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Wave particle duality

Research article

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Abstract

Background:

What are (quantum mechanical) entities, either a particle or a wave or both or none?

Methods:

The most elementary and simple rules of the special relativity theory were used to approach the solution of this matter.

Results:

The particle wave duality has been mathematised very precisely. A relativistic wave equation has been developed.

Conclusion:

Under usual circumstances, a (quantum mechanical) entity is equally both, a particle and a wave.

Keywords: Particle; Wave; Cause; Effect; Causal relationship k; Causality; Causation

1. Introduction

Despite of its important influence as a core part of contemporary physics, there is no general consensus among scientist on the question of what is quantum theory telling us about objective reality at all. Nonetheless, the profound scientific issues raised by today's quantum theory require a repeated and more in-depth treatment of several entities and notions like particle and wave and of other quantum mechanical observables et cetera. Quantum theory itself is to many times dominated by a mathematical formalism which itself has not a very profound and convincing connection to objective reality. In particular, a substantial literature has grown up around the so called 'measurement problem'while today's formal apparatus of quantum mechanics has not solved this issue in a logically consistent manner. However, with what does a possible measuring device interact, how and to what extent? By time, quantum theory is reduced more and more to a measurement problem. Against this, a quantum theory which has reduced itself only to a simple theory of measurement has abolished itself completely. What are we to make of this? In order to approach the solution of this very controversial issue, it is necessary to be clear what is being measured at all, what is the underlying principle of any measurement process and how can this be described in a logically and mathematically satisfactory way. We can not avoid and are forced to take up again the long-lasting and non-ending issue which is deeply embedded into the foundations of quantum mechanics, the relationship between a particle and a wave. Historically, authors like Democritus (5th century BCE), Euclid (4th–3rd century BCE) and many other too, elaborated on this topic, directly or indirectly. In the end, it was the discovery of wave interference of light by Thomas Young (1773–1829), an English physician, in his **double-slit experiment** [1] in 1801 (see Young, 1804) which brought the issue of a particle and a wave to a higher experimental and epistemological level. Scientific progress in this matter soon became an unstoppable force. Based on the preliminary work of Max Karl Ernst Ludwig Planck (1858-1947) and Albert Einstein (1879-1955), Prince Louis Victor Pierre Raymond, 7th Duc de Broglie (1892 – 1987), related wavelength and momentum and postulated 1924 in his PhD thesis (see de Broglie, 1924) that waves might behave as particles and particles themselves might also behave as a wave (a de Broglie wave) with wavelength h/p, where p is the momentum and h is Planck's constant. In other words, in a wave, a particle can be found and vice versa. In a particle, a wave can be found. In the following, Albert Einstein (see Einstein and Infeld, 1938, p. 263) wrote with respect to particle and waves:

> "It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do. "
>
> (see Einstein and Infeld, 1938, p. 263)

To get to the point, particles have wave properties and vice versa. Wave have particle properties. In what follows, I will touch on of view of these topics, with the main goal being to provide a solution of the issue particle wave duality. [2] The causal or hidden variables interpretation of quantum mechanics (the de Broglie-Bohm theory) as discovered by Louis de Broglie (see de Broglie, 1927) in 1927 [3] and rediscovered by David Bohm [4] (see Bohm, 1952) in 1952 will not be analysed in detail.

[1] Young, Thomas (1804). "Bakerian Lecture: Experiments and calculations relative to physical optics". Philosophical Transactions of the Royal Society. 94: 1–16. doi: https://doi.org/10.1098/rstl.1804.0001

[2] Ilija Barukčić, "The Relativistic Wave Equation," International Journal of Applied Physics and Mathematics vol. 3, no. 6, pp. 387-391, 2013.

[3] Louis de Broglie (1927) La mécanique ondulatoire et la structure atomique de la matière et du rayonnement. J. Phys. Radium 8 (5), 225-241 DOI: 10.1051/jphysrad:0192700805022500

[4] Bohm, David, 1952, "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables. I", Physical Review, 85(2): 166–193. doi:10.1103/PhysRev.85.166

2. Material and methods

2.1. Material

2.2. Methods

Avoiding and treatment of logical fallacies addresses one of the most important aspects of any scientific inquiry. Sometimes logically sound definitions are of great help in order to enable us to properly infer **from something known to the something unknown**. It also goes without any need of further saying that a definition as such should be logically consistent and correct. However, theoretically, circumstances might exist under which it is required to *deal with erroneous definitions* [5] too.

2.2.1. Basic definitions of special theory of relativity

Definition 2.1 **(Energy).**

Let E denote energy (see Einstein, 1905a) which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$E = M \times c^2 \tag{1}$$

where M is the matter and c is the speed of the light in vacuum.

Definition 2.2 **(Matter).**

Let M denote matter which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. In our understanding of the matter we follow Einstein's explanations very closely.

> "... '**Materie**'bezeichnet ... nicht nur die **'Materie'im üblichen Sinne**, sondern auch das **elektromagnetische Feld**. " (Einstein, 1916, p. 802/803)

In broken English, '**matter** denotes ... not only **matter in the ordinary sense**, but also the **electromagnetic field**. 'It is worth noting that the equivalence of matter (M) and energy (E) lies at the core of today's physics and has been described by Einstein as follows:

> "Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um L/V^2 ... Die Masse eines Körpers ist ein Maß für dessen Energieinhalt "
>
> (see also Einstein, 1905b, p. 641)

[5] Strobino, Riccardo, "Ibn Sina's Logic", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2018/entries/ibn-sina-logic/.

In general it is

$$M \equiv \frac{E}{c^2} \tag{2}$$

(see also Einstein, 1905b, p. 641)

where M denotes the matter(see also Tolman, 1912) and c is the speed of the light in vacuum. In other words, Einstein is demanding the equivalence of matter and energy as the most important upshot of his special theory of relativity.

"Eines der wichtigsten Resultate der Relativitätstheorie ist die Erkenntnis,

daß jegliche Energie E eine ihr proportionale Trägheit (E/c^2) besitzt"

(see also Einstein, 1912, p. 1062)

Definition 2.3 (**Anti energy**).

Let $\underline{E}$ denote non-energy or anti energy, the other of energy, the complementary of energy, the opposite of energy which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$\underline{E} = S - E \tag{3}$$

Definition 2.4 (**Time**).

Let t denote time, the other of anti-time, the complementary of anti - time, the opposite of anti-time which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. Let $\underline{t}$ denote anti time. It is

$$t = S - \underline{t} \tag{4}$$

Definition 2.5 (**Anti time**).

Let $\underline{t}$ denote non-time or anti-time, the other of time, the complementary of time, the opposite of time which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$\underline{t} = S - t \tag{5}$$

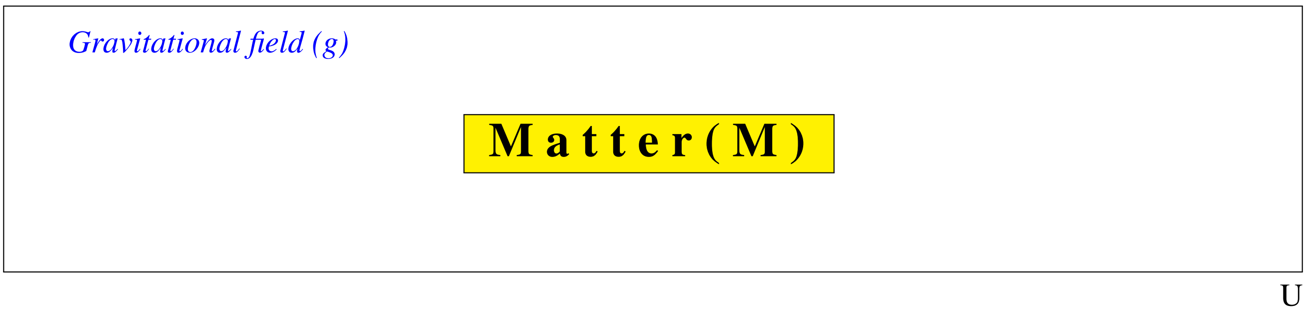
Theoretically, anti-time is the other of time, the complementary of time, the opposite of time.

Definition 2.6 (**Gravitational field**).

Let g denote the gravitational field. The gravitational field g is quite often defined by the gravitational potential. Nonetheless, it is necessary to distinguish the gravitational field and the gravitational potential, both are not identical. Even if it is a little questionable to refer so often to Einstein's position, as long as the same is logically sound, it is also very difficult to simply ignore the same. Although it is much too often overlooked today, let us again refer to Einstein's understanding of the relationship between matter and gravitational field. Einstein defined the gravitational field **ex negativo** as follows.

> "Wir unterscheiden im folgenden zwischen 'Gravitationsfeld'und 'Materie', in dem Sinne, daß **alles außer dem Gravitationsfeld als 'Materie'bezeichnet** wird, also nicht nur die 'Materie'im üblichen Sinne, sondern auch das elektromagnetische Feld. "
>
> (Einstein, 1916, p. 802/803)

Again, Einstein's position translated into English: 'We distinguish in the following between 'gravitational field'and 'matter', in the sense that everything except the gravitational field is regarded as 'matter', that is not only 'matter'in the ordinary sense, but also the electromagnetic field.'The following and only symbolic figure might illustrate the relationship between matter and gravitational field in more detail.



Mathematically, the gravitational field is expressed as follows:

$$g = U - M \tag{6}$$

Definition 2.7 (**Space**).

Let S denote the space which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. We assume that energy and time are determining space. It is

$$S = E + t \tag{7}$$

In the further progress of the research it should be possible to demonstrate beyond any reasonable doubt that

$$S - t = E \tag{8}$$

and that the most general formulation of the Einstein field equations could be

$$\left(S \times g_{\mu\nu}\right) - \left(t \times g_{\mu\nu}\right) = \left(E \times g_{\mu\nu}\right) \tag{9}$$

Definition 2.8 (U)**.**

Let U denote the unity and the struggle between matter and gravitational field which is existing objectively and real outside of human mind and consciousness as viewed from the point of view of the stationary observer R. It is

$$U = \frac{S}{c^2} = M + g \tag{10}$$

2.2.2. Extended definitions of special theory of relativity

Definition 2.9 (Energy and special theory of relativity)**.** *Let $_{r}E_{t}$ denote the total or relativistic (see Lewis and Tolman, 1909) energy [6] of an (quantum mechanical) entity, at a certain run of an experiment t and is dependent on the relative velocity v of an observer. Let $_{0}E_{t}$ denote the rest energy of an entity, at a certain run of an experiment t. The invariant mass $_{0}m_{t}$ (also called rest mass) which is determined by rest energy is an invariant quantity which is the same for all observers in all reference frames. Let $_{w}E_{t}$ denote the electromagnetic wave energy of an entity, at a certain run of an experiment t. Let $_{rp}E_{t}$ denote the relativistic potential energy (see Barukčić, 2013), let $_{rk}E_{t}$ denote the relativistic kinetic energy (see Barukčić, 2013).*

The relativistic momentum, denoted as $_{r}p_{t}$, is defined as

$$\left({}_{r}p_{t}\right) = \left({}_{r}m_{t}\right) \times (v) \tag{11}$$

where v is the relative velocity between observers. The energy of an electromagnetic wave, denoted as $_{w}E_{t}$, is derived as

$$\left({}_{w}E_{t}\right) = \left({}_{r}p_{t}\right) \times (c) = \left({}_{r}m_{t}\right) \times (v) \times (c) \tag{12}$$

where c is the speed of the light in vacuum. In general, it is

$$\left({}_{r}E_{t}\right) = \left({}_{rp}E_{t}\right) + \left({}_{rk}E_{t}\right) \tag{13}$$

and the usual energy momentum relation

$$\left(\left({}_{r}E_{t}\right) \times {}_{rp}E_{t}\right) + \left(\left({}_{r}E_{t}\right) \times {}_{rk}E_{t}\right) = \left({}_{r}E_{t}\right) \times \left({}_{r}E_{t}\right) \tag{14}$$

The invarinat or rest energy (see figure 1), denoted as $\left({}_{0}E_{t}\right)$, is given as

$$\left({}_{0}E_{t}\right)^2 = \left({}_{r}E_{t}\right) \times \left({}_{rp}E_{t}\right) \tag{15}$$

[6] Lewis, Gilbert N. and Tolman, Richard C. (1909), "The Principle of Relativity, and Non-Newtonian Mechanics", Proceedings of the American Academy of Arts and Sciences, 44 (25): 709–726.doi:10.2307/20022495

The relativistic potential energy, ${}_{\mathrm{rp}}E_{\mathrm{t}}$, is given as

$$\left({}_{\mathrm{rp}}E_{\mathrm{t}}\right) = \frac{\left({}_{0}E_{\mathrm{t}}\right)^{2}}{\left({}_{\mathrm{r}}E_{\mathrm{t}}\right)} = \left(1 - \frac{v^{2}}{c^{2}}\right) \times \left({}_{\mathrm{r}}E_{\mathrm{t}}\right) \tag{16}$$

Furthermore, the energy of a electromagnetic wave (see figure 1), denoted as $\left({}_{\mathrm{w}}E_{\mathrm{t}}\right)$, is given as

$$\left({}_{\mathrm{W}}E_{\mathrm{t}}\right)^{2} = \left({}_{\mathrm{r}}E_{\mathrm{t}}\right) \times \left({}_{\mathrm{rk}}E_{\mathrm{t}}\right) \tag{17}$$

The relativistic kinetic energy (see figure 1), denoted as ${}_{\mathrm{rk}}E_{\mathrm{t}}$, is given as

$$\left({}_{\mathrm{rk}}E_{\mathrm{t}}\right) = \frac{\left({}_{\mathrm{W}}E_{\mathrm{t}}\right)^{2}}{\left({}_{\mathrm{r}}E_{\mathrm{t}}\right)} = \frac{\left({}_{\mathrm{r}}m_{\mathrm{t}}\right) \times (v) \times (c) \times \left({}_{\mathrm{r}}m_{\mathrm{t}}\right) \times (v) \times (c)}{\left({}_{\mathrm{r}}m_{\mathrm{t}}\right) \times (c) \times (c)} = \left({}_{\mathrm{r}}m_{\mathrm{t}}\right) \times \left(v^{2}\right) = \left({}_{\mathrm{r}}p_{\mathrm{t}}\right) \times (v) \tag{18}$$

We have very convincing arguments to assume that the concept of **vis viva** (see Leibniz, 1695) as put forward by Leibniz and the notion **relativistic kinetic energy** are identical. The relationship before and their inner connection to the Pythagorean theorem are viewed by figure 1 in more detail.

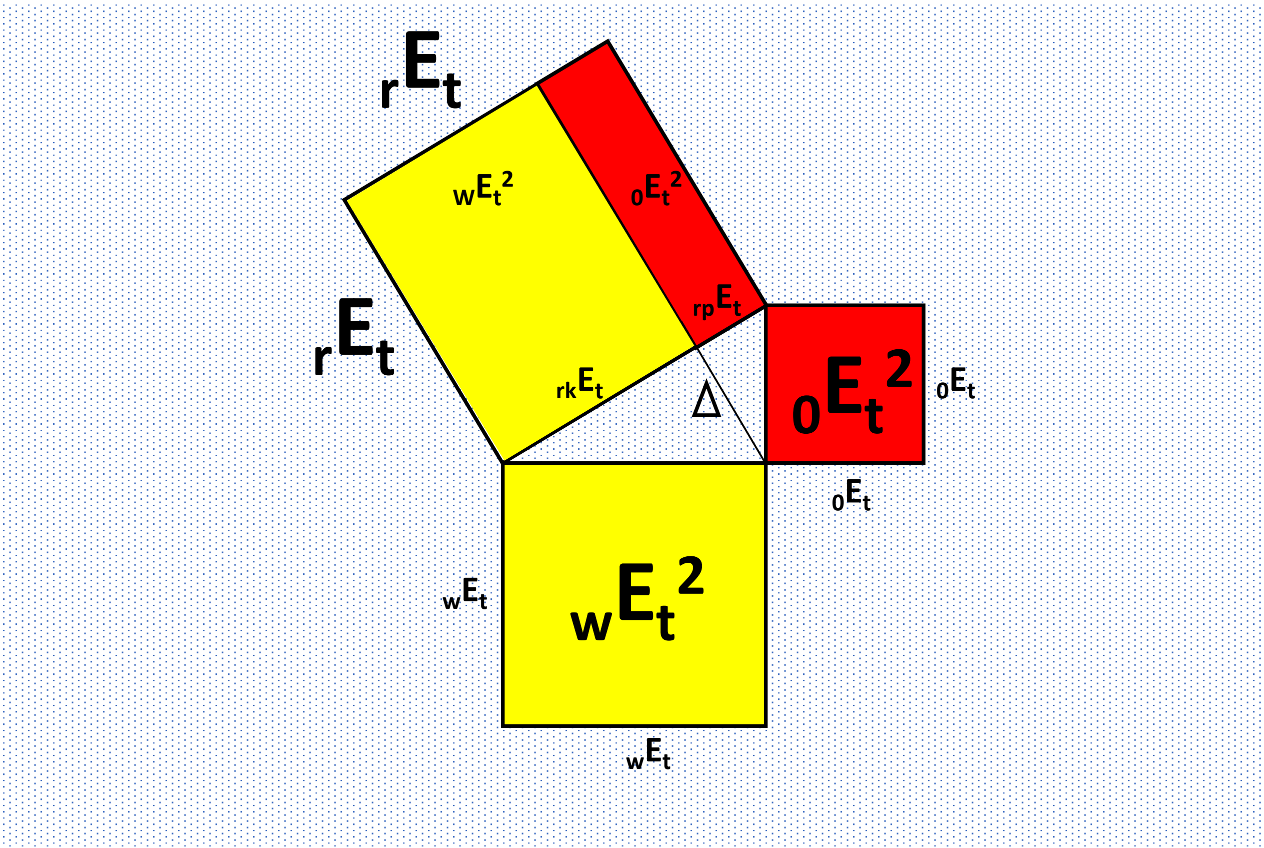


Figure 1. Pythagorean theorem and Einstein's special theory of relativity (Einstein's triangle).

The usual energy momentum relation has been the foundation of many relativistic wave equations. The normalised energy momentum relation is given as

$$\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)+\left(\frac{({}_{\mathrm{w}}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)=\left(\frac{({}_{0}m_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}m_{\mathrm{t}})^{2}}\right)+\left(\frac{(v)^{2}}{(c)^{2}}\right)=+1 \tag{19}$$

while

$$p\left({}_{\mathrm{rp}}E_{\mathrm{t}}\right)=\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)=1-\left(\frac{(v)^{2}}{(c)^{2}}\right) \tag{20}$$

can be understood as the probability of finding a certain particle local. The next figure might provide us with an simplified overview.

$W\,a\,v\,e\;({}_{w}E_{t})^{2}$

P a r t i c l e $({}_{0}E_{\mathrm{t}})^{2}$

Energy $({}_{\mathrm{r}}E_{\mathrm{t}})^{2}$

Depending upon the experimental conditions and the measuring device used, the wave energy $({}_{\mathrm{w}}E_{\mathrm{t}})$ might change, or particle's energy $({}_{0}E_{\mathrm{t}})$ might change (i. e. collision experiments in particle physics) or both et cetera. In any case, the extent of the interaction between a measuring device and an entity to be measured can be determined accurately. Nonetheless, it does not make any sense at all to assume that a measuring device is only a measuring device if the same measuring device generates by an act of measurement the entity which has to be measured. In this respect a remark to the ordinary matter and the electromagnetic field is permissible. It is

$$({}_{\mathrm{r}}E_{\mathrm{t}})=({}_{\mathrm{r}}E_{\mathrm{t}})\times(1+0)=\left(1-\sqrt[2]{\frac{v^{2}}{c^{2}}}+\sqrt[2]{\frac{v^{2}}{c^{2}}}\right)\times({}_{\mathrm{r}}E_{\mathrm{t}})=\underbrace{\left(\left(1-\sqrt[2]{\frac{v^{2}}{c^{2}}}\right)\times({}_{\mathrm{r}}E_{\mathrm{t}})\right)}_{Oridinary\ energy/matter\ {}_{\mathrm{a}}\mathrm{E}_{\mathrm{t}}}+\underbrace{\left(\left(\sqrt[2]{\frac{v^{2}}{c^{2}}}\right)\times({}_{\mathrm{r}}E_{\mathrm{t}})\right)}_{Electromagnetic\ wave\ {}_{\mathrm{w}}\mathrm{E}_{\mathrm{t}}}=({}_{\mathrm{a}}E_{\mathrm{t}})+({}_{\mathrm{w}}E_{\mathrm{t}}) \tag{21}$$

where ${}_{\mathrm{a}}E_{\mathrm{t}}$ is the energy of ordinary matter/energy and ${}_{\mathrm{w}}E_{\mathrm{t}}$ is the energy of the electromagnetic field/wave. Based on equation 21 , it is

$$({}_{\mathrm{a}}E_{\mathrm{t}})=\underbrace{\left(\left(1-\sqrt[2]{\frac{v^{2}}{c^{2}}}\right)\times({}_{\mathrm{r}}E_{\mathrm{t}})\right)}_{Oridinary\ energy/matter\ {}_{\mathrm{a}}\mathrm{E}_{\mathrm{t}}}=\left(1-\sqrt[2]{\frac{v^{2}}{c^{2}}}\right)\times h\times{}_{\mathrm{r}}f_{\mathrm{t}} \tag{22}$$

where ${}_{\mathrm{a}}E_{\mathrm{t}}$ might denote "Alltagsenergie "or ordinary energy/matter, h is Planck's constant and ${}_{\mathrm{r}}f_{\mathrm{t}}$ is the frequency. The total or relativistic energy ${}_{\mathrm{r}}E_{\mathrm{t}}$ is determined as

$${}_{\mathrm{r}}E_{\mathrm{t}}=\frac{{}_{\mathrm{a}}E_{\mathrm{t}}}{\left(1-\sqrt[2]{\frac{v^{2}}{c^{2}}}\right)} \tag{23}$$

The relationship between “rest energy ”, denoted as ${}_{0}E_{\mathrm{t}}$ and ordinary energy ${}_{\mathrm{a}}E_{\mathrm{t}}$ is given as

$$ {}_{0}E_{\mathrm{t}} = \underbrace{\left(\left(\sqrt[2]{1-\frac{v^{2}}{c^{2}}} \right) \times \left({}_{\mathrm{r}}E_{\mathrm{t}} \right) \right)}_{rest\ energy/matter\ {}_{0}\mathrm{E}_{\mathrm{t}}} = \frac{\left(\sqrt[2]{1-\frac{v^{2}}{c^{2}}} \right)}{\left(1-\sqrt[2]{\frac{v^{2}}{c^{2}}} \right)} \times {}_{\mathrm{a}}E_{\mathrm{t}} \tag{24} $$

These relationships are necessary to be considered at any measurement and especially in cosmology. Equation 22 is the natural foundation of the Doppler effect (see Doppler, 1842; Voigt, 1887) and is illustrated by the following picture in more detail.

Electromagnetic wave $({}_{w}E_{t})$

Ordinary matter/energy $({}_{\mathrm{a}}E_{\mathrm{t}})$

Energy $({}_{\mathrm{r}}E_{\mathrm{t}})$

Let us assume just for the sake of the argument that quantum theory is (in principle) a (universal) theory which is applicable (in principle) to all physical systems including our earth-moon system too. This could imply that a linear evolution of quantum states applied to macroscopic objects might routinely lead to superpositions of macroscopically distinct objects. Based on equation 21, such an underlying attitude has not simply been plucked out of the air. Normalising equation 21, it is

$$ \left(\frac{{}_{\mathrm{a}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}} \right) + \left(\frac{{}_{\mathrm{w}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}} \right) = +1 \tag{25} $$

Multiplying by the Schrödinger equation(Schrödinger, Erwin Rudolf Josef Alexander, 1926b), it is

$$ \left(\frac{{}_{\mathrm{a}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}} \times (H \times \Psi) \right) + \left(\frac{{}_{\mathrm{w}}E_{\mathrm{t}}}{{}_{\mathrm{r}}E_{\mathrm{t}}} \times (H \times \Psi) \right) = H \times \Psi \tag{26} $$

It is ${}_{\mathrm{r}}E_{\mathrm{t}} = H = i \times \hbar \times \frac{\partial}{\partial t}$, equation 26 becomes

$$ ({}_{\mathrm{a}}E_{\mathrm{t}} \times \Psi) + ({}_{\mathrm{w}}E_{\mathrm{t}} \times \Psi) = H \times \Psi \tag{27} $$

or in the quantized version

$$ \left(\left(\left(1-\sqrt[2]{\frac{v^{2}}{c^{2}}} \right) \times \left(i \times \hbar \times \frac{\partial}{\partial t} \right) \right) \times \Psi \right) + \left(\left(\left(\sqrt[2]{\frac{v^{2}}{c^{2}}} \right) \times \left(i \times \hbar \times \frac{\partial}{\partial t} \right) \right) \times \Psi \right) = H \times \Psi \tag{28} $$

Among the circumstances in which this might happen are experimental set-ups where two persons, person A like Alice and person B like Bob, are measuring the existence of our moon.

Example.

Person A (i. e. Alice) measures the moon of our earth with his own and opened eyes. At the same place and time, person B (i. e. Bob) measures the same moon of our earth with his own and closed eyes. Thus far, if only open eyes would justify the existence of the moon, person A should not be able to measure anything, because according to the opinion of person B there cannot be anything, his eyes are still closed. At the same point in space-time t both is given, the moon exists (Person A) and the moon does not exist (Person B), which is a contradiction. At this point we must ask the fundamental question for what logically obligatory reason should we have to accept that the moon exists only if we also look at the same.

> "We often discussed **his notions on objective reality**. I recall that during one walk **Einstein** suddenly stopped, turned to me and asked whether I really believed that **the moon exists only when I look at it**. "
>
> (see Pais, 1979, p. 907)

Is our moon there when nobody looks?

Figure 2. Credit: NASA, International Space Station. Heavenly Half Moon. Picture taken by a crew member aboard the International Space Station during Expedition 20.

3. Results

3.1. Theorem. Normalised energy momentum relation

Theorem 1 (Normalised energy momentum relation)**.** *The normalised relativistic energy momentum relation is given as*

$$\left(\frac{({}_0m_t)^{\,2}}{({}_rm_t)^{\,2}}\right)+\left(\frac{(v)^{\,2}}{(c)^{\,2}}\right)=+1 \tag{29}$$

Proof by direct proof. Axiom 1 or

$$+1=+1 \tag{30}$$

is true. Therefore, it is equally true that

$$({}_0m_\mathrm{t})=({}_0m_\mathrm{t}) \tag{31}$$

and that

$$({}_0m_\mathrm{t})=\left(\sqrt[2]{+1-\frac{v^2}{c^2}}\right)\times({}_\mathrm{r}m_\mathrm{t}) \tag{32}$$

Rearranging equation 32, it is

$$({}_0m_\mathrm{t})^{\,2}=\left(+1-\frac{v^2}{c^2}\right)\times({}_\mathrm{r}m_\mathrm{t})^{\,2} \tag{33}$$

and equally

$$\left(\frac{({}_0m_\mathrm{t})^{\,2}}{({}_\mathrm{r}m_\mathrm{t})^{\,2}}\right)=\left(+1-\frac{v^2}{c^2}\right) \tag{34}$$

The normalised relativistic energy momentum relation is given as

$$\left(\frac{({}_0m_\mathrm{t})^{\,2}}{({}_\mathrm{r}m_\mathrm{t})^{\,2}}\right)+\left(\frac{(v)^{\,2}}{(c)^{\,2}}\right)=+1 \tag{35}$$

□

3.2. Theorem. Particle wave duality

Theorem 2 (Particle wave duality)**.** *The particle wave duality is described completely by the equation*

$$\left(\frac{({}_{0}E_{t})^{2}}{({}_{r}E_{t})^{2}}\right)+\left(\frac{({}_{w}E_{t})^{2}}{({}_{r}E_{t})^{2}}\right)=+1 \tag{36}$$

Proof by direct proof. Axiom 1 or

$$+1=+1 \tag{37}$$

is true. Therefore, it is equally true that

$$\left(\frac{({}_{0}m_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}m_{\mathrm{t}})^{2}}\right)+\left(\frac{(v)^{2}}{(c)^{2}}\right)=+1 \tag{38}$$

Rearranging equation 38, it is

$$\left(\frac{({}_{0}m_{\mathrm{t}})^{2}\times c^{4}}{({}_{\mathrm{r}}m_{\mathrm{t}})^{2}\times c^{4}}\right)+\left(\frac{(v)^{2}}{(c)^{2}}\right)=+1 \tag{39}$$

or

$$\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)+\left(\frac{(v)^{2}}{(c)^{2}}\right)=+1 \tag{40}$$

Rearranging equation 40, we obtain

$$\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)+\left(\frac{({}_{\mathrm{r}}m_{\mathrm{t}})^{2}\times(v)^{2}\times(c)^{2}}{({}_{\mathrm{r}}m_{\mathrm{t}})^{2}\times(c)^{2}\times(c)^{2}}\right)=+1 \tag{41}$$

Simplifying equation 41 (see equation 11, p. 11), it is

$$\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)+\left(\frac{({}_{\mathrm{r}}p_{\mathrm{t}})^{2}\times(c)^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)=+1 \tag{42}$$

Simplifying equation 42 (see equation 12, p. 11), we obtain the most general mathematical formulation of the particle wave duality as

$$\left(\frac{({}_{0}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)+\left(\frac{({}_{\mathrm{w}}E_{\mathrm{t}})^{2}}{({}_{\mathrm{r}}E_{\mathrm{t}})^{2}}\right)=+1 \tag{43}$$

□

Any physical entity, including a quantum mechanical entity, is at the same time both, a particle and a wave. In other words, in a particle, the wave can be found and vice versa. In a wave, the particle can be found. However, equation 43 includes conditions (i.e. either a pure particle or a pure wave) too, where an entity is either a particle or a wave. Equation 43 is the foundation of any measurement (i.e. interaction) and observation. A measurement, even if often treated as a privileged source of knowledge, cannot change everything by the process of measurement itself, an act of measurement is relative and not absolute. A quantum mechanical entity which is described completely by equation 43 might undergo some changes by an act of measurements. However, an act of measurement does not affect the rest energy, but only the total energy $({}_{\mathrm{r}}E_{\mathrm{t}})$ and the electromagnetic energy $({}_{0}E_{\mathrm{t}})$. The rest energy $({}_{0}E_{\mathrm{t}})$ is an invariant quantity and is the same for all observers in all reference frames before and after an act of measurement. This fact can be used to calculate the exact degree of interaction between a quantum mechanical entity and a measuring device.

3.3. Theorem. Relativistic wave equation

Various relativistic wave equations (Barukčić, 2013; Dirac, 1928; Gordon, 1926; Klein, 1926) have been proposed. One method consisted in inserting the momentum operator and energy operator into the relativistic energy–momentum relation to obtain a quantized version of the relativistic energy–momentum relation.

Theorem 3 (Relativistic wave equation)**.** *The relativistic wave equation is given as*

$$\left(\left(\left(+1-\frac{(v)^2}{(c)^2}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)+\left(\left(\frac{(v)^2}{(c)^2}\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)=H\times\Psi \tag{44}$$

Proof by direct proof. Axiom 1 or

$$+1=+1 \tag{45}$$

is true. Therefore, it is equally true (see equation 43, p. 17) that

$$\left(\frac{({}_0E_\mathrm{t})^2}{({}_\mathrm{r}E_\mathrm{t})^2}\right)+\left(\frac{({}_\mathrm{w}E_\mathrm{t})^2}{({}_\mathrm{r}E_\mathrm{t})^2}\right)=+1 \tag{46}$$

Equation 46 is identical with the relationship

$$\left(+1-\frac{(v)^2}{(c)^2}\right)+\left(\frac{(v)^2}{(c)^2}\right)=+1 \tag{47}$$

Multiplying equation 47 by the (time-dependent) Schrödinger wave equation(Schrödinger, Erwin Rudolf Josef Alexander, 1926b), it is

$$\left(\left(+1-\frac{(v)^2}{(c)^2}\right)\times H\times\Psi\right)+\left(\left(\frac{(v)^2}{(c)^2}\right)\times H\times\Psi\right)=(H\times\Psi) \tag{48}$$

where H is the Hamiltonian, which corresponds to the total energy of a (quantum mechanical) system and Ψ is the wave function. Energy defined in terms of an energy operator, acting on the wave function of a (quantum mechanical) system is given as

$$H=i\times\hbar\times\frac{\partial}{\partial t} \tag{49}$$

By inserting the energy operator (see equation 49) into equation 48, we obtain **the relativistic wave equation** as

$$\left(\left(\left(+1-\frac{(v)^2}{(c)^2}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)+\left(\left(\frac{(v)^2}{(c)^2}\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)=H\times\Psi \tag{50}$$

where i is the imaginary unit, $\hbar$ is the reduced Planck constant, H is the Hamiltonian operator, Ψ is the wave function, v is the relative velocity and c is the speed of the light in vacuum.

□

The relativistic wave equation (see equation 50) derived as

$$\left(\left(\left(+1-\frac{(v)^2}{(c)^2}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)+\left(\left(\frac{(v)^2}{(c)^2}\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)=H\times\Psi \tag{51}$$

can and should be adapted depending on the individual requirements. Let **x** denote the position operator, let **p** denote the momentum operator. Equation 51 becomes

$$\left(\left(\left(+1-\frac{(v)^2}{(c)^2}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)+\left(\left(\frac{({}_{\mathrm{r}}m_{\mathrm{t}})\times(v)^2}{({}_{\mathrm{r}}m_{\mathrm{t}})\times(c)^2}\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)=H\times\Psi \tag{52}$$

and equally $\left(it\ is\ {}_{\mathrm{r}}E_{\mathrm{t}}=H=i\times\hbar\times\frac{\partial}{\partial t}\right)$

$$\left(\left(\left(+1-\frac{(v)^2}{(c)^2}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)+\left(\left(\frac{({}_{\mathrm{r}}p_{\mathrm{t}})\times(v)}{\left(i\times\hbar\times\frac{\partial}{\partial t}\right)}\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)=H\times\Psi \tag{53}$$

The relativistic wave equation (see equation 53) would simplify in momentum space as

$$\left(\left(\left(+1-\frac{(v)^2}{(c)^2}\right)\times\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\right)\times\Psi\right)+((\mathbf{p}\times v)\times\Psi)=H\times\Psi \tag{54}$$

where **p** is the momentum operator and v is relative velocity, respectively. However, even equation 54 can further be adapted in accordance with the further needs. In order to illustrate the extent of the necessary adjustments, let us take a closer look at the following two cases. In the case of a pure particle it is v = 0 and equation 54 becomes

$$\left(i\times\hbar\times\frac{\partial}{\partial t}\right)\times\Psi=H\times\Psi \tag{55}$$

In the case of a pure wave it is v = c and equation 54 becomes

$$((\mathbf{p}\times c)\times\Psi)=H\times\Psi \tag{56}$$

3.4. Theorem. Time and wave function

Theorem 4 (Time and wave function)**.** *Let ${}_{R}E_{t}$ denote the relativistic (total) energy of a system (viewed from stationary observer R) at a certain run of an experiment t, let ${}_{R}t_{t}$ denote the relativistic time of a system (viewed from stationary observer R) at a certain run of an experiment t. One distinct aspects of the special theory of relativity is the relationship ${}_{0}t_{t} = \left(\sqrt[2]{1-\frac{v^2}{c^2}}\right) \times {}_{R}t_{t}$. In general, it is*

$$ {}_{R}t_{t} = \Psi \tag{57}$$

Proof by direct proof. Axiom 1 or

$$ +1 = +1 \tag{58}$$

is true. In the continuation of this theorem we consider a quantum mechanical system. The total energy of that system is identical with ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$. We obtain

$$ {}_{\mathrm{R}}E_{\mathrm{t}} = {}_{\mathrm{R}}E_{\mathrm{t}} \tag{59}$$

Multiplying the energy of the quantum mechanical system (see equation 59) by ${}_{\mathrm{R}}\mathrm{t}_{\mathrm{t}}$, it is

$$ {}_{\mathrm{R}}E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} = {}_{\mathrm{R}}E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} \tag{60}$$

The quantum mechanical system mentioned previously (see equation 60) can be described without any contradictions with the Schrödinger wave equation. We shall obtain ${}_{\mathrm{t}}$, it is

$$ {}_{\mathrm{R}}E_{\mathrm{t}} \times {}_{\mathrm{R}}t_{\mathrm{t}} = H \times \Psi \tag{61}$$

where Ψ is the wave function and H is the Hamiltonian. The Hamiltonian of a quantum mechanical system is an operator corresponding to the total energy of a quantum mechanical system, including both kinetic energy and potential energy. We have very good reason to assume that ${}_{\mathrm{R}}\mathrm{E}_{\mathrm{t}}$ equals H. Equation 61 can be rearranged as

$$ H \times {}_{\mathrm{R}}t_{\mathrm{t}} = H \times \Psi \tag{62}$$

Under the outlined circumstances (equation 62), we have very high level of evidence that the physical meaning of the wave function is determined as

$$ {}_{\mathrm{R}}t_{\mathrm{t}} = \Psi \tag{63}$$

□

In a more far reaching (see Barukčić, 2016) publication (see Barukčić, 2022) on this matter, it should be possible to provide a proof, that [7]

$$ {}_{\mathrm{R}}t_{\mathrm{t}} = \Psi = g \times c^2 \tag{64}$$

[7] Barukčić, I. (2016) The Physical Meaning of the Wave Function. Journal of Applied Mathematics and Physics, 4, 988-1023. doi: 10.4236/jamp.2016.46106.

3.5. Theorem. General relativity and invariant energy

Theorem 5 (General relativity and invariant energy)**.** *In general relativity, momentum itself is an entity (like mass or energy) that acts as a source of gravity. However, determining the invariant energy ($_{0}E_{t}$) of a massive body in general relativity theory (GRT) is not entirely simple. We must therefore ask ourselves whether we may apply the previous considerations also to areas of the general relativity theory. A general relativity consistent formulation of the particle wave duality is given as*

$$\left(\left(\left(\frac{R}{D}\right)-(R)\right)\times\Psi\right)+\left(\left(\left(\frac{\pi\times[x,p]}{i\times\mathbb{I}\times h}\right)\times(R+(2\times\Lambda))\right)\times\Psi\right)=H\times\Psi \tag{65}$$

Proof by direct proof. Axiom 1 or

$$+1=+1 \tag{66}$$

is true. Therefore, it is equally true that

$$\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}=\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu} \tag{67}$$

Based on the Einstein publications (Einstein, 1915, 1916, 1917, 1935; Einstein and Sitter, 1932) we arrive at the following Einstein's field equations.

$$R_{\mu\nu}-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+(\Lambda\times g_{\mu\nu})=\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu} \tag{68}$$

Taking the trace of both sides of equation 68, it is

$$(R_{\mu\nu}\times g^{\mu\nu})-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\times g^{\mu\nu}\right)+(\Lambda\times g_{\mu\nu}\times g^{\mu\nu})=\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\times g^{\mu\nu} \tag{69}$$

or

$$(R)-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\times g^{\mu\nu}\right)+(\Lambda\times g_{\mu\nu}\times g^{\mu\nu})=\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T \tag{70}$$

and equally

$$(R)-\left(\left(\frac{R}{2}\right)\times D\right)+(\Lambda\times D)=\left(\frac{4\times2\times\pi\times\gamma}{c^4}\right)\times T \tag{71}$$

Changing equation 71, it is

$$\left(\frac{R}{D}\right)-\left(\frac{R}{2}\right)+(\Lambda)=\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{72}$$

Equation 72 can be rearranged as

$$\left(\frac{R}{D}\right)-\left(\frac{R}{2}\right)-\left(\frac{R}{2}\right)+\left(\frac{R}{2}\right)+(\Lambda)=\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{73}$$

or as

$$\left(\frac{R}{D}\right)-(R)+\left(\frac{R}{2}\right)+(\Lambda)=\left(\frac{4\times2\times\pi\times\gamma\times T}{c^4\times D}\right) \tag{74}$$

Normalising equation 74 we obtain (the units (see Barukčić, 2022) cancel out) the relationship

$$\left(\frac{\left(\left(\frac{R}{D}\right)-(R)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}\right)+\left(\frac{\left(\left(\frac{R}{2}\right)+(\Lambda)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}\right)=+1 \tag{75}$$

Multiplying equation 75 by Schrödinger's wave equation $H\times\Psi$, it is

$$\left(\frac{\left(\left(\frac{R}{D}\right)-(R)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}\times H\right)\times\Psi+\left(\frac{\left(\left(\frac{R}{2}\right)+(\Lambda)\right)}{\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)}\times H\right)\times\Psi=H\times\Psi \tag{76}$$

where H is the Hamiltonian and Ψ is the (time dependent/independent) wave function. In general, the Hamiltonian H of a certain system is an operator corresponding to the total energy of that system. The total energy of the system of general relativity is more or less $\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}$ while the units might be adapted. In the following, we consider conditions where

$$H=\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D} \tag{77}$$

equation 76 simplifies as

$$\left(\left(\left(\frac{R}{D}\right)-(R)\right)\times\Psi\right)+\left(\left(\left(\frac{R}{2}\right)+(\Lambda)\right)\times\Psi\right)=H\times\Psi \tag{78}$$

In general, Dirac's/Schrödinger's (see also Dirac, 1926; Dirac and Fowler, 1926; Schrödinger, Erwin Rudolf Josef Alexander, 1926a) constant $\hbar$ is determined as

$$\hbar\equiv\frac{h}{2\times\pi}=\frac{[x,p]}{i\times\mathbb{I}} \tag{79}$$

considered the canonical commutation relation (see Born and Jordan, 1925) which demands that $\hbar=\frac{[x,p]}{i\times\mathbb{I}}$. In other words, it is

$$\frac{1}{2}=\left(\frac{\pi\times[x,p]}{i\times\mathbb{I}\times h}\right) \tag{80}$$

This relationship is substituted into equation 78. A relativistic wave equation (see Barukčić, 2013) is given as

$$\left(\left(\left(\frac{R}{D}\right)-(R)\right)\times\Psi\right)+\left(\left(\left(\frac{\pi\times[x,p]}{i\times\mathbb{I}\times h}\right)\times(R+(2\times\Lambda))\right)\times\Psi\right)=H\times\Psi \tag{81}$$

□

Under circumstances where equation 81 and equation 54 are identical, it would follow that

$$({}_{0}E_{\mathrm{t}})^{2}=\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)\times\left(\left(\frac{R}{D}\right)-\left(\frac{R}{2}\right)\right) \tag{82}$$

and that

$$({}_{\mathrm{w}}E_{\mathrm{t}})^{2}=\left(\frac{4\times 2\times \pi\times \gamma\times T}{c^4\times D}\right)\times\left(\left(\frac{\pi\times[x,p]}{i\times\mathbb{I}\times h}\right)\times(R+(2\times\Lambda))\right) \tag{83}$$

4. Discussion

Interference phenomena as exemplified by the two-slit experiment are one crucial aspect of quantum mechanics. If one repeatedly sends relativistic particles through a screen with two narrow slits a certain kind of a probability distribution of detection of these particles will follow. Equation 50 should be able to describe this situation completely and to solve the measurement problem of quantum mechanics. Depending upon the circumstances , (quantum mechanical) entities (i. e. such as light and electrons et cetera) posses at the same time, at the same run of an experiment both, wavelike and particle-like characteristics. Whether it possible to simultaneously observe wave and particle properties of (quantum mechanical) entity may remain an open question for the present.

5. Conclusion

In a wave, a particle can be found and vice versa. In a particle, a wave can be found.

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Erratum

Unfortunately, some misprints appeared in the previous publications, especially in the section of definitions. Some of the misprints have been brought up to date in this publication as far as possible.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

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I was born October, 1st 1961 in Novo Selo, Bosnia and Herzegovina, former Yogoslavia. I am of Croatian origin. From 1982-1989 C.E., I studied human medicine at the University of Hamburg, Germany. Meanwhile, I am working as a specialist of internal medicine. My basic field of research since my high school days at the Wirtschaftsgymnasium Bruchsal, Baden Württemberg, Germany is the mathematization of the relationship between a cause and an effect valid without any restriction under any circumstances including the conditions of classical logic, probability theory, quantum mechanics, special and general theory of relativity, human medicine et cetera. I endeavour to investigate positions of quantum mechanics, relativity theory, mathematics et cetera, only insofar as these positions put into question or endanger **the general validity of the principle of causality**.

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