \left(\frac{R}{2} - \Lambda \right) \tag{515}$$

Remark 5.1. *In the following of research, it is appropriate to prove the relationship between (1/X) and the complex conjugate of the wave function Ψ^{*} or the identity (1/X)$\equiv \Psi^{*}$.*

Definition 5.49 (**The anti Einstein's curvature tensor or the tensor of non-curvature $\underline{G}_{\mu\nu}$**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of non-curvature is defined/derived/determined (Barukčić, 2020b) as follows:*

$$\underline{G}_{\mu\nu} \equiv R_{\mu\nu} - G_{\mu\nu} \\ \equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) \right) \\ \equiv \left(\frac{R}{2} \right) \times g_{\mu\nu} \\ \equiv b_{\mu\nu} + d_{\mu\nu} \\ \equiv \underline{G} \times g_{\mu\nu} \tag{516}$$

Definition 5.50 **(The 4-index D dimensional stress-energy and momentum tensor $\mathbf{E}_{kl\mu\nu}$).** *The 4-index D dimensional stress-energy-momentum tenosr denoted as $E_{kl\mu\nu}$ is determined in detail as*

$$\begin{aligned} E_{kl\mu\nu} &\equiv \left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{kl\mu\nu} \\ &\equiv R_{kl\mu\nu}-\left(\left(\frac{R}{2}\right)\times g_{kl\mu\nu}\right)+\left(\Lambda\times g_{kl\mu\nu}\right) \\ &\equiv G_{kl\mu\nu}+\left(\Lambda\times g_{kl\mu\nu}\right) \\ &\equiv R_{kl\mu\nu}-\underline{E}_{kl\mu\nu} \\ &\equiv a_{kl\mu\nu}+b_{kl\mu\nu} \\ &\equiv H\times g_{kl\mu\nu}\equiv H_{kl\mu\nu} \\ &\equiv E\times g_{kl\mu\nu} \end{aligned} \tag{517}$$

Definition 5.51 **(The n-index D dimensional stress-energy and momentum tensor $\mathbf{E}_{kl\mu\nu\ldots}$).** *The n-index D dimensional stress-energy-momentum tenosr denoted as $E_{kl\mu\nu\ldots}$ is determined in detail as*

$$\begin{aligned} E_{kl\mu\nu\ldots} &\equiv \left(\frac{8\times\pi\times\gamma\times T}{c^4\times D}\right)\times g_{kl\mu\nu\ldots} \\ &\equiv R_{kl\mu\nu\ldots}-\left(\left(\frac{R}{2}\right)\times g_{kl\mu\nu\ldots}\right)+\left(\Lambda\times g_{kl\mu\nu\ldots}\right) \\ &\equiv G_{kl\mu\nu\ldots}+\left(\Lambda\times g_{kl\mu\nu\ldots}\right) \\ &\equiv R_{kl\mu\nu\ldots}-\underline{E}_{kl\mu\nu\ldots} \\ &\equiv a_{kl\mu\nu\ldots}+b_{kl\mu\nu\ldots} \\ &\equiv H\times g_{kl\mu\nu\ldots}\equiv H_{kl\mu\nu\ldots} \\ &\equiv E\times g_{kl\mu\nu\ldots} \end{aligned} \tag{518}$$

Definition 5.52 **(The tensor of non-energy $\underline{\mathbf{E}}_{\mu\nu}$).** *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of non-energy or the anti tensor of the stress energy tensor is defined/derived/determined as follows:*

$$\begin{aligned} \underline{E}_{\mu\nu} &\equiv R_{\mu\nu}-\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-\left(\Lambda\times g_{\mu\nu}\right) \\ &\equiv \left(\left(\frac{R}{2}-\Lambda\right)\times g_{\mu\nu}\right) \\ &\equiv c_{\mu\nu}+d_{\mu\nu} \\ &\equiv \Psi\times g_{\mu\nu}\equiv\Psi_{\mu\nu} \\ &\equiv \underline{E}\times g_{\mu\nu} \end{aligned} \tag{519}$$

Definition 5.53 (**The 4-index D dimensional tensor of non-energy $\underline{\mathbf{E}}_{\mathbf{kl}\mu\nu}$**). *The 4-index D dimensional tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) of non-energy $\underline{E}_{kl\mu\nu}$ is defined as follows:*

$$\begin{aligned}
\underline{E}_{kl\mu\nu} &\equiv \left(\frac{R}{D} \times g_{kl\mu\nu}\right) - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{kl\mu\nu}\right) \\
&\equiv \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right) - \left(\Lambda \times g_{kl\mu\nu}\right) \\
&\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{kl\mu\nu}\right) \\
&\equiv c_{kl\mu\nu} + d_{kl\mu\nu} \\
&\equiv \Psi \times g_{kl\mu\nu} \equiv \Psi_{kl\mu\nu} \\
&\equiv \underline{E} \times g_{kl\mu\nu}
\end{aligned} \tag{520}$$

Definition 5.54 (**The n-th index D dimensional tensor of non-energy $\underline{\mathbf{E}}_{\mathbf{kl}\mu\nu\ldots}$**). *The n-th index D dimensional tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) of non-energy $\underline{E}_{kl\mu\nu\ldots}$ is defined as follows:*

$$\begin{aligned}
\underline{E}_{kl\mu\nu\ldots} &\equiv \left(\frac{R}{D} \times g_{kl\mu\nu\ldots}\right) - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right) - \left(\Lambda \times g_{kl\mu\nu\ldots}\right) \\
&\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv c_{kl\mu\nu\ldots} + d_{kl\mu\nu\ldots} \\
&\equiv \Psi \times g_{kl\mu\nu\ldots} \equiv \Psi_{kl\mu\nu\ldots} \\
&\equiv \underline{E} \times g_{kl\mu\nu\ldots}
\end{aligned} \tag{521}$$

Definition 5.55 (**The 4-index D dimensional Einstein's curvature tensor $\mathbf{G}_{\mathbf{kl}\mu\nu}$**). *The Riemann tensor $R_{kl\mu\nu}$ does not appear explicitly in Einstein's gravitational field equations. Therefore, the question is justified whether Einstein's equation of gravitation are really the most general equations. Frèdèric Moulin proposed in the year 2017 a kind of a generalized 4-index gravitational field equation which contains the Riemann curvature tensor linearly (Moulin, 2017). Moulin himself ascribed an energy-momentum to the gravitational field itself (Moulin, 2017, p. 5/8) which is not without problems. Besides of all, it is known that the Riemann curvature tensor of general relativity $R_{kl\mu\nu}$ can be split into different ways, including the Weyl conformal tensor $C_{kl\mu\nu}$ and the anti-Weyl conformal tensor $\underline{C}_{kl\mu\nu}$ or in other words the parts which involve only the Ricci tensor $R_{\mu\nu}$ the curvature scalar R. Because of these properties $\left(R_{kl\mu\nu} \equiv C_{kl\mu\nu} + \underline{C}_{kl\mu\nu}\right)$ it is possible to reformulate the famous Einstein equation. The 4-index D dimensional Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) denoted by $G_{kl\mu\nu}$ is defined (see Barukčić, 2020b) as follows:*

$$
\begin{aligned}
G_{kl\mu\nu} &\equiv R_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right) \\
&\equiv \left(\frac{R}{D}\right) \times g_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right) \\
&\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \times g_{kl\mu\nu} \\
&\equiv a_{kl\mu\nu} + c_{kl\mu\nu} \\
&\equiv G \times g_{kl\mu\nu}
\end{aligned}
\tag{522}
$$

Definition 5.56 (The n-index D dimensional Einstein's curvature tensor $\mathbf{G}_{kl\mu\nu\ldots}$). *The n-index D dimensional Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) denoted by $G_{kl\mu\nu\ldots}$ is defined (see Barukčić, 2020b) as follows:*

$$
\begin{aligned}
G_{kl\mu\nu\ldots} &\equiv R_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv \left(\frac{R}{D}\right) \times g_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right) \\
&\equiv \left(\left(\frac{R}{D}\right) - \frac{R}{2}\right) \times g_{kl\mu\nu\ldots} \\
&\equiv a_{kl\mu\nu\ldots} + c_{kl\mu\nu\ldots} \\
&\equiv G \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{523}
$$

Definition 5.57 (The 4-index D dimensional anti Einstein's curvature tensor or the tensor or non-curvature $\underline{\mathbf{G}}_{kl\mu\nu}$). *The 4-index D dimensional anti Einstein's curvature tensor (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) or the tensor of non-curvature denoted as $\underline{G}_{kl\mu\nu}$ is defined/derived/determined (Barukčić, 2020b) as follows:*

$$
\begin{aligned}
\underline{G}_{kl\mu\nu} &\equiv R_{kl\mu\nu} - G_{kl\mu\nu} \\
&\equiv R_{kl\mu\nu} - \left(R_{kl\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu}\right)\right) \\
&\equiv \left(\frac{R}{2}\right) \times g_{kl\mu\nu} \\
&\equiv b_{kl\mu\nu} + d_{kl\mu\nu} \\
&\equiv \underline{G} \times g_{kl\mu\nu}
\end{aligned}
\tag{524}
$$

Definition 5.58 (The n-index D dimensional anti Einstein's curvature tensor or the tensor of non-curvature $\underline{\mathbf{G}}_{kl\mu\nu\ldots}$). *The n-index D dimensional anti Einstein's curvature tensor or the tensor of non-curvature denoted as $\underline{G}_{kl\mu\nu\ldots}$ is defined/derived/determined (Barukčić, 2020b) as follows:*

$$
\begin{aligned}
\underline{G}_{kl\mu\nu\ldots} &\equiv R_{kl\mu\nu\ldots} - G_{kl\mu\nu\ldots}\\
&\equiv R_{kl\mu\nu\ldots} - \left(R_{kl\mu\nu\ldots} - \left(\left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\right)\right)\\
&\equiv \left(\frac{R}{2}\right) \times g_{kl\mu\nu\ldots}\\
&\equiv b_{kl\mu\nu\ldots} + d_{kl\mu\nu\ldots}\\
&\equiv \underline{G} \times g_{kl\mu\nu\ldots}
\end{aligned}
\tag{525}
$$

Definition 5.59 (**The first quadratic Lorentz invariant F_1**). *The inner product of Faraday's electromagnetic field strength tensor yields a Lorentz invariant. The Lorentz invariant does not change from one frame of reference to another. The first quadratic Lorentz invariant, denoted as F_1 is determined as*

$$F_1 \equiv F_{kl} \times F^{kl} \tag{526}$$

The electromagnetic field tensor F_{kl} has two Lorentz invariant quantities. One of the two fundamental Lorentz invariant quantities of the electromagnetic field (Escobar and Urrutia, 2014) is known be $F_{kl} \times F^{kl} = 2 \times \left(B^2 - E^2\right)$ where E denotes the electric E and B the magnetic field in the taken frame of reference.

Definition 5.60 (**The second quadratic Lorentz invariant F_2**). *The second quadratic Lorentz invariant, denoted as F_2 is determined as*

$$F_2 \equiv \varepsilon^{klmn} \times F_{kl} \times F_{mn} \tag{527}$$

Definition 5.61 (**The tensor $b_{\mu\nu}$**). *The co-variant Minkowski's stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two and its components can be displayed by a 4 × 4 matrix too. The trace of energy-momentum tensor of the electromagnetic field is known to be null. Under conditions of Einstein's general theory of relativity (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932), the tensor $b_{\mu\nu}$ denotes the trace-less, symmetric stress-energy tensor for source-free electromagnetic field is defined in cgs-Gaussian units (**depending upon metric signature**) as*

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu}{}^{c}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{528}$$

(see Lehmkuhl, 2011, p. 13) and equally as

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{529}$$

(see Hughston and Tod, 1990, p. 38) [6] *. The co-variant Minkowski's stress-energy tensor of the electromagnetic field is expressed under conditions of D = 4 space-time dimensions more compactly in a coordinate-independent (theorem 3.1, equation 80 Barukčić, 2020b, p. 157) form as*

$$\begin{aligned} b_{\mu\nu} &\equiv \left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\, c}\times F_{\nu\, d}\times g^{cd}\right)+\left(\frac{1}{4}\times g_{\mu\nu}\times F_{de}\times F^{de}\right)\right)\right) \\ &\equiv \left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\, c}\times F^{\mu\, c}\right)+\left(\frac{F_I}{4}\right)\right)\right)\times g_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{D}\right)-a-c-d\right)\times g_{\mu\nu} \\ &\equiv (E-a)\times g_{\mu\nu} \\ &\equiv b\times g_{\mu\nu} \end{aligned} \tag{530}$$

where F_{de} is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 5.62 (The 4-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu}$). *The 4-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu}$ is defined as:*

$$\begin{aligned} b_{kl\mu\nu} &\equiv \left(\left(\frac{R}{D}\right)-a-c-d\right)\times g_{kl\mu\nu} \\ &\equiv (E-a)\times g_{kl\mu\nu} \\ &\equiv b\times g_{kl\mu\nu} \end{aligned} \tag{531}$$

Definition 5.63 (The n-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu\ldots}$). *The n-index D dimensional stress-energy tensor of electromagnetic field $b_{kl\mu\nu\ldots}$ is defined as:*

$$\begin{aligned} b_{kl\mu\nu\ldots} &\equiv \left(\left(\frac{R}{D}\right)-a-c-d\right)\times g_{kl\mu\nu\ldots} \\ &\equiv (E-a)\times g_{kl\mu\nu\ldots} \\ &\equiv b\times g_{kl\mu\nu\ldots} \end{aligned} \tag{532}$$

Definition 5.64 (The tensor $c_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of non-momentum and curvature is defined/derived/determined (Barukčić, 2020b) as follows:*

$$\begin{aligned} c_{\mu\nu} &\equiv b_{\mu\nu}-\left(\Lambda\times g_{\mu\nu}\right) \\ &\equiv (G-a)\times g_{\mu\nu} \\ &\equiv \left(\frac{R}{2}-\Lambda-d\right)\times g_{\mu\nu} \\ &\equiv (b-\Lambda)\times g_{\mu\nu} \\ &\equiv c\times g_{\mu\nu} \end{aligned} \tag{533}$$

[6] L. P. Hughston and K. P. Tod. An introduction to general relativity. Cambridge University Press, Cambridge ; New York, 1990. ISBN 978-0-521-32705-3.

Definition 5.65 (**The 4-index D dimensional tensor $c_{kl\mu\nu}$**). *The 4-index D dimensional $c_{kl\mu\nu}$ is defined as:*

$$\begin{aligned} c_{kl\mu\nu} &\equiv (G-a) \times g_{kl\mu\nu} \\ &\equiv \left(\frac{R}{2} - \Lambda - d\right) \times g_{kl\mu\nu} \\ &\equiv (b-\Lambda) \times g_{kl\mu\nu} \\ &\equiv c \times g_{kl\mu\nu} \end{aligned} \tag{534}$$

Definition 5.66 (**The n-index D dimensional tensor $c_{kl\mu\nu\ldots}$**). *The n-index D dimensional $c_{kl\mu\nu\ldots}$ is defined as:*

$$\begin{aligned} c_{kl\mu\nu\ldots} &\equiv (G-a) \times g_{kl\mu\nu\ldots} \\ &\equiv \left(\frac{R}{2} - \Lambda - d\right) \times g_{kl\mu\nu\ldots} \\ &\equiv (b-\Lambda) \times g_{kl\mu\nu\ldots} \\ &\equiv c \times g_{kl\mu\nu\ldots} \end{aligned} \tag{535}$$

Definition 5.67 (**The tensor of neither curvature nor momentum $d_{\mu\nu}$**). *Under conditions of Einstein's general (Einstein, 1915, 1916, 1917, 1935, Einstein and Sitter, 1932) theory of relativity, the tensor of neither curvature nor momentum is defined/derived/determined (Barukčić, 2020b) as follows:*

$$\begin{aligned} d_{\mu\nu} &\equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - c_{\mu\nu} \\ &\equiv \left(\frac{\left(\left(\frac{R}{D}\right) \times D\right)}{2} - b\right) \times g_{\mu\nu} \\ &\equiv \left(\frac{\left(\left(\frac{R}{D}\right) \times D\right)}{2} - \Lambda - c\right) \times g_{\mu\nu} \\ &\equiv \frac{R}{D} \times {}_{gw}g_{\mu\nu} \\ &\equiv d \times g_{\mu\nu} \end{aligned} \tag{536}$$

There may exist circumstances where this tensor might indicate something like the density of gravitational waves. In detail, it is

$$d_{\mu\nu} \equiv \frac{R}{D} \times {}_{gw}g_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd}\right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{537}$$

Under these circumstances, the metric tensor of the gravitational waves ${}_{gw}g_{\mu\nu}$ would follow as

$$ {}_{d}g_{\mu\nu} \equiv {}_{gw}g_{\mu\nu} \equiv \frac{D}{R} \times \left(\left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd} \right) + \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de} \right) \right) \right) \right) \tag{538} $$

The cosmic microwave background (CMBR) radiation (Penzias and Wilson, 1965) is an electromagnetic radiation which is part of the tensor $b_{\mu\nu}$.

Definition 5.68 (The 4-index D dimensional d $_{kl\mu\nu}$). *The 4-index D dimensional d $_{kl\mu\nu}$ is defined as:*

$$ \begin{aligned} d_{kl\mu\nu} &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - b \right) \times g_{kl\mu\nu} \\ &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - \Lambda - c \right) \times g_{kl\mu\nu} \\ &\equiv d \times g_{kl\mu\nu} \end{aligned} \tag{539} $$

Definition 5.69 (The n-index D dimensional d $_{kl\mu\nu\,\ldots}$). *The n-index D dimensional d $_{kl\mu\nu\,\ldots}$ is defined as:*

$$ \begin{aligned} d_{kl\mu\nu\,\ldots} &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - b \right) \times g_{kl\mu\nu\,\ldots} \\ &\equiv \left(\frac{\left(\left(\frac{R}{D} \right) \times D \right)}{2} - \Lambda - c \right) \times g_{kl\mu\nu\,\ldots} \\ &\equiv d \times g_{kl\mu\nu\,\ldots} \end{aligned} \tag{540} $$

5.5. Axioms

Whether science needs new and obviously generally valid statements (axioms) which are able to assure the truth of theorems proved from them may remain an unanswered question. In order to be accepted, a new axiom candidate (see Easwaran, 2008) should be at least as simple as possible and logically consistent to enable advances in our knowledge of nature. The importance of axioms is particularly emphasized by Albert Einstein. "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**." (see Einstein, 1919, p. 17). In general, lex identitatis, lex contradictionis and lex negationis have the potential to denote the most simple, the most general and the most far-reaching axioms of science, the foundation of our today's and of our future scientific inquiry.

5.5.1. Principium identitatis (Axiom I)

Principium identitatis or **lex identitatis** or axiom I, is closely related to central problems of metaphysics, epistemology and of science as such. It turns out that it is more than rightful to assume that

$$+1 \equiv +1 \tag{541}$$

is true, otherwise there is every good reason to suppose that nothing can be discovered at all.

Identity as the epitome of a self-identical or of self-reference is at the same time different from difference, identity is free from difference, identity is not difference, identity is at the same time the other of itself, identity is non-identity. Identity as simple equality with itself is determined by a non-being, by a non-being of its own other, by a non-being of difference, identity is different from difference. Identity is in its very own nature different and is in its own self the opposite of itself (symmetry). It is equally

$$-1 \equiv -1 \tag{542}$$

In general, +1 and -1 are distinguished, however these distinct are related to one and the same 1. Identity as a vanishing of otherness, therefore, is this distinguishedness in one relation. It is

$$0 \equiv +1 - 1 \equiv 0 \times 1 \equiv 0 \tag{543}$$

Identity, as the unity of something and its own other is in its own self a separation from difference, and as a moment of separation might pass over into an equivalence relation which itself is reflexive, symmetric and transitive. Nonetheless, backed by thousands of years of often bitter human experience, the scientific development has taught us all that human knowledge is relative too. Even if experiments and other suitable proofs are of help to encourage us more and more in our belief of the correctness of a theory, it is difficult to prove the correctness of a theorem or of a theory et cetera once and for all. The challenge for all the science is the need to comply with Einstein's position: **"Niemals aber kann die Wahrheit einer Theorie erwiesen werden. Denn niemals weiß man, daß auch in Zukunft eine Erfahrung bekannt werden wird, die Ihren Folgerungen widerspricht..."** (Einstein, 1919).

Albert Einstein's position translated into English: 'But the truth of a theory can never be proven. For one never knows if future experience will contradict its conclusion; and furthermore, there are always other conceptual systems imaginable which might coordinate the very same facts.'Our human experience tells us that everything in life is more or less transitory, and that nothing lasts. As a result of our knowledge and experience, several scientific theories have a glorious past to look back on, but all the glory of such scientific theories might remain in the past if scientist don't continue to innovate. In a word, theories can be refuted by time.

"No amount of experimentation can ever prove me right;
a single experiment can prove me wrong."

(Albert Einstein according to: Robertson, 1998, p. 114)

In the light of the foregoing, it is clear that appropriate axioms and conclusions derived from the same are a main logical foundation of any 'theory'.

"**Grundgesetz (Axiome)** und **Folgerungen** zusammen bilden das was man **eine 'Theorie'** nennt. "

(Einstein, 1919)

However, another point is worth being considered again. One single experiment can be enough to refute a whole theory. Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, an axiom as a free creation of the human mind which precedes all science should be like all other axioms, as simple as possible and as self-evident as possible. Historically, the earliest documented use of **the law of identity** can be found in Plato's dialogue Theaetetus (185a) as "... each of the two is different from the other and the same as itself "[7] . However, Aristotle (384–322 B.C.E.), Plato's pupil and equally one of the greatest philosophers of all time, elaborated on the law of identity too. In Metaphysica, Aristotle wrote:

"... all things ... have some unity and identity. "

(see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica, Chapter IV, 999a, 25-30, p. 66)

[7]Plato's dialogue Theaetetus (185a), p. 104.

In Prior Analytics, [8], [9] Aristotle, a tutor of Alexander, the thirteen-year-old son of Philip, the king of Macedon, is writing: "When A applies to the whole of B and of C, and is other predicated of nothing else, and B also applies to all C, A and B must be convertible. For since A is stated only of B and C, and B is predicated both of itself and of C, it is evident that B will also be stated of all subjects of which A is stated, except A itself. "[10], [11] For the sake of completeness, it should be noted at the outset that Aristotle himself preferred **the law of contradiction** and **the law of excluded middle** as examples of fundamental axioms. Nonetheless, it is worth noting that **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally the most simple axiom of science but has been treated by Aristotle in an inadequate manner without having any clear and determined meaning for Aristotle himself. Nonetheless, something which is really just itself is equally different from everything else. In point of fact, is such an equivalence (Degen, 1741) which everything has to itself inherent or must the same be constructed by human mind and consciousness. Can and how can something be **identical with itself** (Förster and Melamed, 2012, Hegel, Georg Wilhelm Friedrich, 1812a, Koch, 1999, Newstadt, 2015) and in the same respect different from itself. An increasingly popular view on identity is the one advocated by Gottfried Wilhelm Leibniz (1646-1716):

"Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra
A est A,
B est B. "
(Leibniz, 1765, p. 327)

or **A = A**, **B = B** or **+1 = +1**. In other words, a thing is what it is (Leibniz, 1765, p. 327). Leibniz' **principium identitatis indiscernibilium** (p.i.i.), the principle of the indistinguishable, occupies a central position in Leibniz' logic and metaphysics and was formulated by Leibniz himself in different ways in different passages (1663, 1686, 1704, 1715/16). All in all, Leibniz writes:

"C'est
le principe des indiscernables,
en vertu duquel
il ne saurait exister dans la nature deux êtres identiques.
...
Il n'y a point deux individus indiscernables. "
(see Leibniz, Gottfried Wilhelm, 1886, p. 45)

Exactly in complete compliance with Leibniz, Johann Gottlieb Fichte (1762 - 1814) elaborates on this subject as follows:

[8] Aristotle, Prior Analytics, Book II, Part 22, 68a
[9] Kenneth T. Barnes. Aristotle on Identity and Its Problems. Phronesis. Vol. 22, No. 1 (1977), pp. 48-62 (15 pages)
[10] Aristotle, Prior Analytics, Book II, Part 22, 68a, p. 511.
[11] Ivo Thomas. On a passage of Aristotle. Notre Dame J. Formal Logic 15(2): 347-348 (April 1974). DOI: 10.1305/ndjfl/1093891315

"Each thing is what it is ;
it has those realities which are posited when it is posited,
(A = A.) "
(Fichte, 1889)

Georg Wilhelm Friedrich Hegel (1770 – 1831) himself objected the Law of Identity by claiming that "A = A is ... an empty tautology. "(see Hegel, Georg Wilhelm Friedrich, 1991, p. 413) provided an example of his own mechanical understanding of the Law of Identity. "the empty tautology: nothing is nothing; ... from nothing only nothing becomes ... nothing remains nothing. "(see Hegel, Georg Wilhelm Friedrich, 1991, p. 84). Nonetheless, Hegel preferred to reformulate an own version of Leibniz principium identitatis indiscernibilium in his own way by writing that "All things are different, or: there are no two things like each other. "(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422). Much of the debate about identity is still a matter of controversy. This issue has attracted the attention of many authors and has been discussed by Hegel too. In this context, it is worth to consider Hegel's radical position on identity.

"The other expression of the law of identity: A cannot at the same time be A and not-A, has a negative form; it is called
the law of contradiction. "
(Hegel, Georg Wilhelm Friedrich, 1991, p. 416)

We may, usefully (see Barukčić, 2019), state Russell's position with respect to the identity law as mentioned in his book 'The problems of philosophy ' (see Russell, 1912). In particular, according to Russell,

"...principles have been singled out by tradition under the name of 'Laws of Thought.' They are as follows:
(1) **The law of identity**: 'Whatever is,is.
(2)**The law of contradiction**: 'Nothing can both be and not be.'
(3) The law of excluded middle: 'Everything must either be or not be.'
These three laws are samples of self-evident logical principles, but are not really more fundamental or more self-evident than various other similar principles: for instance, the one we considered just now, which states that what follows from a true premise is true. The name 'laws of thought' is also misleading, for what is important is not the fact that we think in accordance with these laws, but the fact that **things behave in accordance with them**; "

(see Russell, 1912, p. 113)

Russell's critique, that we tend too much to focus only on the formal aspects of the 'Laws of Thoughts' with the consequence that "... we thing in accordance with these laws" (see Russell, 1912, p. 113) is justified. Judged solely in terms of this aspect, it is of course necessary to think in accordance with the 'Laws of Thoughts'. But this is not the only aspect of the 'Laws of Thoughts'. The other and may be much more important aspect of these 'Laws of Thoughts'is the fact that quantum mechanical objects or that "... things behave in accordance with them" (see Russell, 1912, p. 113).

5.5.2. Principium contradictionis (Axiom II)

Principium contradictionis or **lex contradictionis** [12], [13], [14] or axiom II, the other of lex identitatis, the negative of lex identitatis, the opposite of lex identitatis, a complementary of lex identitatis, can be expressed mathematically as

$$+0 \equiv 0 \times 1 \equiv +1 \tag{544}$$

In addition to the above, from the point of view of mathematics, axiom II (equation 544) is equally the most simple mathematical expression and formulation of a contradiction. However, there is too much practical and theoretical evidence that a lot of 'secured'mathematical knowledge and rules differ too generously from real world processes, and the question may be asked whether mathematical truths can be treated as absolute truths at all. Many of the basic principle of today's mathematics allow every single author defining the real world events and processes et cetera in a way as everyone likes it for himself. Consequentially, a resulting dogmatic epistemological subjectivism and at the end agnosticism too, after all, is one of the reasons why we should rightly heed the following words of wisdom of Albert Einstein.

> **"I don't believe in mathematics."**
>
> (Albert Einstein cited according to Brian, 1996, p. 76)

In the long term, however, the above attitude of mathematics is not sustainable. History has taught us time and time again that objective reality has the potential to correct wrong human thinking slowly but surely, and many more than this. Objective reality has demonstrably corrected wrong human thinking again and again in the past.

[12] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

[13] Barukčić I. Aristotle's law of contradiction and Einstein's special theory of relativity. Journal of Drug Delivery and Therapeutics (JDDT). 15Mar.2019;9(2):125-43. https://jddtonline.info/index.php/jddt/article/view/2389

[14] Barukčić, Ilija. (2020, December 28). The contradiction is exsiting objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

Despite all the adversities, it is necessary and crucial to consider that a self-identical as the opposite of itself is no longer only self-identity but a difference of itself from itself within itself. In other words, “All things are different, or: there are no two things like each other ... is, in fact, opposed to the law of identity ...”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422) Each on its own and without any respect to the other is distinctive within itself and from itself and not only from another. As the opposite of its own something, is no longer only self-identity, but also a negation of itself out of itself and therefore a difference of itself from itself within itself. In other words, in opposition, a self-identical is able to return into simple unity with itself, with the consequence that even as a self-identical the same self-identical is inherently self-contradictory. A question of fundamental theoretical importance is, however, why should something be itself and at the same time the other of itself, the opposite of itself, not itself? Is something like this even possible at all and if so, why and how? These and similar questions have occupied many thinkers, including Hegel.

> “Something is therefore
> alive only in so far as it contains contradiction within it,
> and moreover is this power to
> hold and endure the contradiction within it. ”
>
> (see Hegel, Georg Wilhelm Friedrich, 1991, p. 440)

However, as directed against identity, contradiction itself is also at the same time a source of self-changes of a self-identical out of itself.

> “... contradiction
> is the root of all movement and vitality;
> it is only in so far as something has a contradiction within it
> that it moves, has an urge and activity. ”
>
> (see Hegel, Georg Wilhelm Friedrich, 1991, p. 439)

The further advance of science will throw any contribution to scientific progress of each of us back into scientific insignificance, as long as principium contradictionis is not given enough and the right attention. **The contradiction [15] is existing objectively and real and is the heartbeat of every self-identical.** We have reason to be delighted by the fact that very different aspects of principium contradictionis have been examined since centuries from different angles by various authors. According to Aristotle, principium contradictionis applies to everything that is, it is the first and the firmest of all principles of philosophy.

[15] Barukčić, Ilija. (2020, December 28). The contradiction is existing objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

> "... the same ... cannot at the same time belong and not belong to the same
> ... in the same respect ... This, then, is
> the most certain of all principles "
>
> (see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaph., IV, 3, 1005b, 16–22)

Principium contradictionis or axiom II has many facets. As long as we follow Leibniz in this regard, we should consider that "**Le principe de contradiction est en general ...** "(Leibniz, 1765, p. 327). Scientist inevitably have false beliefs and make mistakes. In order to prevent scientific results from falling into logical inconsistency or logical absurdity, it is necessary to posses among other the methodological possibility to start a reasoning with a (logical) contradiction too. However and in contrast to the way of reasoning with inconsistent premises as proposed by para-consistent (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) and other logic, in the absence of technical and other errors of reasoning, the contradiction itself need to be preserved. In other words, **from a contradiction does not anything follows but the contradiction itself** while the theoretical question is indeed justified "What is so Bad about Contradictions? " (Priest, 1998). Historically, **the principle of (deductive) explosion** (Carnielli and Marcos, 2001, Priest, 1998, Priest et al., 1989), coined by 12th-century French philosopher William of Soissons, demand us to accept that anything, including its own negation, can be proven or can be inferred from a contradiction. In short, according to **ex falso sequitur quodlibet**, a (logical) contradiction implies anything. Respecting the principle of explosion, the existence of a contradiction (or the existence of logical inconsistency) in a scientific theorem, rule et cetera is disastrous. However, the historical development of science shows that scientist inevitably revise the theories, false positions and claims are identified once and again, and we all make different kind of mistakes. In order to avert disproportionately great damage to science and to prevent reducing science into pure subjective belief, a negation of the principle of explosion is required. Nonetheless, a justified negation of the **ex contradictione quodlibet principle** (Carnielli and Marcos, 2001) does not imply the correctness of para consistent logic (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) as such as advocated especially by the Peruvian philosopher Francisco Miró Quesada (Quesada, 1977) and other (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989). In general, scientific theories appear to progress from lower and simpler to higher and more complex levels. However, high level theories cannot be taken for granted because high level theories are grounded on a lot of assumptions, definitions and other procedures and may rest upon too much erroneous stuff even if still not identified. Therefore, it should be considered to check at lower at simpler levels like with like.

5.5.2.1. Zero power zero

Theorem 37 (Erroneous operation zero power zero)**.** *In general, the relationship*

$$+0^{+2} \equiv +0 \tag{545}$$

is false.

Proof by modus inversus. The premise

$$+0 \equiv +1 \tag{546}$$

is false. In the following, any rearrangement of the premise which is free of (technical) errors, need to end up at a contradiction. In other words, the contradiction will be preserved. We obtain

$$+0 \times +0 \equiv +1 \times +0 \tag{547}$$

Equation 547 becomes

$$+0^{+2} \equiv +0 \tag{548}$$

□

5.5.2.2. Zero divided by zero

Theorem 38 (Erroneous division by zero)**.** *In general, the relationship*

$$\frac{1}{0} \equiv \frac{0}{0} \tag{549}$$

is false.

Proof by modus inversus. If the premise

$$+1 \equiv +0 \tag{550}$$

is false, then the relationship

$$\frac{1}{0} \equiv \frac{0}{0} \tag{551}$$

is also false.

□

5.5.3. Principium negationis (Axiom III)

Lex negationis or axiom III, is often mismatched with simple opposition. However, from the point of view of philosophy and other sciences, identity, contradiction, negation and similar notions are equally mathematical descriptions of the most simple laws of objective reality. What sort of natural process is negation at the end? Mathematically, we define principium negationis or lex negationis or axiom III as

$$\text{Negation}(0) \;\times 0 \equiv \neg(0) \times 0 \equiv +1 \tag{552}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, Fritz H., 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990). In this context, there is some evidence that

$$\text{Negation}(1) \;\times 1 \equiv \neg(1) \times 1 = 0 \tag{553}$$

Logically, it follows that

$$\text{Negation}(1) \;\equiv 0 \tag{554}$$

In the following we assume that axiom I is universal. Under this assumption, the following theorem follows inevitably.

Theorem 39 (Zero divided by zero)**.** *According to classical logic, it is*

$$\frac{0}{0} \equiv 1 \tag{555}$$

Proof by direct proof. The premise

$$1 \equiv 1 \tag{556}$$

is true. It follows that

$$\begin{aligned} 0 &\equiv 0 \\ &\equiv 0 \times 1 \end{aligned} \tag{557}$$

In the following, we rearrange the premise (see equation 552, p. 138). We obtain

$$0 \times (\text{Negation}(0) \;\times 0) \equiv 0 \tag{558}$$

Equation 558 changes slightly (see equation 553, p. 138). It is

$$(\text{Negation}(1) \;\times 1) \times (\text{Negation}(0) \;\times 0) \equiv 0 \tag{559}$$

Equation 559 demands that

$$(\text{Negation}(1)\;) \times (\text{Negation}(0)\;) \times 0 \equiv 0 \tag{560}$$

Equation 560 is logically possible (see equation 543, p. 130) only if

$$(\text{Negation}(1)\ ) \times (\text{Negation}(0)\ ) \equiv 1 \tag{561}$$

(see theorem 37, equation 545) whatever the meaning of Negation(1) or of Negation(0) might be, equation 561 demands that

$$\text{Negation}(0) \ \equiv \frac{1}{\text{Negation}(1)} \tag{562}$$

and that

$$\text{Negation}(1) \equiv \frac{1}{\text{Negation}(0)} \tag{563}$$

Equation 562 simplifies as (see equation 554, p. 138)

$$\begin{aligned} \text{Negation}(0) \ &\equiv \frac{+1}{\text{Negation}(1)} \\ &\equiv \frac{+1}{+0} \end{aligned} \tag{564}$$

It follows that

$$\neg(0) \times 0 \equiv \frac{1}{0} \times 0 \equiv \frac{0}{0} \equiv 1 \tag{565}$$

To bring it to the point. Classical logic, assumed as generally valid, demands that

$$\frac{0}{0} \equiv 1 \tag{566}$$

□

Concepts like identity, difference, negation, opposition et cetera engaged the attention of scholars at least over the last twenty-three centuries (see also Horn, 1989, Speranza and Horn, 2010). As long as we first and foremost follow Josiah Royce, negatio or negation "is one of the simplest and most fundamental relations known to the human mind. For the study of logic, no more important and fruitful relation is known." (see also Royce, 1917, p. 265) But, do we really know what, for sure, what negation is? Based on what we know about negation, Aristotle (see also Wedin, 1990) has been one of the first to present a theory of negation, which can be found in discontinuous chunks in his works the Metaphysics, the Categories, De Interpretatione, and the Prior Analytics (see also Horn, 1989, p. 1). Negation (see also Newstadt, 2015) as a fundamental philosophical concept found its own very special melting point especially in Hegel's dialectic and is more than just a formal logical process or operation which converts true to false or false to true. Negation as such is a natural process too and equally '**an engine of changes of objective reality** " (see also Barukčić, 2019). However, it remains an open question to establish a generally accepted link between this fundamental philosophical concept and an adequate counterpart in physics, mathematics and mathematical statistics et cetera. Especially the relationship between creation and conservation or creatio ex nihilio (see also Donnelly, 1970, Ehrhardt, 1950, Ford, 1983), determination and negation (see also Ayer, 1952, Hedwig, 1980, Heinemann, Fritz H., 1943, Kunen, 1987) has been discussed in science since ancient (see also Horn, 1989, Speranza and Horn, 2010) times too. Why and how does an event occur or why and how is an event created (creation), why and how does an event maintain its own existence over time (conservation)? The development of the notion of negation leads from Aristotle to Meister Eckhart (see also Eckhart, 1986) von Hochheim (1260-1328), commonly known as Meister Eckhart (see also Tsopurashvili, 2012) or Eckehart, to Spinoza (1632 – 1677), to Immanuel Kant (1724-1804) and finally to Georg Wilhelm Friedrich Hegel (1770-1831) and other authors too. One point is worth being noted, even if it does not come as a surprise, it was especially Benedict de Spinoza (1632 – 1677) as one of the philosophical founding fathers of the Age of Enlightenment who addressed the relationship between determination and negation in his lost letter of June 2, 1674 to his friend Jarig Jelles (see also Förster and Melamed, 2012) by the discovery of his fundamental insight that " **determinatio negatio est**" (see also Spinoza, 1674, p. 634). Hegel went even so far as to extended the slogan raised by Spinoza into to "Omnis determinatio est negatio" (see also Hegel, Georg Wilhelm Friedrich, 1812b, 2010, p. 87). Finally, it did not take too long, and the notion of negation entered the world of mathematics and mathematical logic at least with Boole's (see also Boole, 1854) publication in the year 1854. "Let us, for simplicity of conception, give to the symbol x the particular interpretation of men, then 1 - x will represent the class of 'not-men'." (see also Boole, 1854, p. 49). Finally, the philosophical notion negation found its own way into physics by the contributions of authors like Woldemar Voigt (see Voigt, 1887), George Francis FitzGerald (see FitzGerald, 1889), Hendrik Antoon Lorentz (see Lorentz, 1892, 1899), Joseph Larmor (see Larmor, 1897), Jules Henri Poincaré (see Poincaré, 1905) and Albert Einstein (see Einstein, 1905b) by contributions to the physical notion "Lorentz factor".

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I was born October, 1st 1961 in Novo Selo, Bosnia and Herzegovina, former Yogoslavia. I am of Croatian origin. From 1982-1989 C.E., I studied human medicine at the University of Hamburg, Germany. Meanwhile, I am working as a specialist of internal medicine. My basic field of research since my high school days at the Wirtschaftsgymnasium Bruchsal, Baden Württemberg, Germany is the mathematization of the relationship between a cause and an effect valid without any restriction under any circumstances including the conditions of classical logic, probability theory, quantum mechanics, special and general theory of relativity, human medicine et cetera. I endeavour to investigate positions of quantum mechanics, relativity theory, mathematics et cetera, only insofar as these positions put into question or endanger **the general validity of the principle of causality**.

[a]https://orcid.org/0000-0002-6988-2780
[b]https://cel.webofknowledge.com/InboundService.do?app=wos&product=CEL&Func=Frame&SrcApp=Publons&SrcAuth=Publons_CEL&locale=en-US&SID=F4r5Tsr3OcrmFbYrqiF&customersID=Publons_CEL&smartRedirect=yes&mode=FullRecord&IsProductCode=Yes&Init=Yes&action=retrieve&UT=WOS%3A000298855300006
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