CAUSATION, 17(10): 5–127

DOI:110.5281/zenodo.6799453
Received: June 26, 2022
Accepted: June 26, 2022
Published: July 5, 2022
Deutsche Nationalbibliothek Frankfurt

BCG vaccination excludes Crohn's disease relapse.

Research article

Ilija Barukčić[1]

[1] Internist, Horandstrasse, 26441 Jever, Germany

* **Correspondence:** E-Mail: Barukcic@t-online.de;
Tel: +49-4466-333; Fax: +49-4466-333.

Abstract

Background:

The incidence of nontuberculous mycobacterial (NTM) infections including Mycobacterium avium subspecies paratuberculosis (MAP) infections is increasing worldwide. Should we rely on the protective effect of BCG vaccination of humans against NTM too?

Methods:

The relationship between BCG vaccination and Crohn's disease relapse has been re-analysed.

Results:

The data of the study of Burnham et al. are to some extent biased. Nonetheless, the evidence provided by the study of Burnham et al. cannot be ignored.

Conclusion:

BCG vaccination excludes Crohn's disease relapse.

Keywords: BCG vaccination; Crohn's disease relapse; Cause; Effect; Causal relationship k; Causality; Causation

1. Introduction

Crohn's (see Crohn et al., 1932) disease (CD) and ulcerative colitis (UC) are two related but distinct disorders of inflammatory bowel diseases (IBD). In contrast to UC, CD is a transmural patchy inflammation of one or more segments of the digestive tract. Remissions and relapses are typical for CD. Crohn's disease is an inflammatory disorder of known (see Barukčić, 2018) but still not accepted etiology with immunologic, genetic, and various other environmental influences discussed. The incidence and prevalence of Crohn's disease is increasing worldwide and with it the number of therapeutic options. However, besides of the rapidly increasing therapeutic options, a definite cure of CD is still not in sight. Exclusive enteral nutrition and corticosteroids (success rate around 80%) are used for induction of remission. [1] , [2] The success rate of various treatment options for induction and

[1] Büller HA. Objectives and outcomes in the conventional treatment of pediatric Crohn's disease. J Pediatr Gastroenterol Nutr. 2001 Sep;33 Suppl 1:S11-8. doi: 10.1097/00005176-200109001-00003. PMID: 11685970.

[2] Benchimol EI, Seow CH, Steinhart AH, Griffiths AM. Traditional corticosteroids for induction of remission in Crohn's disease. Cochrane Database Syst Rev. 2008 Apr 16;2008(2):CD006792. doi: 10.1002/14651858.CD006792.pub2. PMID: 18425970; PMCID: PMC6718222.

maintenance of remission for CD like 5-ASA, azathioprine/6-MP [3], methotrexate [4, 5] and biologics including TNF [6, 7, 8] alpha antagonists varies extremely. Amazingly and curious enough, it has been observed that incidence rates of human TB in counties in England were lowest where exposure to bovine TB was highest [9, 10] and so much more. By time, TB rates dropped and several countries discontinued BCG vaccination. It is unmistakably the case that soon after, the incidence rates of disease due to nontuberculosis mycobacteria increased. [11] Behr et al. found [12] that declining rates of incidence of tuberculosis were accompanied with increasing incidence of Crohn's disease. To this end, we present our own representation (see figure 1 and figure 2) based on the graphic of Behr et al. [13] All these findings suggested to some extent that a mycobacterial infection may have a role in the etiology of CD. Indeed, meanwhile we all know it better than anyone, Mycobacterium is not only involved in the pathogenesis of Crohn's disease, **Mycobacterium avium subspecies paratuberculosis (MAP) is the cause of Crohn's disease** (see Barukčić, 2018). Unfortunately it appears that we are completely powerless against MAP, the cause of CD. In point of fact, it should be noted, however, that there is a possible vaccination [14] against a mycobacterial infection. Bacille Calmette–Guerin (BCG) vaccination [15] has been prepared from attenuated live bovine tuberculosis bacillus Mycobatrium bovis and is still the only vaccine to fight down tuberculosis (TB), the result of infection by Mycobacterium tuberculosis. BCG is at the same time the most widely used vaccine in human history. [16] In particular, it is worth pointing out, lastly, that Bacillus Calmette Guerin vaccine itself stimulates especially cellular

[3] Chande N, Townsend CM, Parker CE, MacDonald JK. Azathioprine or 6-mercaptopurine for induction of remission in Crohn's disease. Cochrane Database Syst Rev. 2016 Oct 26;10(10):CD000545. doi: 10.1002/14651858.CD000545.pub5. PMID: 27783843; PMCID: PMC6464152.

[4] McDonald JW, Wang Y, Tsoulis DJ, MacDonald JK, Feagan BG. Methotrexate for induction of remission in refractory Crohn's disease. Cochrane Database Syst Rev. 2014 Aug 6;2014(8):CD003459. doi: 10.1002/14651858.CD003459.pub4. PMID: 25099640; PMCID: PMC7154581.

[5] Wang Y, MacDonald JK, Vandermeer B, Griffiths AM, El-Matary W. Methotrexate for maintenance of remission in ulcerative colitis. Cochrane Database Syst Rev. 2015 Aug 11;2015(8):CD007560. doi: 10.1002/14651858.CD007560.pub3. PMID: 26263042; PMCID: PMC6486092.

[6] Akobeng AK, Zachos M. Tumor necrosis factor-alpha antibody for induction of remission in Crohn's disease. Cochrane Database Syst Rev. 2004;2003(1):CD003574. doi: 10.1002/14651858.CD003574.pub2. PMID: 14974022; PMCID: PMC8721561.

[7] Sandborn WJ, Rutgeerts P, Enns R, Hanauer SB, Colombel JF, Panaccione R, D'Haens G, Li J, Rosenfeld MR, Kent JD, Pollack PF. Adalimumab induction therapy for Crohn disease previously treated with infliximab: a randomized trial. Ann Intern Med. 2007 Jun 19;146(12):829-38. doi: 10.7326/0003-4819-146-12-200706190-00159. Epub 2007 Apr 30. PMID: 17470824.

[8] Hanauer SB, Sandborn WJ, Rutgeerts P, Fedorak RN, Lukas M, MacIntosh D, Panaccione R, Wolf D, Pollack P. Human anti-tumor necrosis factor monoclonal antibody (adalimumab) in Crohn's disease: the CLASSIC-I trial. Gastroenterology. 2006 Feb;130(2):323-33; quiz 591. doi: 10.1053/j.gastro.2005.11.030. PMID: 16472588.

[9] FRANCIS J. Control of infection with the bovine tubercle bacillus. Lancet. 1950 Jan 7;1(6593):34-9. doi: 10.1016/s0140-6736(50)90236-4. PMID: 15398223.

[10] Behr MA, Bruere P, Oxlade O. Global rates of Crohn's disease. Inflamm Bowel Dis. 2008 Aug;14(8):1170-2. doi: 10.1002/ibd.20435. PMID: 18340644.

[11] Romanus V, Hallander HO, Whlén P, Olinder-Nielsen AM, Magnusson PH, Juhlin I. Atypical mycobacteria in extrapulmonary disease among children. Incidence in Sweden from 1969 to 1990, related to changing BCG-vaccination coverage. Tuber Lung Dis. 1995 Aug;76(4):300-10. doi: 10.1016/s0962-8479(05)80028-0. PMID: 7579311.

[12] Behr MA, Bruere P, Oxlade O. Global rates of Crohn's disease. Inflamm Bowel Dis. 2008 Aug;14(8):1170-2. doi: 10.1002/ibd.20435. PMID: 18340644.

[13] Behr MA, Bruere P, Oxlade O. Global rates of Crohn's disease. Inflamm Bowel Dis. 2008 Aug;14(8):1170-2. doi: 10.1002/ibd.20435. PMID: 18340644.

[14] Dow CT. Proposing BCG Vaccination for Mycobacterium avium ss. paratuberculosis (MAP) Associated Autoimmune Diseases. Microorganisms. 2020 Feb 5;8(2):212. doi: 10.3390/microorganisms8020212. PMID: 32033287; PMCID: PMC7074941.

[15] Calmette A. Preventive Vaccination Against Tuberculosis with BCG. Proc R Soc Med. 1931 Sep;24(11):1481-90. PMID: 19988326; PMCID: PMC2182232.

[16] McShane H. Tuberculosis vaccines: beyond bacille Calmette-Guerin. Philos Trans R Soc Lond B Biol Sci. 2011 Oct 12;366(1579):2782-9. doi: 10.1098/rstb.2011.0097. PMID: 21893541; PMCID: PMC3146779.

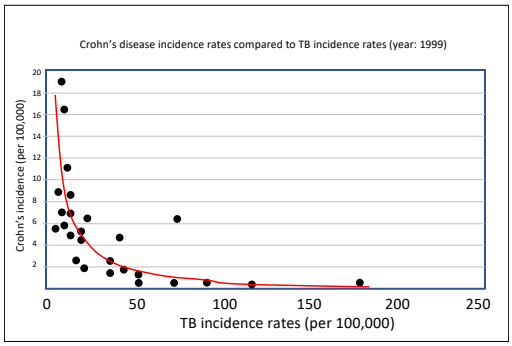


Figure 1. CD incidence rates and TB incidence rates, 1999

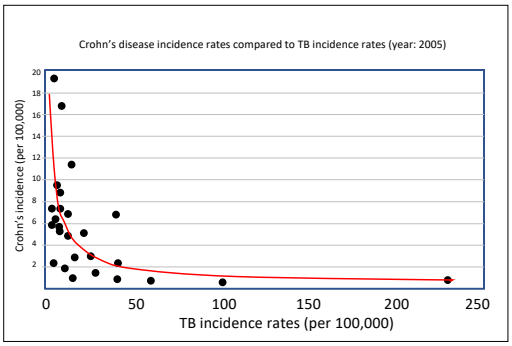


Figure 2. CD incidence rates and TB incidence rates, 2005

immunity [17] and is effective not only against Mycobacterium tuberculosis but against other [18] , [19] , [20] pathogenic mycobacteria like Mycobacterium avium, Mycobacterium leprae and Mycobacterium ulcerans too. [21] Nonetheless, theoretically, BCG vaccine may have a role in the treatment of CD. Thus far, it is hardly surprising that Geffroy et al. [22] used BCG vaccine as immunotherapy for the treatment of CD and claimed to have been successful. However, other studies [23] failed to show any significant therapeutic effect of (oral) BCG vaccine over placebo in CD. Discussion raised the issue of whether Bacillus Calmette Guerin vaccine [24] is of any help at all in the management of Crohn's disease. Is there a truth in this matter, and if so, where can we find the same?

Let $\mathrm{Mb_t}$ denote the number of all Myocobacterium positive cases in a human population at a certain (period) time t. Let $\mathrm{Mtb_t}$ denote the number of Myocobacterium tuberculosis positive cases in a human population of the same (period) time t. Let $\underline{\mathrm{Mtb}}_\mathrm{t}$ denote the number of all Myocobacterium positive cases in a human population of the same (period) time t which are not Myocobacterium tuberculosis positive, in other words all other Mycobacterium cases like MAP et cetera but not Myocobacterium tuberculosis. Is there any relationship between mycobacterial infection with a population?

$$Mb_\mathrm{t} \equiv Mb_\mathrm{t} - Mtb_\mathrm{t} + Mtb_\mathrm{t} \equiv \underline{Mtb}_\mathrm{t} + Mbt_\mathrm{t} \tag{1}$$

[17] Rousseau MC, Parent ME, St-Pierre Y. Potential health effects from non-specific stimulation of the immune function in early age: the example of BCG vaccination. Pediatr Allergy Immunol. 2008 Aug;19(5):438-48. doi: 10.1111/j.1399-3038.2007.00669.x. Epub 2007 Dec 27. PMID: 18167158.

[18] Mande R. Les nouvelles vaccinations [The new vaccinations]. Cah Coll Med Hop Paris. 1968 Jun;9(7):667-70. French. PMID: 5678820.

[19] Mande R. Les nouvelles vaccinations [The new vaccinations]. Cah Coll Med Hop Paris. 1968 Jun;9(7):667-70. French. PMID: 5678820.

[20] Zimmermann P, Finn A, Curtis N. Does BCG Vaccination Protect Against Nontuberculous Mycobacterial Infection? A Systematic Review and Meta-Analysis. J Infect Dis. 2018 Jul 24;218(5):679-687. doi: 10.1093/infdis/jiy207. PMID: 29635431.

[21] Dow CT. Proposing BCG Vaccination for Mycobacterium avium ss. paratuberculosis (MAP) Associated Autoimmune Diseases. Microorganisms. 2020 Feb 5;8(2):212. doi: 10.3390/microorganisms8020212. PMID: 32033287; PMCID: PMC7074941.

[22] Geffroy Y, Colin R, Segrestin M, Hecketsweiler P. B.C.G. treatment in Crohn's disease. Lancet. 1970 Sep 12;2(7672):571-2. doi: 10.1016/s0140-6736(70)91381-4. PMID: 4195236.

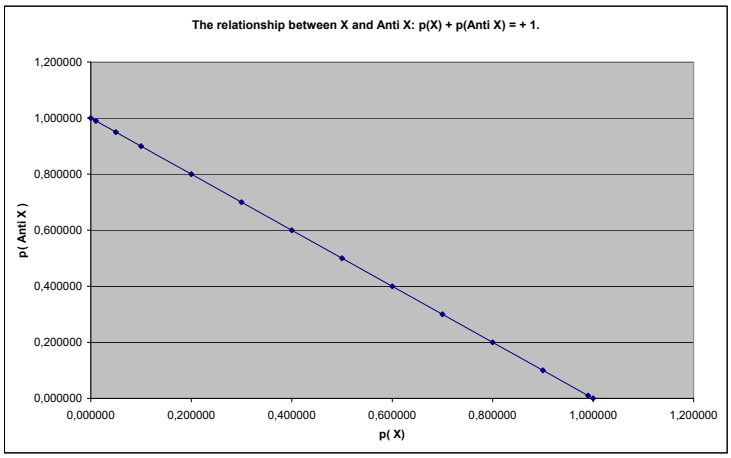
[23] El-Matary W, Yap J, Deora V, Singh H (2018) Bacillus Calmette Guerin (BCG) Vaccine for Inducing and Maintaining Remission in Crohn's Disease: A Systematic Review. J Clin Gastroenterol Hepatol Vol.2: No.1:6.

[24] Burnham WR, Lennard-Jones JE, Hecketsweiler P, Colin R, Geffroy Y. Oral BCG vaccine in Crohn's disease. Gut. 1979 Mar;20(3):229-33. doi: 10.1136/gut.20.3.229. PMID: 374194; PMCID: PMC1412307.

Normalising equation 1 (as illustrated by figure 1 and figure 2), we obtain

$$\frac{Mtb_{\mathrm{t}}}{Mb_{\mathrm{t}}} + \frac{Mtb_{\mathrm{t}}}{Mb_{\mathrm{t}}} \equiv \frac{Mb_{\mathrm{t}}}{Mb_{\mathrm{t}}} \equiv +1 \tag{2}$$

In other words, the higher the frequency or probability of Mycobacterium tuberculosis cases $\frac{Mtb_{\mathrm{t}}}{Mb_{\mathrm{t}}}$, the lower the frequency or probability of other mycobacterial cases $\frac{Mtb_{\mathrm{t}}}{Mb_{\mathrm{t}}}$ and vice versa. The question that springs to mind here is why is there such a dependence or relationship. MAP infections should occur independently of any Mtb infections which is not the case in reality (see figure 1 and figure 2). Even in these less than perfect circumstances we need to consider the possibility that a Mtb infection can protect against other non Mtb infections and vice versa. Behr et al. [25] provided the first epidemiological proof (see figure 1 and figure 2) that myocbacterium is part of the etiology of CD.



Under ideal conditions, the fundamental relationship between X and Anti X is given as

p(X) + p(Anti X) = + 1.

Figure 3. The relationship between X and Anti X.

2. Material and methods

Scientific knowledge and objective reality are more than only interrelated. It cannot be repeated often enough that objective reality or processes of objective reality is the foundation of any scientific knowledge. Our human experience teaches us however that seen by light, grey is never merely simply grey, and looked at from different angles, many paths may lead to climb up a certain mountain. In general, it is appropriate to ensure as much as possible a broader consideration of a research question and to take into account the different facets and viewpoints of an issue investigated in order to reach a goal.

[25] Behr MA, Bruere P, Oxlade O. Global rates of Crohn's disease. Inflamm Bowel Dis. 2008 Aug;14(8):1170-2. doi: 10.1002/ibd.20435. PMID: 18340644.

2.1. Material

2.1.1. The 1979 study of Burnham et al.

Burnham et al. conducted a randomised double-blind trial over one year to compare oral BCG group (verum) with a control (placebo) in the treatment of Crohn's disease. "Of the 50 patients ⋯ 24 were allocated to BCG and 26 to placebo ⋯ The disease relapsed in three patients taking BCG and seven taking the placebo "[26] Burnham et al. concluded that no significant benefit from oral BCG in the treatment of Crohn's disease has been demonstrated. The data of Burnham et al. have been re-analysed.

2.1.2. Statistical methods

The probability of the exclusion (Barukčić, 2021c) relationship(see also Barukčić, 2021a) p(EXCL) has been calculated and tested for statistical significance. The chi-square goodness of fit test with one degree of freedom has been used to test whether the sample data published fit a certain theoretical distribution in the population. Additionally, the P Value has been calculated approximately (see also Barukčić, 2019e). The causal relationship k (Barukčić, 2016b, 2020a, 2021c) has been calculated to evaluate a possible causal relationship between the events. The hyper-geometric (Fisher, 1922, Gonin, 1936, Huygens and van Schooten, 1657, Pearson, 1899) distribution (HGD) has been used to test the one-sided significance of the causal relationship k. Bringing different studies together for analysing them or doing a meta-analysis is not without problems. Due to several reasons, there is variability in the data of the studies and there will be differences found. Usually, the heterogeneity among the studies is assessed through I^2 statistics [27, 28, 29]. Under usual circumstances, an I^2 value of 25%, 50% and 75% are regarded as low, moderate and high heterogeneity[30]. In this publication, the study (design) bias and the heterogeneity among the studies has been controlled by IOI, the index of independence (Barukčić, 2019c) and IOU, the index of unfairness (Barukčić, 2019d). All the data were analysed using MS Excel (Microsoft Corporation, USA).

P values less than 0.05 were considered statistically significant.

[26] Burnham WR, Lennard-Jones JE, Hecketsweiler P, Colin R, Geffroy Y. Oral BCG vaccine in Crohn's disease. Gut. 1979 Mar;20(3):229-33. doi: 10.1136/gut.20.3.229. PMID: 374194; PMCID: PMC1412307.

[27] Cochran WG. The combination of estimates from different experiments. Biometrics 1954; 10(1): 101-29.

[28] Higgins JP, Thompson SG. Quantifying heterogeneity in a meta-analysis. Stat Med. 2002 Jun 15;21(11):1539-58. doi: 10.1002/sim.1186. PMID: 12111919.

[29] Higgins JP, Thompson SG, Deeks JJ, Altman DG. Measuring inconsistency in meta-analyses. BMJ. 2003 Sep 6;327(7414):557-60. doi: 10.1136/bmj.327.7414.557. PMID: 12958120; PMCID: PMC192859.

[30] Higgins JP, Thompson SG, Deeks JJ, Altman DG. Measuring inconsistency in meta-analyses. BMJ. 2003 Sep 6;327(7414):557-60. doi: 10.1136/bmj.327.7414.557. PMID: 12958120; PMCID: PMC192859.

2.2. *Methods*

Definitions should help us to provide and assure a systematic approach to a scientific issue. It also goes without the need of further saying that a definition as such need to be logically consistent and correct.

2.2.1. Random variables

As highlighted especially by Albert Einstein (1879-1955) and his coworkers Boris Yakovlevich Podolsky (1896-1966) and Nathan Rosen (1909-1995), "... objective reality ... is independent of any theory ... "[31] (see Einstein et al., 1935, p. 777), objective reality is independent of any observer and of any perceiving subject, objective reality is independent of any measurements. Let us carry this point to epistemological extremes, objective reality is existing independently and outside of human mind and consciousness. However, in its own self-sameness objective reality is different from a random variable too and self-contradictory. Nonetheless, in its difference, in its own contradiction, a random variable itself is self-identical and is in its own self a transition of itself into the other of itself and vice versa. Lastly, a random variable as such is in its own self the opposite of itself. More or less, a random variable is in its own self the unity of identity and difference and finds its own completion in the determinate relationship of self-identity and difference. A random variable as such is in its own self self-identical and different. This has at least a twofold aspect, identity and difference constitute the determinations of a random variable itself. These two moments of a random variable which are merely different in one and the same identity are constituting as moments of difference the determinations of an opposition too. A self-identical and a different constitute equally the interior nature of itself in relation to one another. The self-identical, determined with reference to an otherness, has within itself the reference-to-other which is the determinateness of the self-identical itself. The difference contains within itself the reference to its non-being, to identity, and vice versa. Identity contains within itself the reference to its non-being, to difference. However, a random variable as such is itself and its other and the identity of difference with itself is at the end a self-reference too. Consequently, a random variable as such has its own determinateness not in an other, but in its own self, it is self-referred, while the reference to its other manifests itself as a self-reference. The other of itself which a random variable as such contains is also the non-being of that in which it is supposed to be contained only as a moment. A random variable as such therefore is, only in so far as its non-being is, and is in an identical relationship with it. The moments of a random variable are different in one and the same identity and as moments of difference are constituting the determinations of an opposition. Closer consideration shows that a random variable as such is only in so far as the same contains a reference to its non-being, to its own other moment (i.e. local hidden variable). A self-identical which is equally a different too is thus far determining an opposition as such. While the one is not as yet self-identical, the other is not as yet different. However, both are different to one another. Nonetheless, the indifference of a random variable as such towards another random variable distinguished from the same has no influence on the fact that a random variable as such is in its own self the unity of identity and difference. At the end, a

[31] Einstein, A; B Podolsky; N Rosen (1935-05-15). "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" (PDF). Physical Review. 47 (10): 777–780. DOI:https://doi.org/10.1103/PhysRev.47.777

random variable as such is, only in so far as the other is; it is what it is, through the other, through its own non-being. A random variable is, in so far as the other is not; it is what it is, through the non-being of its own other.

The notion something is widely taken for granted as a foundation of axioms, theorems and theories. But, very broadly put, there are many different kinds of very concrete, single entities with real world implications. Thus far, what is something, what is its own other? In the most general way, there are circumstances where something and its own other existing independently and outside of human mind and consciousness is described mathematically by the notion random variable. Let a **random variable** (Gosset, 1914) X denote something like a function defined on a probability space, which itself maps from the sample space (Neyman and Pearson, 1933) to the real numbers.

2.2.1.1. The Expectation of a Random Variable

Definition 2.1 (**The First Moment Expectation of a Random Variable**). *Summaries of an entire distribution of a random variable (see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 22) X, such as the expected value, or average value, are useful in order to identify where X is expected to be without describing the entire distribution. For practical and other reasons, we shall limit ourselves here to discrete random variables, while the basic properties of the expectation value of a random variable X will not be investigated. Thus far, let X be a discrete random variable with the probability p(X). The relationship between the first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of X, denoted by E(X), and the probability p(X), is given by the equation:*

$$\begin{aligned} E(X) &\equiv X \times p(X) \\ &\equiv \Psi(X) \times X \times \Psi^{*}(X) \end{aligned} \tag{3}$$

where $\Psi(X)$ is the wave-function (see Born, 1926, Schrödinger, Erwin Rudolf Josef Alexander, 1926) of X, $\Psi^{*}(X)$ is the complex conjugate wave-function of X. Under conditions where $p(X) \equiv +1$ equation 3 (see p. 12) becomes

$$E(X) \equiv X \tag{4}$$

but not general. The first moment expectation value squared of a random variable X follows as

$$\begin{aligned} E(X)^{2} &\equiv p(X) \times X \times p(X) \times X \\ &\equiv p(X) \times p(X) \times X \times X \\ &\equiv (p(X) \times X)^{2} \\ &\equiv E(X) \times E(X) \end{aligned} \tag{5}$$

The ongoing progress with artificial intelligence has the potential to transform human society far beyond any imaginable border of human recognition and can help even to solve problems that otherwise would not be tractable. No wonder, scientist and systems are confronted with large volumes of data (big data) of various natures and from different sources. The use of tensor technology can simplify and

accelerate Big data analysis. In other words, let $X_{kl\mu\nu\ldots}$ denote an n-th index co-variant tensor with the probability $p(X_{kl\mu\nu\ldots})$. The first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of $X_{kl\mu\nu\ldots}$, denoted by $E(X_{kl\mu\nu\ldots})$, is a number defined as follows:

$$E\left(X_{kl\mu\nu\ldots}\right) \equiv p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \equiv p\left(X_{kl\mu\nu\ldots}\right) \cap X_{kl\mu\nu\ldots} \tag{6}$$

while $\times$ or $\cap$ might denote the commutative multiplications of tensors. The first moment expectation value squared of a random variable X follows as

$$\begin{aligned} {}^2E\left(X_{kl\mu\nu\ldots}\right) &\equiv p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \times p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \\ &\equiv p\left(X_{kl\mu\nu\ldots}\right) \times p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \times X_{kl\mu\nu\ldots} \\ &\equiv {}^2\left(p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots}\right) \\ &\equiv E\left(X_{kl\mu\nu\ldots}\right) \times E\left(X_{kl\mu\nu\ldots}\right) \end{aligned} \tag{7}$$

Definition 2.2 (**The Second Moment Expectation of a Random Variable**). *The second (see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42) moment expectation value (or more or less arithmetic mean) of a (large) number of independent realizations of a random variable X follows as:*

$$\begin{aligned} E\left(X^2\right) &\equiv p\left(X\right) \times X^2 \\ &\equiv \left(p\left(X\right) \times X\right) \times X \\ &\equiv E\left(X\right) \times X \\ &\equiv X \times E\left(X\right) \end{aligned} \tag{8}$$

From the point of view of tensor algebra it is

$$\begin{aligned} E\left({}^2X_{kl\mu\nu\ldots}\right) &\equiv p\left(X_{kl\mu\nu\ldots}\right) \times {}^2X_{kl\mu\nu\ldots} \\ &\equiv \left(p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \\ &\equiv E\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \\ &\equiv X_{kl\mu\nu\ldots} \times E\left(X_{kl\mu\nu\ldots}\right) \end{aligned} \tag{9}$$

Definition 2.3 (**The n-th Moment Expectation of a Random Variable**). *The n-th (see Barukčić, 2020a, 2021c) moment expectation value of a (large) number of independent realizations of a random variable X follows as:*

$$\begin{aligned} E\left(X^n\right) &\equiv p\left(X\right) \times X^n \\ &\equiv \left(p\left(X\right) \times X\right) \times X^{n\text{-}1} \\ &\equiv E\left(X\right) \times X^{n\text{-}1} \end{aligned} \tag{10}$$

2.2.1.2. Probability of a Random Variable What is the nature of the probability of an event, or what is the relationship between probability and geometry or between the probability of an event and notions like false or true. At a first pass, various authors answer this question, one way or another. For authors like De Morgan, probability is only a degree of confidence, or credences or of belief. "By degree of probability, we really mean, or ought to mean, degree of belief" (see De Morgan, 1847, p. 172). Such a purely subjective (or personalist or Bayesian (see Bayes, 1763)) interpretation of probabilities as degrees of confidence, or credences finds its own scientific opposition, moreover, in Kolmogorov's axiomatization of probability theory. However, perhaps we can do better, then, to think that Kolmogorov's axiomatization of probability theory is the last word spoken on probability theory. Nobody seriously considers that Kolmogorov's conceptual apparatus of probability theory has solved the basic problem of any probability theory, the relationship between classical logic or geometry and probability theory. One very massive disadvantage of Kolmogorov's axiomatization of probability theory is that it is very silent especially on this issue. Any unification of geometry and probability theory into one unique mathematical framework might prove very difficult as long as we rely purely on Kolmogorov's understanding of probability theory. It's not surprising that the probability of an event bear at least directly, and sometimes indirectly, upon central philosophical and scientific concerns. A correct understanding of probability is one of the most important foundational scientific problems. Now let us strengthen our position with respect to the probability of an event. In our understanding, the probability of an event is something objectively and real. The probability of an event is the truth value of something or the degree to which something, i.e. a random variable X, is determined by its own expectation value. The probability p(X) of a random variable X follows as (see equation 3)

$$\begin{aligned} p(X) &\equiv \frac{X \times p(X)}{X} \equiv \frac{E(X)}{X} \equiv p(X) \\ &\equiv \frac{X \times X \times p(X)}{X \times X} \equiv \frac{X \times E(X)}{X \times X} \equiv \frac{E\left(X^2\right)}{X^2} \\ &\equiv \frac{E(X)}{X} \equiv \frac{E(X) \times E(X)}{X \times E(X)} \equiv \frac{E(X)^2}{E\left(X^2\right)} \\ &\equiv \frac{E(X)}{X} \equiv \frac{E(X) \times E(\underline{X})}{X \times E(\underline{X})} \equiv \frac{\sigma(X)^2}{X \times X \times (1 - p(X))} \equiv \frac{\sigma(X)^2}{E\left(\underline{X}^2\right)} \\ &\equiv \Psi(X) \times \Psi^*(X) \end{aligned} \tag{11}$$

where $\Psi(X)$ is the wave-function of X, $\Psi^*(X)$ is the complex conjugate wave-function of X. As soon as the probability p(X) of an event X is determined, the probability of its own other, 1 - p(X), the

complementary of X, the opposite of X, anti X, is determined too. We obtain

$$\begin{aligned}
1-p(X) &\equiv 1-\frac{X\times p(X)}{X}\equiv 1-\frac{E(X)}{X}\equiv \frac{X}{X}-\frac{E(X)}{X}\equiv \frac{X-E(X)}{X}\equiv \frac{E(\underline{X})}{X}\equiv p(\underline{X})\\
&\equiv 1-\frac{X\times X\times p(X)}{X\times X}\equiv 1-\frac{X\times E(X)}{X\times X}\equiv 1-\frac{E\left(X^2\right)}{X^2}\equiv \frac{X^2}{X^2}-\frac{E\left(X^2\right)}{X^2}\equiv \frac{X^2-E\left(X^2\right)}{X^2}\\
&\equiv 1-\frac{E(X)}{X}\equiv 1-\frac{E(X)\times E(X)}{X\times E(X)}\equiv 1-\frac{E(X)^2}{E\left(X^2\right)}\\
&\equiv 1-\frac{E(X)}{X}\equiv 1-\frac{E(X)\times E(\underline{X})}{X\times E(\underline{X})}\equiv 1-\frac{\sigma(X)^2}{X\times X\times(1-p(X))}\equiv 1-\frac{\sigma(X)^2}{E\left(\underline{X}^2\right)}\\
&\equiv 1-\Psi(X)\times\Psi^{*}(X)
\end{aligned} \tag{12}$$

2.2.1.3. Variance of a Random Variable

Definition 2.4 (The Variance of a Random Variable). *Johann Carl Friedrich Gauß (1777-1855) introduced the normal distribution and the error of mean squared in his 1809 monograph (see Gauß, Carl Friedrich, 1809). In the following, Karl Pearson (1857-1936) coined the term "standard deviation"in 1893. Pearson is writing: "Then σ will be termed its standard-deviation (error of mean square)." (see Pearson, 1894, p. 80). Finally, the term variance was introduced by Sir Ronald Aylmer Fisher (1890-1962) in the year 1918.*

> *"The ... deviations of a ... measurement from its mean ... may be ... measured by the standard deviation corresponding to the square root of the mean square error ... It is ... desirable **in analysing the causes** ... to deal with the square of the standard deviation as the measure of variability. We shall term this quantity the Variance... "*
>
> *(see Fisher, Ronald Aylmer, 1919, p. 399)*

The deviation of a random variable X from its population mean or sample mean E(X) has a central role in statistics and is one important measure of dispersion. The variance $\sigma(X)^2$ (see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42), the second central moment of a distribution, is the expectation value of the squared deviation of a random variable X from its own expectation value E(X) and is determined in general as (see equation 8):

$$\begin{aligned}\sigma(X)^2 &\equiv E\left(X^2\right) - E(X)^2 \\ &\equiv (X \times E(X)) - E(X)^2 \\ &\equiv E(X) \times (X - E(X)) \\ &\equiv E(X) \times E(\underline{X})\end{aligned} \tag{13}$$

while $E(\underline{X}) \equiv X - E(X)$. From the point of view of tensor algebra, it is

$$\begin{aligned}{}^2\sigma\left(X_{kl\mu\nu\ldots}\right) &\equiv E\left({}^2X_{kl\mu\nu\ldots}\right) - {}^2E\left(X_{kl\mu\nu\ldots}\right) \\ &\equiv \left(X_{kl\mu\nu\ldots} \times E\left(X_{kl\mu\nu\ldots}\right)\right) - {}^2E\left(X_{kl\mu\nu\ldots}\right) \\ &\equiv E\left(X_{kl\mu\nu\ldots}\right) \times \left(X_{kl\mu\nu\ldots} - E\left(X_{kl\mu\nu\ldots}\right)\right) \\ &\equiv E\left(X_{kl\mu\nu\ldots}\right) \times E\left(\underline{X}_{kl\mu\nu\ldots}\right)\end{aligned} \tag{14}$$

while $E\left(\underline{X}_{kl\mu\nu\ldots}\right) \equiv X_{kl\mu\nu\ldots} - E\left(X_{kl\mu\nu\ldots}\right)$. As demonstrated by equation 14, variance depends not just on the expectation value of what has actually been observed $E\left(\left(X_{kl\mu\nu\ldots}\right)\right)$, but also on the expectation value that could have been observed but were not $\left(E\left(\underline{X}_{kl\mu\nu\ldots}\right)\right)$. There are circumstances in quantum mechanics where this fact is called the local hidden variable. Even if his might strike us as

peculiar, variance [32] is primarily a mathematical method which is of use in order to evaluate specific hypotheses in the light of some empirical facts. However, as a mathematical tool or method, variance is also a scientific description of a certain part of objective reality too. In this context, as a general mathematical principle, one fundamental meaning of variance is to provide a logically consistent link between something and its own other, between X and anti X.

> "The variance in this sense is a measure of the inner contradictions of a random variable, of changes, of struggle within this random variable itself, or the greater $\sigma(X)^2$ of a random variable, the greater the inner contradictions of this random variable. "
>
> (see Barukčić, 2006a, p. 57)

All things considered, we can safely say that, on the whole, **the variance is a mathematical description of the philosophical notion of the inner contradiction of a random variable X** (see Hegel, Georg Wilhelm Friedrich, 1812a, 1813, 1816) . Based on equation 13, it is

$$E\left(X^2\right) \equiv E(X)^2 + \sigma(X)^2 \tag{15}$$

or

$$\frac{E(X)^2}{E(X^2)} + \frac{\sigma(X)^2}{E(X^2)} \equiv p(X) + \frac{\sigma(X)^2}{E(X^2)} \equiv +1 \tag{16}$$

In other words, the variance (see Barukčić, 2006b) of a random variable is a determining part of the probability of a random variable. The wave function Ψ follows in general, as

$$\begin{aligned}
\Psi(X) &\equiv \frac{1}{\Psi^*(X)} - \frac{\sigma(X)^2}{(\Psi^*(X) \times E(X^2))} \\
&\equiv \frac{\left(E\left(X^2\right) - \sigma(X)^2\right)}{(\Psi^*(X) \times E(X^2))} \\
&\equiv \frac{1}{(\Psi^*(X) \times E(X^2))} \times \left(E\left(X^2\right) - \sigma(X)^2\right) \\
&\equiv \frac{1}{(\Psi^*(X) \times E(X^2))} \times E(X)^2 \\
&\equiv \frac{1}{\Psi^*(X)} \times \frac{E(X)^2}{E(X^2)} \\
&\equiv \frac{1}{\Psi^*(X) \times X} \times E(X)
\end{aligned} \tag{17}$$

The wave function (see Born, 1926) of a quantum-mechanical system is a central determining part of the Schrödinger wave equation (see Schrödinger, Erwin Rudolf Josef Alexander, 1926, 1929, 1952).

[32] Romeijn, Jan-Willem, "Philosophy of Statistics", The Stanford Encyclopedia of Philosophy (Spring 2022 Edition), Edward N. Zalta (ed.), forthcoming URL = https://plato.stanford.edu/archives/spr2022/entries/statistics/.

Definition 2.5 (**The First Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**). *In general, let $E(\underline{X})$ be defined as*

$$E(\underline{X}) \equiv X - E(X) \equiv X - (X \times p(X)) \equiv X \times (+1 - p(X)) \tag{18}$$

and denote an expectation value of a (discrete) random variable anti X with the probability

$$p(\underline{X}) \equiv 1 - p(X) \tag{19}$$

The first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of anti X, denoted as E($\underline{X}$), is a number defined as follows:

$$E(\underline{X}) \equiv X - (X \times p(X)) \equiv X \times (1 - p(X)) \equiv X \times p(\underline{X}) \tag{20}$$

The first moment expectation value squared of a random variable anti X follows as

$$\begin{aligned} E(\underline{X})^2 &\equiv p(\underline{X}) \times X \times p(\underline{X}) \times X \\ &\equiv p(\underline{X}) \times p(\underline{X}) \times X \times X \\ &\equiv (p(\underline{X}) \times X)^2 \\ &\equiv E(\underline{X}) \times E(\underline{X}) \end{aligned} \tag{21}$$

Definition 2.6 (**The Second Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**). *The second (see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42) moment expectation value (or more or less arithmetic mean) of a (large) number of independent realizations of a random variable anti X follows as:*

$$\begin{aligned} E(\underline{X}^2) &\equiv p(\underline{X}) \times X^2 \\ &\equiv (p(\underline{X}) \times X) \times X \\ &\equiv E(\underline{X}) \times X \\ &\equiv X \times E(\underline{X}) \end{aligned} \tag{22}$$

Definition 2.7 (**The n-th Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**). *The n-th (see Barukčić, 2020a, 2021c) moment expectation value of a (large) number of independent realizations of a random variable anti X follows as:*

$$\begin{aligned} E(\underline{X}^n) &\equiv p(\underline{X}) \times X^n \\ &\equiv (p(\underline{X}) \times X) \times X^{n\text{-}1} \\ &\equiv E(\underline{X}) \times X^{n\text{-}1} \end{aligned} \tag{23}$$

Definition 2.8 (**The Co-Variance of a Random Variable**). *Sir Ronald Aylmer Fisher (1890 -1962) introduced the term covariance (see Bailey, 1931) in the year 1930 in his book as follows:*

> *"It is obvious too that where a considerable fraction of the variance is contributed by chance causes, the variance of any group of individuals will be inflated in comparison with the covariances between related groups ... "*
>
> *(see Fisher, Ronald Aylmer, 1930, p. 195)*

In general, the co-variance is defined as given by equation 24.

$$\sigma(X,Y) \equiv E(X,Y) - (E(X) \times E(Y)) \tag{24}$$

From the point of view of tensor algebra, it is

$$\sigma\left(X_{kl\mu\nu\dots}, Y_{kl\mu\nu\dots}\right) \equiv E\left(X_{kl\mu\nu\dots}, Y_{kl\mu\nu\dots}\right) - \left(E\left(X_{kl\mu\nu\dots}\right) \times E\left(Y_{kl\mu\nu\dots}\right)\right) \tag{25}$$

2.2.2. Normal distribution

The origins of the normal distribution, also known as the Gaussian distribution, the second law of Laplace, the law of error et cetera, has been studied at least since the 18th century and can be traced back even to a French mathematician Abraham de Moivre. Johann Carl Friedrich Gauß's (1777-1855) presented 1809 the normal distribution (see Gauß, Carl Friedrich, 1809, p. 244) while illustrating the method of least squares. In the following, Karl Pearson (1857-1936) popularised a new name for Gauß distribution. Pearson wrote: "A frequency-curve, which for practical purposes, can be represented by the error curve, will for the remainder of this paper be termed a normal curve." (see Pearson, 1894, p. 72).

$$p({}_{\mathrm{R}}X_{\mathrm{t}}) = \left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \tag{26}$$

The standard normal distribution is illustrated by figure 4.

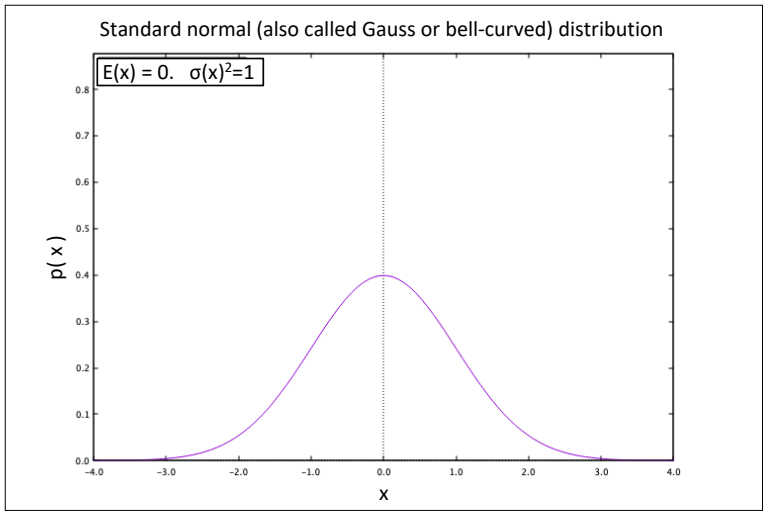
Sir Ronald Aylmer Fisher (1890-1962) [33] , a very influential statistician of the first half of the 20th century, presented the case of a normal (see Fisher, Ronald Aylmer, 1912, p. 157) distribution with non-zero mean (see Fisher, Ronald Aylmer, 1920, p. 758) as a typical case. The probability density function (pdf) of an anti-normal distribution is given as

$$p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) = 1 - \left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \tag{27}$$

as illustrated by figure 5. In general, it is

$$p({}_{\mathrm{R}}X_{\mathrm{t}}) + p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) = 1 \tag{28}$$

[33] R. A. Fisher Digital Archive, The University of Adelaide. 5005 AUSTRALIA. copyright@adelaide.edu.au



Normal distribution
with
E(x) = 0
and
$\sigma(\ x\ )^2 = 1$

Figure 4. Normal distribution

The variance of a Gaussian distributed random variable is given as

$$\begin{aligned}\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2 &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \times p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \\ &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) \right)\end{aligned} \tag{29}$$

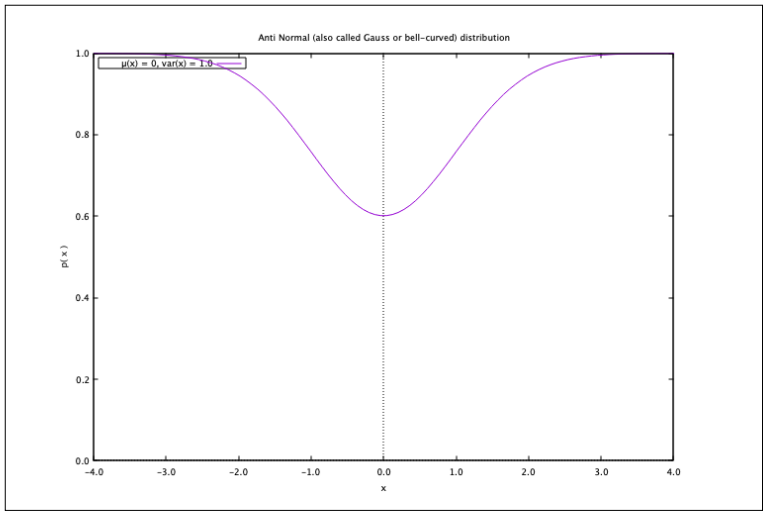
Under conditions where $\mathrm{E}({}_{\mathrm{R}}X_{\mathrm{t}}) = 0$ and $\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2 = 1$, equation 29 becomes

$$\begin{aligned}\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2 &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \times p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \\ &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) \right) \\ &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi \times 1}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - 0)^2}{2 \times 1}} \right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi \times 1}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - 0)^2}{2 \times 1}} \right) \right) \\ &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}})^2}{2}} \right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}})^2}{2}} \right) \right)\end{aligned} \tag{30}$$

Standard normal distribution

In general, a normal distribution with mean 0 and variance 1 is called the standard normal distribution. Modern publications often write the density function for the standard normal distribution, 'bell-shaped curve', as

$$p(z) = \left(\frac{1}{\sqrt{2\pi}} \right) e^{-\frac{z^2}{2}} \tag{31}$$



Anti-normal distribution
with
E(x) = 0
and
$\sigma(\ x\ )^2 = 1$

Figure 5. Anti-normal distribution

The density function for the anti-standard normal distribution is given as

$$p(\underline{z}) = 1 - p(z) = 1 - \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{z^2}{2}} \tag{32}$$

It is

$$p(z) + p(\underline{z}) = 1 \tag{33}$$

and is illustrated by figure 6.

Truman Lee Kelley (1884–1961) introduced statistical methods into psychological studies [34] and defined the z-score (see Kelley, 1924, p. 115). In mathematical statistics, a random variable ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ is standardised by subtracting its expected value $\mathrm{E}({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})$ and dividing the difference by its standard deviation $\sigma({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})$. The z-score or standard score, denoted as $\mathrm{z}({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})$, is defined as

$$z({}_{\mathrm{R}}X_{\mathrm{t}}) = \frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))}{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})} \tag{34}$$

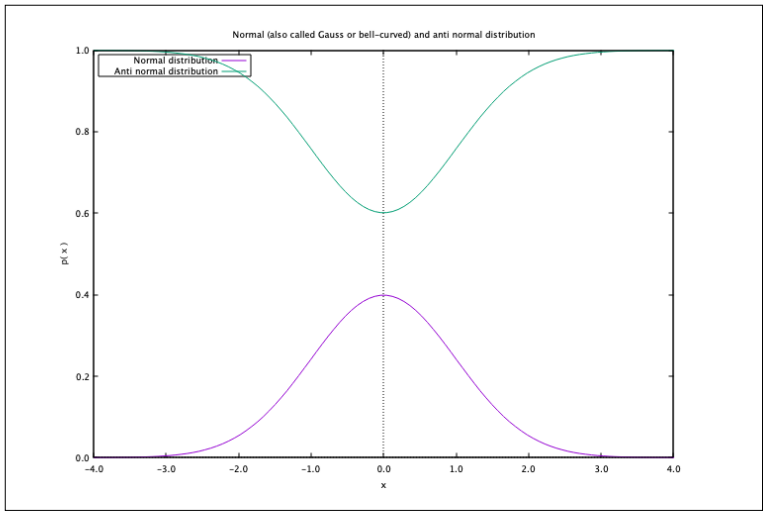
Simply put, a z-score (also called a standard score) describes how many standard deviations a given quantum mechanical observable or a random variable lies above or below a specific value. Equation 34 changes to

$$z({}_{\mathrm{R}}X_{\mathrm{t}})^2 = \frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})^2}{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}{E({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{{}_{\mathrm{R}}X_{\mathrm{t}} \times (1 - p({}_{\mathrm{R}}X_{\mathrm{t}}))}{{}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{(1 - p({}_{\mathrm{R}}X_{\mathrm{t}}))}{p({}_{\mathrm{R}}X_{\mathrm{t}})} \tag{35}$$

Equation 35 simplifies as

$$E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times E({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{36}$$

[34] McClure WE. Speed and Accuracy of the Feebleminded on Performance Tests. Psychol Clin. 1931 Feb;19(9):265-274. PMID: 28909304; PMCID: PMC5138284.



Normal and anti-normal distribution
with
E(x) = 0
and
$\sigma(\mathrm{x})^2 = 1$

Figure 6. Normal and anti-normal distribution

We can imagine drawing figure 6 in n dimensions. Under these circumstances we would obtain something similar to an **Einstein–Rosen bridge** or Einstein–Rosen wormhole [35] formulated in terms of the framework of probability theory. Attention should be drawn to circumstances especially of quantum mechanics, where E(${}_{\mathrm{R}}\underline{X}_{\mathrm{t}}$) indicates something like the expectation value of a 'local hidden variable '. Equation 36 changes slightly. It is

$$ {}_{\mathrm{R}}X_{\mathrm{t}} \times (1 - p({}_{\mathrm{R}}X_{\mathrm{t}})) = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times {}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{37} $$

and

$$ (1 - p({}_{\mathrm{R}}X_{\mathrm{t}})) = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{38} $$

Equation 38 is rearranged as

$$ 1 = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times p({}_{\mathrm{R}}X_{\mathrm{t}}) + p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{39} $$

or

$$ 1 = (z({}_{\mathrm{R}}X_{\mathrm{t}})^2 + 1) \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{40} $$

At the end, it follows that

$$ p({}_{\mathrm{R}}X_{\mathrm{t}}) = \frac{1}{z({}_{\mathrm{R}}X_{\mathrm{t}})^2 + 1} \tag{41} $$

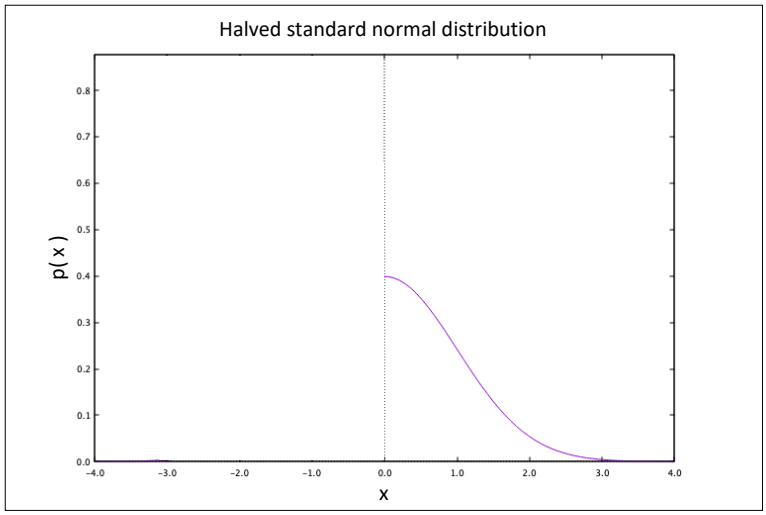
From equation 35 follows that

$$ z({}_{\mathrm{R}}X_{\mathrm{t}})^2 = \frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})^2}{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}{E({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \times E({}_{\mathrm{R}}X_{\mathrm{t}})}{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}{E({}_{\mathrm{R}}X_{\mathrm{t}})^2} \tag{42} $$

Thus far, it is equally

$$ \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2 = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times E({}_{\mathrm{R}}X_{\mathrm{t}})^2 \tag{43} $$

[35] Cramer JG, Forward RL, Morris MS, Visser M, Benford G, Landis GA. Natural wormholes as gravitational lenses. Phys Rev D Part Fields. 1995 Mar 15;51(6):3117-3120. doi: 10.1103/physrevd.51.3117. PMID: 10018782.



Halved normal distribution
with
E(x) = 0
and
$\sigma(\mathrm{x})^2 = 1$

Figure 7. Halved normal distribution

or

$$\sigma({}_{\mathrm{R}}X_{\mathrm{t}}) = z({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{44}$$

Per definition, it is

$$E({}_{\mathrm{R}}X_{\mathrm{t}}) = \frac{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})}{z({}_{\mathrm{R}}X_{\mathrm{t}})} \tag{45}$$

The probability density of a halved normal distribution for positive x is given as

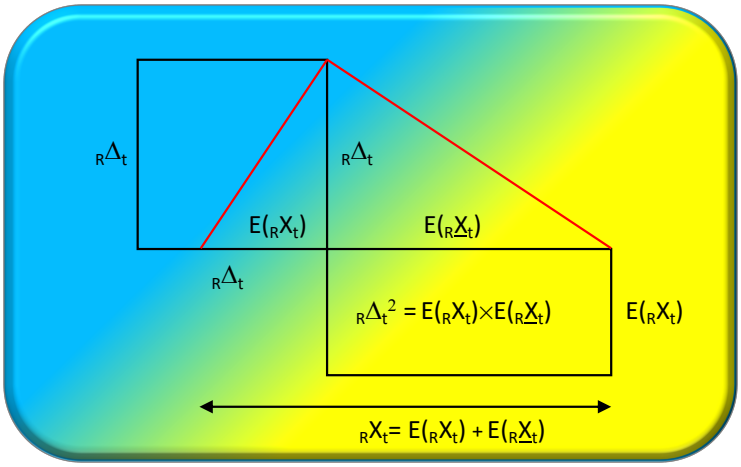
$$p({}_{\mathrm{R}}X_{\mathrm{t}}) = \left(\frac{2}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \tag{46}$$

and illustrated by figure 7.

2.2.3. Geometry

2.2.3.1. Euclid's theorem Various theories of geometry, including Euclidean geometry and non-Euclidean geometry, are based on definitions, axioms, theorems, proofs et cetera which themselves are derived more or less to some extent from knowledge of the objective reality too. Recalling Einstein's profound position

> "... Geometrie is offenbar eine Naturwissenschafft ... Ihre Aussagen beruhen im wesentlichen auf Induktion aus der Erfahrung, nicht aber auf logischen Schlüssen. "
>
> (see Einstein, 1921, p. 6)



Euclid's theorem
with
$_RX_t = E(_RX_t) + E(_R\underline{X}_t)$
as the foundation of
the relationship between
$_RX_t$ and $_R\underline{X}_t$
where
$E(_RX_t)$ is the expectation value
of $_RX_t$, and
$E(_R\underline{X}_t)$ is the expectation value
of anti $_RX_t$

Figure 8. Euclid's theorem.

and translating the same into simple English as "Geometry ... is ... a natural science ... in fact ... it ... rest essentially on induction from experience"we cannot avoid considering the limitations of geometry. In other words, explaining objective reality completely as a something like a complicated interplay between basic properties like points or lines might turn out to be stigmatized to some extent by imperfection. However, in logic, there is geometry, in geometry, there is logic. Both interpenetrate each other. The logic of geometry is determined by the geometry of logic and vice versa. Nonetheless, even if a detailed examination of geometry as presented by Euclid might reveal a number of problems, some of Euclid's theorems are still valid. In this context, it is worth considering Euclid's (ca. 360-280 BC) so-called right triangle theorem or Euclid's altitude theorem or Euclid's geometric mean theorem or simply Euclid's theorem, published as a corollary to proposition 8 in Book VI of Euclid's Elements (see roposition 8 in Book VI: Euclid, of Alexandria (300 B. C. E.), 1908, p. 209) and used in proposition 14 of Book II (see Book II, proposition 14: Euclid, of Alexandria (300 B. C. E.), 1908, pp. 409-410) to square a rectangle is defined (see Barukčić, 2013, 2015, 2016c) as

$$\begin{aligned} {}_R\Delta_t^2 &\equiv E\left({}_RX_t\right) \times E\left({}_R\underline{X}_t\right) \\ &\equiv \frac{\left(E\left({}_RX_t\right) \times {}_RX_t\right) \times \left(E\left({}_R\underline{X}_t\right) \times {}_RX_t\right)}{{}_RX_t \times {}_RX_t} \\ &\equiv \frac{\left({}_Ra_t\right)^2 \times \left({}_Rb_t\right)^2}{{}_RX_t^2} \\ &\equiv \sigma\left(X_t\right)^2 \end{aligned} \tag{47}$$

where $\sigma\left(X_t\right)^2$ is the variance of the random variable X_t. The variance ${}_R\Delta_t^2 \equiv \sigma\left(X_t\right)^2$ of a right-angled triangle is illustrated by Fig. 8 in more detail. It is

$$\begin{aligned} {}_R\Delta_t &\equiv \frac{\left({}_Ra_t\right) \times \left({}_Rb_t\right)}{{}_RX_t} \\ &\equiv \sigma\left(X_t\right) \end{aligned} \tag{48}$$

2.2.3.1.1. Euclid's theorem and expectation value It should be remembered, moreover, that Euclid's theorem is related to Thales of Miletus (ca. 624/623–ca.548/545 BCE) theorem. We may now apply Euclid's theorem to the relative latecomer in scientific history, the expectation values (see also fig. 8).

Theorem 2.1 (Euclid's theorem and expectation values)**.** *In general and according to Euclid's theorem, any random variable ${}_{R}X_{t}$ has the potential of being in a state of superposition as*

$$ {}_{R}X_{t} = E\left({}_{R}X_{t}\right) + E\left({}_{R}\underline{X}_{t}\right) \tag{49} $$

where ${}_{R}\Delta_{t}$ denotes the altitude in a right triangle and $E({}_{R}X_{t})$ and $E({}_{R}\underline{X}_{t})$ the segments on the hypotenuse ${}_{R}X_{t}$ in a right-angle triangle. In general, something, denoted by ${}_{R}X_{t}$, is self-contradictory. According to Euclid's theorem, it is equally the unity and the struggle between itself $E({}_{R}X_{t})$ and its own other $E({}_{R}\underline{X}_{t})$.

Proof by direct proof. The premise

$$ +1 \equiv +1 \tag{50} $$

is true. In the following, we rearrange the premise. We obtain

$$ +1+0 \equiv +1+0 \tag{51} $$

or

$$ +1 \equiv +1 - p\left(X_{\mathrm{t}}\right) + p\left(X_{\mathrm{t}}\right) \tag{52} $$

Equation 52 simplifies as

$$ +1 \equiv p\left(X_{\mathrm{t}}\right) + \left(+1 - p\left(X_{\mathrm{t}}\right)\right) \tag{53} $$

Multiplying equation 53 by the random variable $\mathrm{X_t}$, it is

$$ X_{\mathrm{t}} \equiv \left(X_{\mathrm{t}} \times p\left(X_{\mathrm{t}}\right)\right) + \left(X_{\mathrm{t}} \times \left(+1 - p\left(X_{\mathrm{t}}\right)\right)\right) \tag{54} $$

Equation 54 becomes (see equation 3, p. 12)

$$ X_{\mathrm{t}} \equiv E\left(X_{\mathrm{t}}\right) + \left(X_{\mathrm{t}} \times \left(+1 - p\left(X_{\mathrm{t}}\right)\right)\right) \tag{55} $$

Equation 55 changes (see equation 18, p. 18) further. Based on Euclid's theorem, any random variable $\mathrm{X_t}$ is more or less in a state of superposition as given by the equation

$$ X_{\mathrm{t}} \equiv E\left(X_{\mathrm{t}}\right) + E\left(\underline{X}\right) \tag{56} $$

□

2.2.3.1.2. Euclid's theorem and normalisation of expectation values The expectation values can be normalised.

Theorem 2.2 (Euclid's theorem and normalisation of expectation values)**.** *In general, the expectation values are normalised as*

$$+1 \equiv \frac{E\left({X_t}^2\right)}{{X_t}^2} + \frac{E\left(\underline{X}^2\right)}{{X_t}^2} \tag{57}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{58}$$

is true. In the following, we rearrange the premise. We obtain

$$X_{\mathrm{t}} \equiv X_{\mathrm{t}} \tag{59}$$

Equation 59 changes (see equation 56, p. 25) slightly. It is

$$X_{\mathrm{t}} \equiv E\left(X_{\mathrm{t}}\right) + E\left(\underline{X}\right) \tag{60}$$

Multiplying equation 60 by X_{t}, it is

$$X_{\mathrm{t}} \times X_{\mathrm{t}} \equiv \left(X_{\mathrm{t}} \times E\left(X_{\mathrm{t}}\right)\right) + \left(X_{\mathrm{t}} \times E\left(\underline{X}\right)\right) \tag{61}$$

or (see equation 8, p. 13) and equation 22, p. 18)

$${X_{\mathrm{t}}}^2 \equiv E\left({X_{\mathrm{t}}}^2\right) + E\left(\underline{X}^2\right) \tag{62}$$

Normalising the relationships of equation 62, it is

$$+1 \equiv \frac{{X_{\mathrm{t}}}^2}{{X_{\mathrm{t}}}^2} \equiv \frac{E\left({X_{\mathrm{t}}}^2\right)}{{X_{\mathrm{t}}}^2} + \frac{E\left(\underline{X}^2\right)}{{X_{\mathrm{t}}}^2} \tag{63}$$

□

2.2.3.1.3. Euclid's theorem and normalisation of probabilities

Theorem 2.3. *Euclid's theorem can be normalized. In general, it is*

$$p\left({}_R X_t\right) + p\left({}_R \underline{X}_t\right) = +1 \tag{64}$$

Proof. **If** the premise

$$+1 = +1 \tag{65}$$

is true, **then** the conclusion

$$p\left({}_{\mathrm{R}} X_{\mathrm{t}}\right) + p\left({}_{\mathrm{R}} \underline{X}_{\mathrm{t}}\right) = +1 \tag{66}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying Eq. 65 by $+_{R}X_{t}$ it is

$$+_{R}X_{t} = +_{R}X_{t} \tag{67}$$

Rearranging Eq. 67, we obtain

$$+_{R}X_{t} - E(_{R}X_{t}) + E(_{R}X_{t}) = +_{R}X_{t} + 0 \tag{68}$$

while it is necessary that $E(_{R}X_{t})$ is for sure one determining part of $+_{R}X_{t}$, whatever $E(_{R}X_{t})$ and $+_{R}X_{t}$ may denote. In general, we consider without an exception all but $E(_{R}X_{t})$ at a certain period of or point in time t as anti $E(_{R}X_{t})$. Anti $E(_{R}X_{t})$ is denoted by $E(_{R}\underline{X}_{t})$. Arithmetically, we define $E(_{R}\underline{X}_{t})$as

$$E\left(_{R}\underline{X}_{t}\right) \equiv +\left(_{R}X_{t}\right) - E\left(_{R}X_{t}\right) \tag{69}$$

Eq. 68 changes in perfect agreement with 69 to

$$+E\left(_{R}\underline{X}_{t}\right) + E\left(_{R}X_{t}\right) = {}_{R}X_{t} \tag{70}$$

By rearranging Eq. 70, we obtain the general normalized form of Euclid's theorem as

$$+\left(\frac{E(_{R}X_{t})}{_{R}X_{t}}\right) + \left(\frac{E(_{R}\underline{X}_{t})}{_{R}X_{t}}\right) = \left(\frac{_{R}X_{t}}{_{R}X_{t}}\right) = +1 \tag{71}$$

From the point of view of geometry, the probability of a single event, an entity, a quantity, a number et cetera is the extent to which $E(_{R}X_{t})$, this single event, entity, quantity, number et cetera, is a determining part of ${}_{R}X_{t}$. In general, it is

$$p\left(_{R}X_{t}\right) \equiv \frac{E(_{R}X_{t})}{_{R}X_{t}} \tag{72}$$

From the point of view of geometry, the probability of a single anti-event, an anti-entity, an anti-quantity, an anti-number et cetera is the extent to which $E(_{R}\underline{X}_{t})$, this single anti-event, an anti-entity, an anti-quantity, an anti-number et cetera, is a determining part of ${}_{R}X_{t}$. In general, it is

$$p\left(_{R}\underline{X}_{t}\right) \equiv \frac{E(_{R}\underline{X}_{t})}{_{R}X_{t}} = 1 - \frac{(_{R}X_{t}) \times p(_{R}X_{t})}{_{R}X_{t}} = 1 - \frac{E(_{R}X_{t})}{_{R}X_{t}} = 1 - p\left(_{R}X_{t}\right) \tag{73}$$

Taking into account the previous definitions (Eq. 72 and Eq. 73) then Eq. 71 changes to

$$p\left(_{R}X_{t}\right) + p\left(_{R}\underline{X}_{t}\right) = +1 \tag{74}$$

□

Theorem 2.4 (THE APPROXIMATE PROBABILITY OF AN EVENT).

In general, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \exp^{-\left(p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)} \tag{75}$$

Proof. **If** the premise

$$+1 = +1 \tag{76}$$

is true, **then** the conclusion

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \exp^{-\left(p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)} \tag{77}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying Eq. 76 by the probability $p({}_{\mathrm{R}}X_{\mathrm{t}})$ of an event ${}_{\mathrm{R}}X_{\mathrm{t}}$ at the (period of) time / Bernoulli trial t, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{78}$$

Eq. 78 changes according to Eq. 74 into

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \left(+1 - p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right) \tag{79}$$

or to

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \left(+1 - \left(\frac{n \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right) \tag{80}$$

Assumed that the probability is constant from trial to trial while the number of observations increases, we obtain the following.

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \equiv \left(+1 - \left(\frac{n \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right)^{\mathrm{n}} \tag{81}$$

Eq. 81 can be simplified as

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \equiv \left(+1 - \left(\frac{E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right)^{\mathrm{n}} \tag{82}$$

From elementary calculus (see also DeGroot and Schervish, 2005, p. 195) it is known that

$$\lim_{n \to +\infty} \left(+1 - \left(\frac{E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right)^{\mathrm{n}} \equiv \exp^{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)} \tag{83}$$

According to Eq. 83, Eq. 82 is rearranged as

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \equiv \exp^{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)} \tag{84}$$

The probability of a single event follows as

$$\begin{aligned} p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) &\equiv \sqrt[n]{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}}} \\ &\equiv \sqrt[n]{\exp^{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}} \\ &\equiv \exp^{\frac{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}} \\ &\equiv \exp^{-\frac{\left(n \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)}{n}} \end{aligned} \tag{85}$$

Finally, the probability of a single event (see Barukčić, 2019e, pp. 1843-1844) is given by

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \exp^{-\left(p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)} \tag{86}$$

□

Our sun has risen every day for a long time in the past. However, will the same sun rise tomorrow, for sure? In the light of such empirical facts, any inference from the known or observed to the unknown or unobserved, has become known as "inductive inferences". Inductive inference is often overshadowed by the possibility of being mistaken and is associated with a certain level of significance (see Arbuthnot, John, 1710, Venn, 1888), often denoted as the p-value (see Pearson, 1900b). Historically, it was especially David Hume [36] who put into question in his 1739 Book 'A Treatise of Human Nature, part iii, section 6' (see Hume, 1739) any justification in which humans form knowledge which became known as Hume's 'problem of induction '.

[36] Henderson, Leah, "The Problem of Induction", The Stanford Encyclopedia of Philosophy (Spring 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2020/entries/induction-problem/.

2.2.3.2. Pythagorean theorem Pythagoras of Samos (c. 570 – c. 495 BCE) lived on the island of Samos in the Aegean Sea, in Egypt, Babylon and southern Italy. In mathematics, Pythagoras is credited with several scientific and mathematical discoveries, including the Pythagorean theorem, or Pythagoras' theorem, too. However, the history of Pythagorean theorem is more or less the subject of much debate while neither the date of first discovery of Pythagoras' theorem nor the date of the first proof of Pythagoras' theorem is certain. At present, there are quite a few publications available suggesting that the Pythagorean theorem was known in ancient Babylon, Egypt and India (the Baudhayana Shulba Sutra), too. Yet astonishingly enough, there are reports that the Pythagorean theorem was found on an old Babylonian tablet meanwhile known as Plimpton 322 (see Friberg, 1981, Maor, 2007), written between 1790 and 1750 BCE during the reign of King Hammurabi the Great.

Definition 2.9 (**The right-angled triangle**). *A right-angled triangle is a triangle in which one angle is a 90-degree angle. Let ${}_RX_t$ denote the hypotenuse, the side opposite the right angle (side ${}_RX_t$ inside figure 9). The sides ${}_Ra_t$ and ${}_Rb_t$ are called legs of the triangle. In a right-angled triangle ABC, the side AC, which is abbreviated as ${}_Rb_t$, is the side which is adjacent to the angle α, while the side CB, denoted as ${}_Ra_t$, is the side opposite to the angle α. Figure 9 might illustrate a right-angled triangle (see Bettinger and Englund, 1960). The relation between the sides and angles of a right-angled triangle are known to be the basis for trigonometry, but are the basis of probability theory too.*

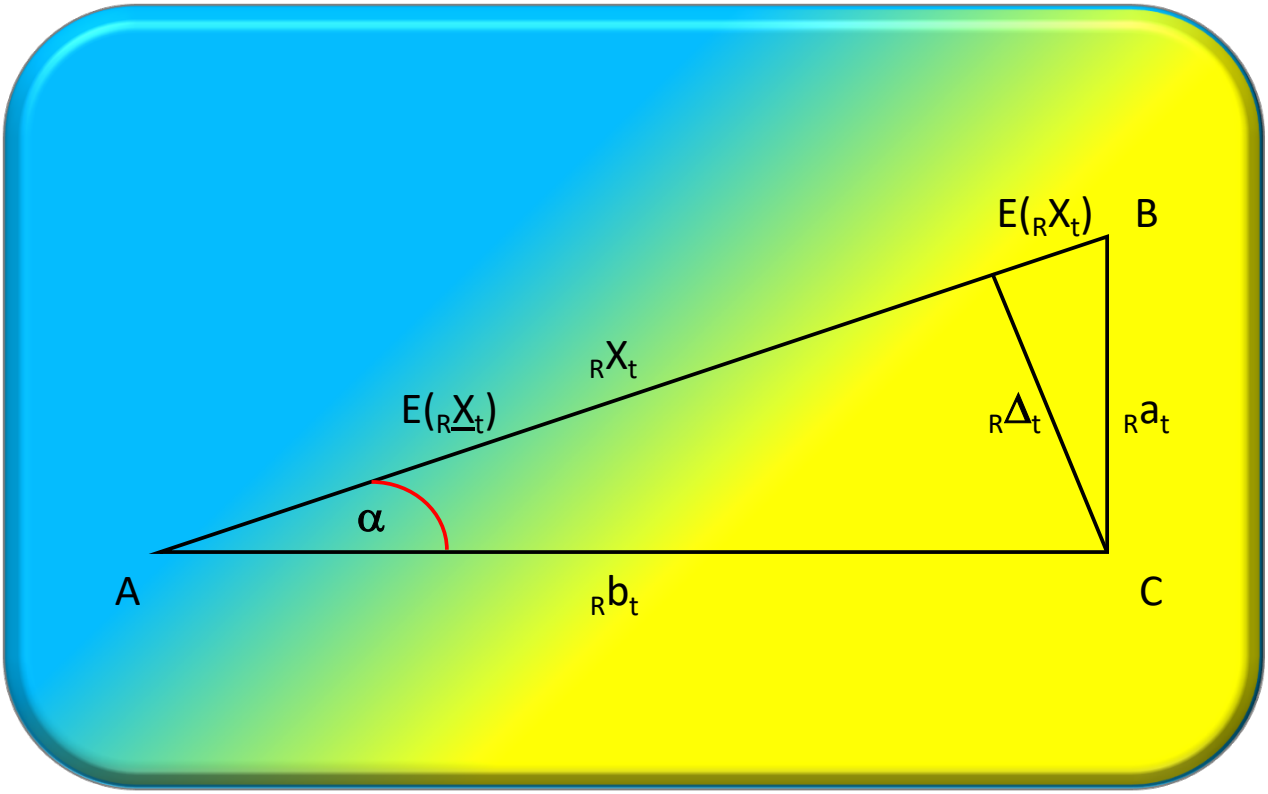


Figure 9. Right-angled triangle. ${}_RX_t$ is hypotenuse, ${}_Ra_t$ and ${}_Rb_t$ are called triangles legs.

Again, ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ is in the state of superposition, a law which has been re-formulated by the Danish geologist Nicolaus Steno (see Stenonis, Nicolai, 1669) in his 1696 book 'De Solido Intra Naturaliter Contento Dissertationis Prodomus '. Thus far, how big is the chance or probability that three random points like A, B, C in space-time are able to form a certain, stable right-angled triangle? Problems of similar type have been studied in the 18th century under the notion of geometric probability (see Milman, Vitali D., 2008, Solomon, 1978). Geometry and probability are deeply interrelated.

There is something extremely simple and deeply hidden even inside Einstein's masterpiece, the tremendously complex general theory of relativity. That turns out to be the right-angled triangle (ninety-degree angle at one of its corner).

2.2.3.2.1. Pythagorean theorem in general

Definition 2.10 (**The Pythagorean theorem**)**.**

The famous Pythagorean theorem of Euclidean geometry is attributed to the Greek thinker Pythagoras of Samos (ca. 570 – ca. 495 BCE). However, even if attributed to Pythagoras, the theorem has been known to the Babylonians (see Maor, 2007) more than a thousand years before Pythagoras. In general, the Pythagorean theorem is defined as

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} + {}_{\mathrm{R}}b_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \tag{87} $$

where ${}_{0}$ may denote the point of view of a co-moving observer, while ${}_{\mathrm{R}}$ may denote the point of view of a stationary observer at a certain point in space-time ${}_{\mathrm{t}}$. Fig. 10 is illustrating the Pythagorean theorem in all its splendour and beauty in more detail.

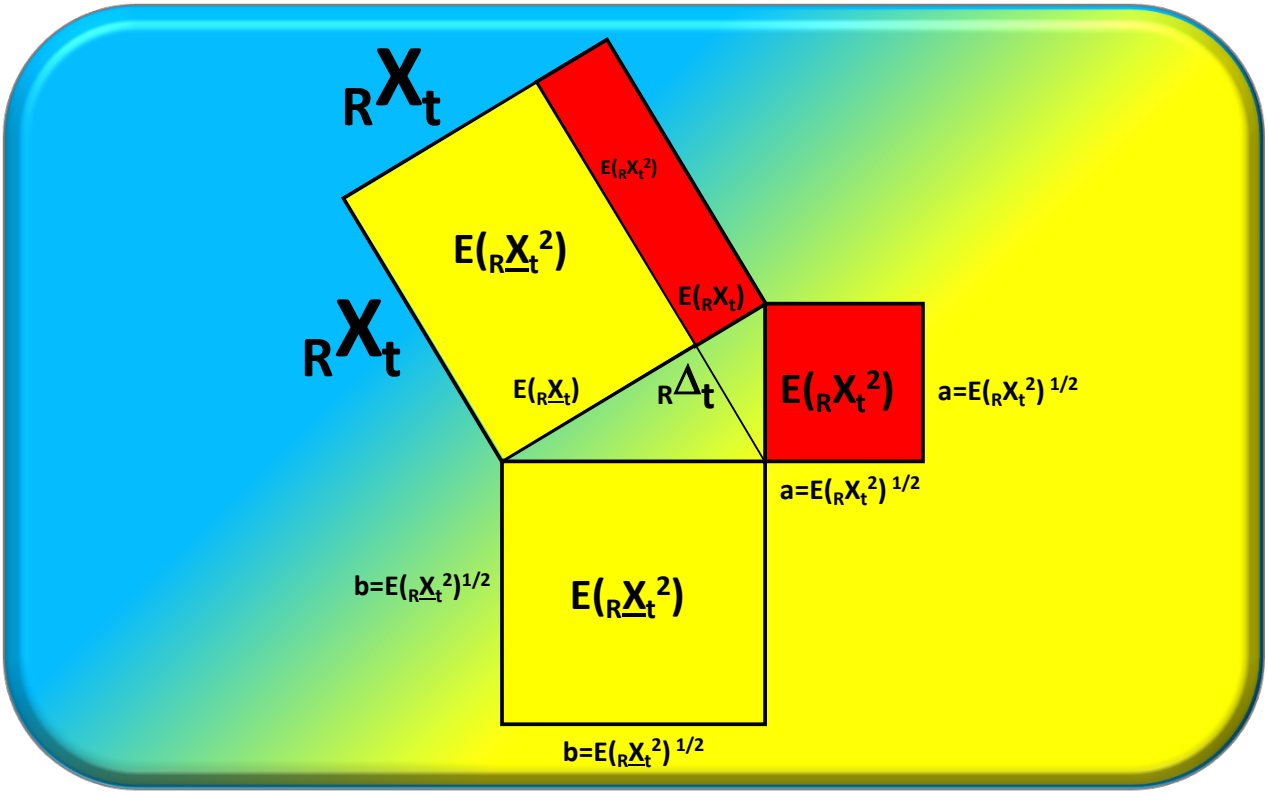


Figure 10. The Pythagorean theorem.

2.2.3.2.2. Pythagorean theorem normalised

Theorem 2.5. *The normalised Pythagorean theorem is determined as*

$$\frac{{}_{R}a_{t}^{2}}{{}_{R}X_{t}^{2}} + \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}} \equiv +1^{+2} \tag{88}$$

Proof. **If** the premise

$$+1 \equiv +1 \tag{89}$$

is true, **then** the conclusion

$$\frac{{}_{R}a_{t}^{2}}{{}_{R}X_{t}^{2}} + \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}} \equiv +1^{+2} \tag{90}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. The Pythagorean theorem is proofed (see Eq. 87) as ${}_{R}a_{t}^{2} + {}_{R}b_{t}^{2} \equiv {}_{R}X_{t}^{2}$. Eq. 89 changes to

$${}_{R}X_{t}^{2} \equiv {}_{R}X_{t}^{2} \tag{91}$$

and finally to

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} + {}_{\mathrm{R}}b_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \equiv C^{2} \tag{92} $$

In the following, We set $C = {}_{\mathrm{R}}X_{\mathrm{t}}$ and $a = {}_{\mathrm{R}}a_{\mathrm{t}}$ and $b = {}_{\mathrm{R}}b_{\mathrm{t}}$. Dividing Eq. 92 by ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}{}^{2}$ under conditions where this is possible and allowed, we obtain the normalized form of the Pythagorean theorem (see equation 11, p. 33) as

$$ \frac{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} \equiv \frac{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + 1^{+2} \tag{93} $$

□

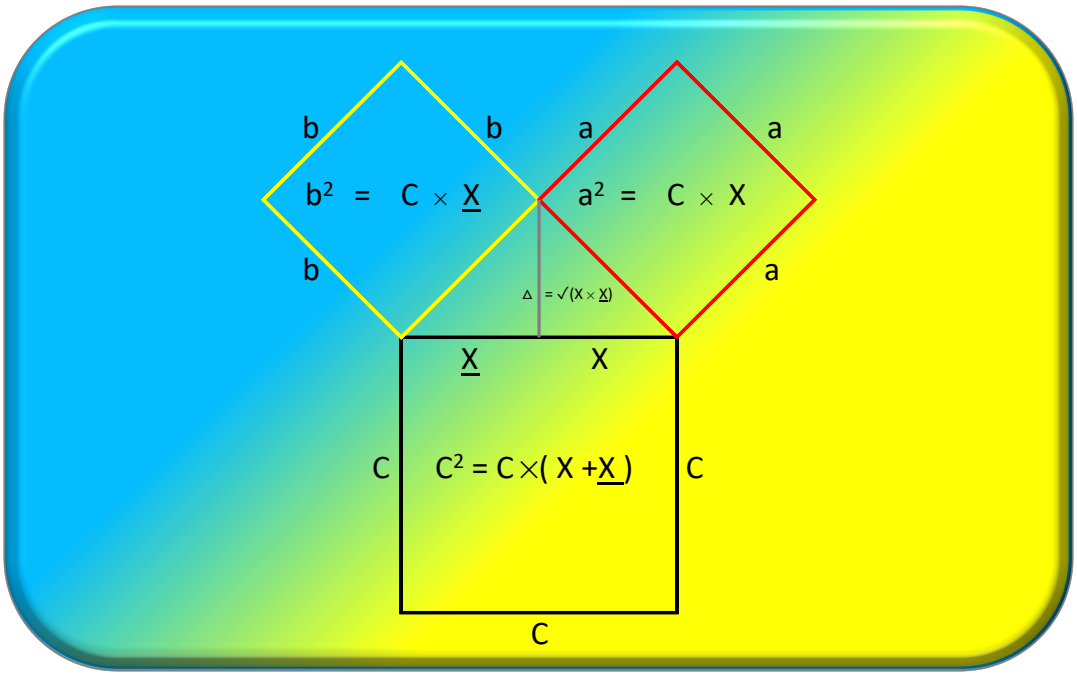


Figure 11. Geometry and probability theory.

In our understanding, there are conditions where probability theory / statistics is related with geometry (i.e. Pythagorean theorem, Euclid's theorem et cetera) (see also figure 11) by the equation:

$$ a^{2} \equiv E\left(X^{2}\right) \tag{94} $$

Further research should be able and might provide convincing evidence whether - and to what extent - equation 94 makes any sense at all. However, none of these reliefs us of our duty to seriously consider

the possibility of negative probabilities (see theorem 3.38 Barukčić, 2019b, pp. 67-68) like

$$-p(X) \equiv \frac{-E(X)}{-X} \tag{95}$$

It is

$$+1 \equiv p(X) + 1 - p(X) \equiv p(X) + p(\underline{X}) \equiv C \tag{96}$$

as illustrated by figure 12

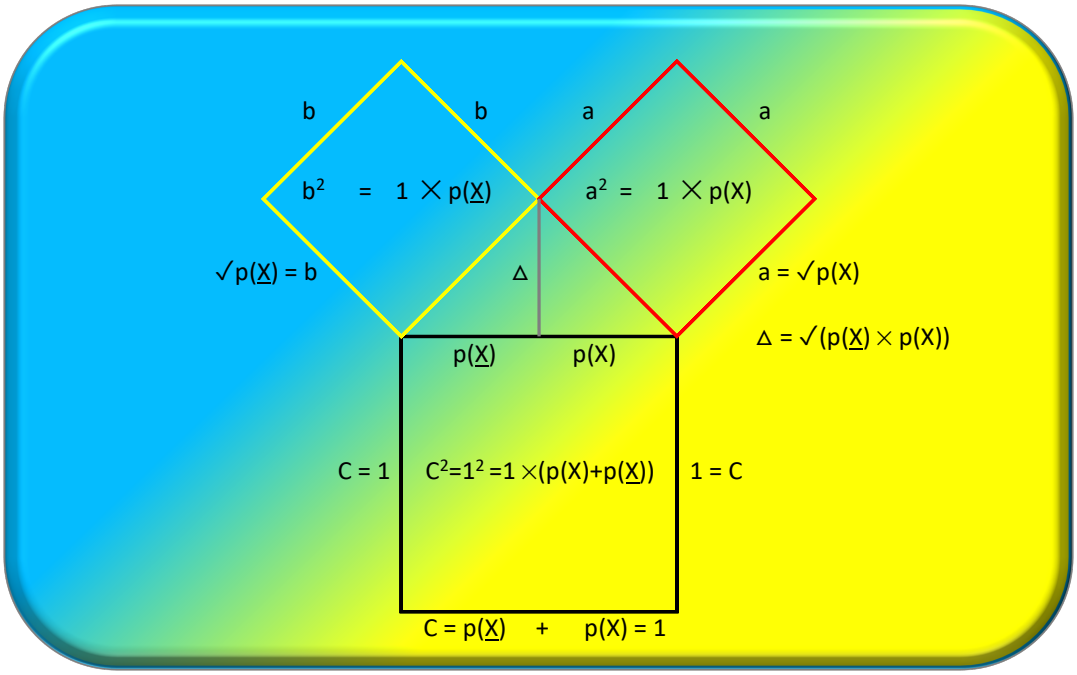


Figure 12. Geometry and probability theory.

and equally

$$\begin{aligned} +1^{+2} &\equiv (1 \times p(X)) + (1 \times (1 - p(X))) \equiv (1 \times (p(X) + p(\underline{X}))) \equiv C^2 \\ &\equiv a^2 + b^2 \equiv C^2 \\ &\equiv C^2 \end{aligned} \tag{97}$$

2.2.3.2.3. Geometry and probability The distributions of properties of geometric objects like length, area, volume, etc. is studied by **geometric probability** (see Klain and Rota, 1997, Milman, Vitali D., 2008, Solomon, 1978) too. In other words, probability is involved in geometry. **Example**. Let the length of a line C be C = 10 cm. Let X denote the length of a sub-line of C. Let X = 5 cm. The probability p(C = X) = 5 / 10 = 1/2. However, as can be seen by figure 11, probability and geometry are not only deeply interrelated. In contrast to Menger's approach to **probabilistic geometry** (see Menger, 1951, 2003, Milman, Vitali D., 2008, Špaček, 1956), probability theory can be defined by geometry, completely and potentially vice versa too. The trigonometrical functions are the geometric way to formulate probability.

In consideration of the preceding of figure 12 before and the general definition of the trigonometric function sine (see also Abu Dscha'far Muhammad ibn Musa al Chwarizmi's Algebra, written around 825 CE translated by Gerard of Cremona (1114-1187) from Arabic into Latin), denoted as sin, it is

$$sin\ \alpha \equiv \frac{a}{c} \equiv \frac{a}{1} \equiv \frac{\sqrt[2]{p(X)}}{1} \equiv \sqrt[2]{p(X)} \tag{98}$$

and

$$sin^2\ \alpha \equiv (sin\ \alpha)^2 \equiv sin\ \alpha \times sin\ \alpha \equiv \left(\frac{a}{c}\right)^2 \equiv \left(\frac{a}{1}\right)^2 \equiv a^2 \equiv p(X) \equiv \Psi(X) \times \Psi^*(X) \tag{99}$$

Against the background of figure 12 and the general definition of the function cosecant, denoted as csc, it is

$$csc\ \alpha \equiv \frac{c}{a} \equiv \frac{1}{a} \equiv \frac{1}{\sqrt[2]{p(X)}} \tag{100}$$

and equally

$$csc^2\ \alpha \equiv (csc\ \alpha)^2 \equiv csc\ \alpha \times csc\ \alpha \equiv \left(\frac{c}{a}\right)^2 \equiv \left(\frac{1}{a}\right)^2 \equiv \frac{1}{a^2} \equiv \frac{1}{p(X)} \tag{101}$$

In general it is

$$sin\ \alpha \times csc\ \alpha \equiv +1 \tag{102}$$

In the light of figure 12 above, and the definition the function cosine, denoted as cos, it is

$$cos\ \alpha \equiv \frac{b}{c} \equiv \frac{b}{1} \equiv \frac{\sqrt[2]{p(\underline{X})}}{1} \equiv \sqrt[2]{p(\underline{X})} \tag{103}$$

and at the same time

$$cos^2\ \alpha \equiv (cos\ \alpha)^2 \equiv cos\ \alpha \times cos\ \alpha \equiv \left(\frac{b}{c}\right)^2 \equiv \left(\frac{b}{1}\right)^2 \equiv b^2 \equiv p(\underline{X}) \equiv 1 - \Psi(X) \times \Psi^*(X) \tag{104}$$

Claudius Ptolemy (c. 85 – c. 165 CE) was the most influential Greek astronomers of his time, Ptolemy developed a geocentric theory of our solar system that prevailed for more than 1400 years

until overthrown by Isaac Newton's (see Newton, 1687) worldview. Already Ptolemy knew about the relationship

$$sin^2\,\alpha + cos^2\,\alpha \equiv +1 \tag{105}$$

which is meanwhile identified in more detail as

$$sin^2\,\alpha + cos^2\,\alpha \equiv p\,(X) + p\,(\underline{X}) \equiv +1 \tag{106}$$

We are justified in asking whether the expectation value of an angle α, denoted as E(α), might be given by the equation

$$E\,(\alpha) \equiv \alpha \times \left(sin^2\,\alpha\right) \tag{107}$$

whether E(α^2) would be given by the equation

$$E\left(\alpha^2\right) \equiv \alpha \times \alpha \times \left(sin^2\,\alpha\right) \tag{108}$$

Under these assumptions, the variance $\sigma(\alpha)^2$ of an angle would follow as

$$\sigma\,(\alpha)^{\,2} \equiv E\left(\alpha^2\right) - E\,(\alpha)^{\,2} \equiv \alpha \times \alpha \times \left(sin^2\,\alpha\right) \times \left(1 - \left(sin^2\,\alpha\right)\right) \tag{109}$$

Having regard to figure 12 above and on the basis of the definition of the function secant, denoted by sec, it is

$$sec\;\alpha \equiv \frac{c}{b} \equiv \frac{1}{b} \equiv \frac{1}{\sqrt[2]{p\,(\underline{X})}} \tag{110}$$

and equally

$$sec^2\;\alpha \equiv (sec\;\alpha)^{\,2} \equiv sec\;\alpha \times sec\;\alpha \equiv \left(\frac{c}{b}\right)^{\,2} \equiv \left(\frac{1}{b}\right)^{\,2} \equiv \frac{1}{p\,(\underline{X})} \tag{111}$$

$$cos\;\alpha \times sec\;\alpha \equiv +1 \tag{112}$$

On the basis of a presentation by figure 12 and the known definition of the function tangent, denoted as tan, it is

$$tan\;\alpha \equiv \frac{sin\;\alpha}{cos\;\alpha} \equiv \frac{\frac{a}{c}}{\frac{b}{c}} \equiv \frac{a}{b} \equiv \frac{\sqrt[2]{p\,(X)}}{\sqrt[2]{p\,(\underline{X})}} \equiv \sqrt[2]{\frac{p\,(X)}{p\,(\underline{X})}} \tag{113}$$

and equally

$$tan^2\;\alpha \equiv (tan\;\alpha)^{\,2} \equiv (tan\;\alpha) \times (tan\;\alpha) \equiv \left(\frac{sin\;\alpha}{cos\;\alpha}\right) \times \left(\frac{sin\;\alpha}{cos\;\alpha}\right) \equiv \frac{sin^2\;\alpha}{cos^2\;\alpha} \equiv \frac{\frac{a^2}{c^2}}{\frac{b^2}{c^2}} \equiv \frac{a^2}{b^2} \equiv \frac{p\,(X)}{p\,(\underline{X})} \tag{114}$$

In view of figure 12 and the definition of cotangent, denoted as cot, it is

$$cot\ \alpha \equiv \frac{b}{a} \equiv \frac{\sqrt[2]{p\left(\underline{X}\right)}}{\sqrt[2]{p\left(X\right)}} \equiv \sqrt[2]{\frac{p\left(\underline{X}\right)}{p\left(X\right)}} \tag{115}$$

Furthermore, it is

$$cot^2\ \alpha \equiv \left(cot\ \alpha\right)^2 \equiv \left(cot\ \alpha\right) \times \left(cot\ \alpha\right) \equiv cot^2\ \alpha \equiv \frac{b^2}{a^2} \equiv \frac{p\left(\underline{X}\right)}{p\left(X\right)} \tag{116}$$

Based on the findings as explained before and by figure 12 it is

$$tan\ \alpha \times cot\ \alpha \equiv +1 \tag{117}$$

An undeniable consequence of the previous explanations is that the "local hidden variable" (see Bohm, 1952, De Broglie, Louis, 1927), denoted as $E\left(\underline{X}\right)$, is determined by the relationship

$$E\left(\underline{X}\right) \equiv X \times cos^2\ \alpha \equiv \frac{\sigma\left(X\right)^2}{E\left(X\right)} \equiv \frac{\sigma\left(X\right)^2}{\Psi\left(X\right) \times X \times \Psi^*\left(X\right)} \tag{118}$$

while the variance from the point of view of geometry is given as

$$\begin{aligned} \sigma\left(X\right)^2 &\equiv E\left(X^2\right) - E\left(X\right)^2 \\ &\equiv \left(X \times p\left(X\right)\right) \times X \times \left(1 - p\left(X\right)\right) \\ &\equiv \left(X \times sin^2\ \alpha\right) \times X \times \left(1 - sin^2\ \alpha\right) \\ &\equiv \left(X \times sin^2\ \alpha\right) \times X \times \left(cos^2\ \alpha\right) \\ &\equiv X^2 \times \left(sin^2\ \alpha\right) \times \left(cos^2\ \alpha\right) \end{aligned} \tag{119}$$

From the point of view of tensor algebra, we obtain

$$\begin{aligned} p\left(X_{kl\mu\nu\ldots}\right) &\equiv \frac{X_{kl\mu\nu\ldots} \times p\left(X_{kl\mu\nu\ldots}\right)}{X_{kl\mu\nu\ldots}} \equiv \frac{E\left(X_{kl\mu\nu\ldots}\right)}{X_{kl\mu\nu\ldots}} \\ &\equiv \frac{X_{kl\mu\nu\ldots} \times X_{kl\mu\nu\ldots} \times p\left(X_{kl\mu\nu\ldots}\right)}{X_{kl\mu\nu\ldots} \times X_{kl\mu\nu\ldots}} \equiv \frac{E\left({}^2X_{kl\mu\nu\ldots}\right)}{{}^2X_{kl\mu\nu\ldots}} \\ &\equiv \frac{E\left(X_{kl\mu\nu\ldots}\right) \times E\left(X_{kl\mu\nu\ldots}\right)}{E\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots}} \equiv \frac{{}^2E\left(X_{kl\mu\nu\ldots}\right)}{E\left({}^2X_{kl\mu\nu\ldots}\right)} \\ &\equiv \Psi\left(X_{kl\mu\nu\ldots}\right) \times \Psi^*\left(X_{kl\mu\nu\ldots}\right) \end{aligned} \tag{120}$$

where $\Psi\left(X_{kl\mu\nu\ldots}\right)$ is the wave-function tensor of $X_{kl\mu\nu\ldots}$, $\Psi^*\left(X_{kl\mu\nu\ldots}\right)$ is the complex conjugate wave-function tensor of $X_{kl\mu\nu\ldots}$.

2.2.3.2.4. Pythagorean theorem and negation

Theorem 2.6. *In general, ${}_{R}a_{t}$ is the negation of ${}_{R}b_{t}$ and vice versa. It is*

$$ {}_{R}a_{t}{}^{2} = \neg({}_{R}b_{t}) \times {}_{R}X_{t}{}^{2} \tag{121} $$

Proof. **If** the premise

$$ +1 = +1 \tag{122} $$

is true, **then** the conclusion

$$ {}_{\mathrm{R}}a_{\mathrm{t}} = \sqrt[2]{\left(+1^{+2} - \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\right)} \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{123} $$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Eq. 122 is rearranged as

$$ +1^{+2} = +1^{+2} \tag{124} $$

The normalized form of the Pythagorean theorem is proofed as (see theorem 2.5, Eq. 93) as $\frac{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} = +1^{+2}$. Eq. 124 changes to

$$ \frac{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} = +1^{+2} \tag{125} $$

Rearranging Eq. 125

$$ \frac{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + = \left(+1^{+2} - \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\right) \tag{126} $$

Simplifying Eq. 126, it is

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} \times +1^{+2} = \left(+1^{+2} - \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \tag{127} $$

Eq. 127 changes to

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} = \left(+1^{+2} - \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \tag{128} $$

and to

$$ {}_{\mathrm{R}}a_{\mathrm{t}} = \sqrt[2]{\left(+1^{+2} - \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\right)} \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{129} $$

We define in general

$$ \neg({}_{\mathrm{R}}b_{\mathrm{t}}) \equiv \left(+1^{+2} - \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\right) \tag{130} $$

Eq. 128 changes to

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} = \neg({}_{\mathrm{R}}b_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \tag{131} $$

□

The negation of ${}_{\mathrm{R}}\mathrm{b_t}$ need to be calculated similarly. We will obtain

$$\neg\left({}_{\mathrm{R}}a_{\mathrm{t}}\right) \equiv \left(+1^{+2} - \frac{{}_{\mathrm{R}}{a_{\mathrm{t}}}^2}{{}_{\mathrm{R}}{X_{\mathrm{t}}}^2}\right) \tag{132}$$

Under conditions of Einstein's special relativity where ${}_{\mathrm{R}}\mathrm{a_t}$ does denote the rest-mass and where ${}_{\mathrm{R}}\mathrm{X_t}$ does denote the relativistic mass, we obtain the identity with reciprocal Lorentz factor or Lorentz term (see also Lorentz, 1899, p. 432) as $\left(\sqrt[2]{\left(+1^{+2} - \frac{{}_{\mathrm{R}}{b_{\mathrm{t}}}^2}{{}_{\mathrm{R}}{X_{\mathrm{t}}}^2}\right)}\right) \equiv \left(\sqrt[2]{\left(1 - \frac{v^2}{c^2}\right)}\right)$ (see also Barukčić, 2019a).

2.2.3.2.5. The n-dimensional Pythagorean theorem The n-dimensional Pythagorean theorem can be derived in a simple and logically consistent way.

Theorem 2.7. *The n-dimensional Pythagorean theorem is determined as*

$${}_{R}{a_t}^{2n} + {}_{R}{b_t}^{2n} \equiv {}_{R}{X_t}^{2n} \tag{133}$$

Proof. **If** the premise

$$+1 = +1 \tag{134}$$

is true, **then** the conclusion

$${}_{\mathrm{R}}{a_{\mathrm{t}}}^{2\mathrm{n}} + {}_{\mathrm{R}}{b_{\mathrm{t}}}^{2\mathrm{n}} \equiv {}_{\mathrm{R}}{X_{\mathrm{t}}}^{2\mathrm{n}} \tag{135}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Eq. 134 change quickly to

$${}_{\mathrm{R}}{X_{\mathrm{t}}}^2 = {}_{\mathrm{R}}{X_{\mathrm{t}}}^2 \tag{136}$$

Multiplying Eq. 136 by Eq. 88 of theorem 2.5 known to be derived as $\left(\frac{{}_{\mathrm{R}}{a_{\mathrm{t}}}^2}{{}_{\mathrm{R}}{X_{\mathrm{t}}}^2} + \frac{{}_{\mathrm{R}}{b_{\mathrm{t}}}^2}{{}_{\mathrm{R}}{X_{\mathrm{t}}}^2} = {}_{\mathrm{R}}{X_{\mathrm{t}}}^2\right)$ yields

$$\frac{{}_{\mathrm{R}}{a_{\mathrm{t}}}^2}{{}_{\mathrm{R}}{X_{\mathrm{t}}}^2} + \frac{{}_{\mathrm{R}}{b_{\mathrm{t}}}^2}{{}_{\mathrm{R}}{X_{\mathrm{t}}}^2} = {}_{\mathrm{R}}{X_{\mathrm{t}}}^2 \tag{137}$$

Several properties of the Pythagorean theorem are already identified. In general, it is proofed that

$${}_{\mathrm{R}}{a_{\mathrm{t}}}^2 \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{138}$$

or that

$${}_{\mathrm{R}}{a_{\mathrm{t}}}^{2\mathrm{n}} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \times {}_{\mathrm{R}}{X_{\mathrm{t}}}^{\mathrm{n}} \equiv \left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \tag{139}$$

Furthermore, it is

$${}_{\mathrm{R}}{b_{\mathrm{t}}}^2 \equiv E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{140}$$

and equally

$${}_{\mathrm{R}}{b_{\mathrm{t}}}^{2\mathrm{n}} \equiv E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)^{\mathrm{n}} \times {}_{\mathrm{R}}{X_{\mathrm{t}}}^{\mathrm{n}} \equiv \left(E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \tag{141}$$

where n might denote the number of dimensions. Rearranging Eq. 137 according to the relationship of Eq. 138 it is

$$\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})\times {}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}+\frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}=+1^{2} \tag{142}$$

Rearranging Eq. 142 according to the relationship of Eq. 140 it is

$$\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})\times {}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}+\frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})\times {}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}=+1^{2} \tag{143}$$

Eq. 143 simplifies further as

$$\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}}+\frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})1}{{}_{\mathrm{R}}X_{\mathrm{t}}}=+1^{2} \tag{144}$$

Simplifying Eq. 144 it is

$$E({}_{\mathrm{R}}X_{\mathrm{t}})+E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})={}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}\times 1^{1}\times 1^{1}=({}_{\mathrm{R}}X_{\mathrm{t}}\times 1\times 1)^{1}={}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}={}_{\mathrm{R}}X_{\mathrm{t}} \tag{145}$$

As known, it is $\left(U^{1}\times U^{0}\equiv U^{+1}\equiv U\right)$. However, Eq. 145 simplifies further. The most simple and most general form of the Pythagorean theorem (see Barukčić, 2016c) is based on the fundamental relationship,

$$E({}_{\mathrm{R}}X_{\mathrm{t}})+E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \tag{146}$$

In particular, the Pythagorean theorem can be extended to higher dimensions (see Yeng et al., 2008) too. In the n-dimensional case (see Barukčić, 2020b), the relationship before becomes

$$\left(E({}_{\mathrm{R}}X_{\mathrm{t}})+E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})\right)^{\mathrm{n}}\equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \tag{147}$$

Multiplying Eq. 147 by ${}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}$, the Pythagorean theorem becomes something like

$$\left(E({}_{\mathrm{R}}X_{\mathrm{t}})+E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})\right)^{\mathrm{n}}\times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \tag{148}$$

or as

$$\underbrace{E({}_{\mathrm{R}}X_{\mathrm{t}})^{\mathrm{n}}\times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}}_{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2\mathrm{n}}}+\underbrace{\cdots}_{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2\mathrm{n}}}\equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2\mathrm{n}} \tag{149}$$

In general, the n-dimensional Pythagorean theorem is determined as

$${}_{\mathrm{R}}a_{\mathrm{t}}{}^{2\mathrm{n}}+{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2\mathrm{n}}\equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2\mathrm{n}} \tag{150}$$

□

2.2.3.2.6. Pythagorean theorem and probability of an event

Theorem 2.8 (**PYTHAGOREAN THEOREM AND PROBABLITIY OF AN EVENT**).

Under conditions of special theory of relativity (see also Einstein, 1905), the probability that ${}_{\mathrm{R}}\mathrm{E_t}$ is determined by ${}_{0}\mathrm{E_t}$ is given by

$$p\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) \equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right) \tag{151}$$

by direct proof. According to Eq. 138 on page 39 it is

$${}_{\mathrm{R}}{a_{\mathrm{t}}}^2 \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{152}$$

Eq. 152 is equivalent with

$${}_{\mathrm{R}}{a_{\mathrm{t}}}^2 \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{153}$$

Dividing Eq. 153 by ${}_{\mathrm{R}}{\mathrm{X_t}}^2$ it is

$$\frac{{}_{\mathrm{R}}{a_{\mathrm{t}}}^2}{\left({}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv \frac{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left({}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}}\right)}{\left({}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{154}$$

Let us consider conditions of the special theory of relativity where ${}_{\mathrm{R}}{a_{\mathrm{t}}}^2 \equiv {}_{0}{E_{\mathrm{t}}}^2 \equiv {}_{\mathrm{R}}{E_{\mathrm{t}}}^2 \times \left(+1-\left(\frac{v^2}{c^2}\right)\right)$. Furthermore, there are conditions where ${}_{\mathrm{R}}X_{\mathrm{t}} \equiv {}_{\mathrm{R}}E_{\mathrm{t}}$ and it follows that Eq. 154 changes to

$$\begin{aligned} p\left({}_{\mathrm{R}}E_{\mathrm{t}}\right) &\equiv \frac{{}_{\mathrm{R}}{a_{\mathrm{t}}}^2}{\left({}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}}\right)} \\ &\equiv \frac{{}_{0}{E_{\mathrm{t}}}^2}{\left({}_{\mathrm{R}}{E_{\mathrm{t}}}^2\right)} \equiv \frac{{}_{\mathrm{R}}{E_{\mathrm{t}}}^2 \times \left(+1-\left(\frac{v^2}{c^2}\right)\right)}{\left({}_{\mathrm{R}}{E_{\mathrm{t}}}^2\right)} \\ &\equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right) \end{aligned} \tag{155}$$

Under conditions of special theory of relativity, the probability that total energy (relativistic energy et cetera) ${}_{\mathrm{R}}\mathrm{E_t}$ is determined by the rest-energy ${}_{0}\mathrm{E_t}$ is given by

$$p\left({}_{\mathrm{R}}E_{\mathrm{t}} = {}_{0}E_{\mathrm{t}}\right) \equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right) \tag{156}$$

□

Remark 2.1. *It need not be noisy to consider whether there exist any circumstances which might permit us to conclude that* $p\left({}_{R}E_{t} = {}_{0}E_{t}\right) \equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right)$ *indicates the probability to which a quantum mechanical entity can be regarded as being local.*

2.2.3.2.7. Pythagorean theorem and the wave function Ψ Especially in order to compute how a wave propagates and behaves like in quantum mechanics, the application of the superposition principle is of advantage. There is some evidence that **the superposition principle** has been stated by Daniel Bernoulli (1700 – 1782) in 1753 ("**Later (1753), Daniel Bernoulli formulated the principle of superposition ...**" (see Leon Brillouin, 1946, p. 2)).

Theorem 2.9. *In general, it is*

$$\left(\frac{\Psi\left({}_{R}X_{t}\right)\times\Psi\left(E\left({}_{R}X_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)\times\Psi\left({}_{R}X_{t}\right)}\right)+\left(\frac{\Psi\left({}_{R}X_{t}\right)\times\Psi\left(E\left({}_{R}\underline{X}_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)\times\Psi\left({}_{R}X_{t}\right)}\right)\equiv+1^{2} \tag{157}$$

Proof. **If** the premise

$$+1=+1 \tag{158}$$

is true, **then** the conclusion

$$\left(\frac{\Psi\left({}_{R}X_{t}\right)\times\Psi\left(E\left({}_{R}X_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)\times\Psi\left({}_{R}X_{t}\right)}\right)+\left(\frac{\Psi\left({}_{R}X_{t}\right)\times\Psi\left(E\left({}_{R}\underline{X}_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)\times\Psi\left({}_{R}X_{t}\right)}\right)\equiv+1^{2} \tag{159}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying Eq. 158 by ${}_{R}X_{t}$, it is

$${}_{R}X_{t}={}_{R}X_{t} \tag{160}$$

Based on theorem 2.7, Eq. 146, Eq. 160 changes to

$$E\left({}_{R}X_{t}\right)+E\left({}_{R}\underline{X}_{t}\right)\equiv{}_{R}X_{t} \tag{161}$$

Theoretically it is necessary to consider the possibility that there are conditions where ${}_{R}X_{t}$ is in a state of superposition of $E({}_{R}X_{t})$ and $E({}_{R}\underline{X}_{t})$. Thus far, under conditions where Eq. 161 can be described by a (linear) function $\Psi\left({}_{R}X_{t}\right)$ which satisfies the superposition principle, it is equally

$$\Psi\left(E\left({}_{R}X_{t}\right)\right)+\Psi\left(E\left({}_{R}\underline{X}_{t}\right)\right)\equiv\Psi\left(E\left({}_{R}X_{t}\right)+E\left({}_{R}\underline{X}_{t}\right)\right)\equiv\Psi\left({}_{R}X_{t}\right) \tag{162}$$

The principle of superposition and the Pythagorean theorem are the two sides of the same coin. It is

$$\Psi\left(E\left({}_{R}X_{t}\right)\right)+\Psi\left(E\left({}_{R}\underline{X}_{t}\right)\right)\equiv\Psi\left({}_{R}X_{t}\right) \tag{163}$$

Normalizing the relationship before, Eq. 163 changes slightly. It is

$$\frac{\Psi\left(E\left({}_{R}X_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)}+\frac{\Psi\left(E\left({}_{R}\underline{X}_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)}\equiv\frac{\Psi\left({}_{R}X_{t}\right)}{\Psi\left({}_{R}X_{t}\right)}\equiv+1^{1} \tag{164}$$

Multiplying Eq. 164 by $\left(\frac{\Psi\left({}_{R}X_{t}\right)}{\Psi\left({}_{R}X_{t}\right)}\right)$ it is,

$$\left(\frac{\Psi\left({}_{R}X_{t}\right)\times\Psi\left(E\left({}_{R}X_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)\times\Psi\left({}_{R}X_{t}\right)}\right)+\left(\frac{\Psi\left({}_{R}X_{t}\right)\times\Psi\left(E\left({}_{R}\underline{X}_{t}\right)\right)}{\Psi\left({}_{R}X_{t}\right)\times\Psi\left({}_{R}X_{t}\right)}\right)\equiv+1^{2} \tag{165}$$

□

Theorem 2.10 (THE GENERAL CONTRADICTION LAW). *In general, it is*

$$E\left({}_{R}\underline{X}_{t}\right) \leq \left(\frac{{}_{R}X_{t}{}^{2} \times \pi^{2} \times \hbar^{2}}{E\left({}_{R}X_{t}\right) \times h^{2}}\right) \tag{166}$$

Proof. **If** the premise

$$+1 = +1 \tag{167}$$

is true, **then** the conclusion

$$E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \leq \left(\frac{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \times \pi^{2} \times \hbar^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times h^{2}}\right) \tag{168}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying Eq. 167 by the variance of ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ denoted as $\sigma({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})^{2}$ (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42), it is

$$\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} \equiv \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} \tag{169}$$

The variance of ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$, denoted as $\sigma({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})^{2}$ (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42), is defined or has been proved as

$$\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{170}$$

In general, according to theorem 2.3, Eq. 72, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} \tag{171}$$

while theorem 2.3, Eq. 73 demands that

$$p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \equiv \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} = 1 - \frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} = 1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{172}$$

Therefore, Eq. 170 changes to

$$\begin{aligned} \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} &\equiv \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \\ &\equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}} - E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)^{2} \\ &\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \times \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(1 - \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)\right)\right) \\ &\equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \end{aligned} \tag{173}$$

Eq. 173 simplifies as

$$\begin{aligned} \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} &\equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \\ &\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \times \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(1 - \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)\right)\right) \end{aligned} \tag{174}$$

Under conditions, where the probability of a single event is not known, it is

$$\left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(1 - \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)\right)\right) \leq \frac{1}{4} \tag{175}$$

Eq. 174 changes slightly to

$$\frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\leq\left(\frac{1}{2}\times\frac{1}{2}\right) \tag{176}$$

From quantum theory, it is known that

$$\frac{1}{2}\equiv\frac{\pi\times\hbar}{h} \tag{177}$$

Eq. 176 changes to

$$\frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\leq\left(\frac{\pi^{2}\times\hbar^{2}}{h^{2}}\right) \tag{178}$$

The expectation value of anti ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$, denoted as $\mathrm{E}({}_{\mathrm{R}}\underline{\mathrm{X}}_{\mathrm{t}})$, follows approximately as

$$E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\leq\left(\frac{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\times\pi^{2}\times\hbar^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\times h^{2}}\right) \tag{179}$$

□

Eq. 179 does not give any reason for the assumption that there is a kind of uncertainty between ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ and ${}_{\mathrm{R}}\underline{\mathrm{X}}_{\mathrm{t}}$ and do not constitute in no way a new uncertainty principle. Under conditions of 4 space-time dimensions of general relativity, it is

$$\frac{1}{g_{\mu\nu}\times g^{\mu\nu}}\equiv\frac{1}{4} \tag{180}$$

Eq. 176 changes under these conditions of general relativity to

$$\frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}}\leq\frac{1}{g_{\mu\nu}\times g^{\mu\nu}} \tag{181}$$

or to

$${}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\geq E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\times g_{\mu\nu}\times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\times g^{\mu\nu} \tag{182}$$

Furthermore, under conditions where

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\equiv{}_{\mathrm{R}}X_{\mathrm{t}} \tag{183}$$

we obtain, the identity (see also Barukčić, 2020a,b, 2021c) of

$${}_{\mathrm{R}}\Delta_{\mathrm{t}}{}^{2}\equiv\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} \tag{184}$$

Especially, general relativity is related to the Pythagorean theorem. General relativity is a theory of the geometrical properties of space-time to, while the metric tensor $\mathrm{g}_{\mu\nu}$ itself is of fundamental importance for general relativity. An important differentiation with respect to the metric tensor $\mathrm{g}_{\mu\nu}$ is necessary. The metric tensor $\mathrm{g}_{\mu\nu}$ does not describe above all the gravitational field, but the gravitational potential. Einstein himself worded this fact excellently.

"... die ... Komponenten des Gravitationspotentials $g_{\mu\nu}$ ... "

(see also Einstein, 1916, p. 818)

In English: '... the ... components of the gravitational potential $g_{\mu\nu}$ ...'. The metric tensor $g_{\mu\nu}$ is something like the generalization of the Pythagorean theorem. Thus far, it does not appear to be necessary to restrict the validity of the Pythagorean theorem only to certain situations. The question is justified why the Riemannian geometry should be oppressed by the quadratic restriction. In this context, **Finsler geometry**, named after Paul Finsler (1894 - 1970) who studied it in his doctoral thesis (see Finsler, 1918) in 1918, appears to be a kind of metric generalization of Riemannian geometry without the quadratic restriction and justifies the attempt to systematize and to extend the possibilities of general relativity.

2.2.4. A circle and a right-angled triangle

2.2.4.1. The equivalence of a circle and a right-angled triangle Let ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ denote the circumference of a circle (see Book 3 of Euclid's Elements). It is

$$ {}_{\mathrm{R}}U_{\mathrm{t}} \equiv 2 \times \pi \times {}_{\mathrm{R}}r_{\mathrm{t}} \equiv \pi \times {}_{\mathrm{R}}d_{\mathrm{t}} \tag{185} $$

Let,

$$ {}_{\mathrm{R}}d_{\mathrm{t}} \equiv 2 \times {}_{\mathrm{R}}r_{\mathrm{t}} \tag{186} $$

where ${}_{\mathrm{R}}d_{\mathrm{t}}$ is the diameter of a circle and ${}_{\mathrm{R}}r_{\mathrm{t}}$ is the radius of a circle, the distance between any point of a circle and the centre of the same circle.

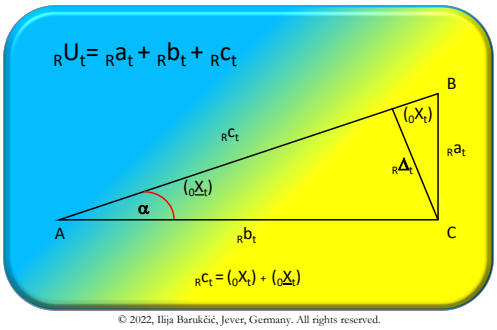


Figure 13. Circumference: right-angled triangle

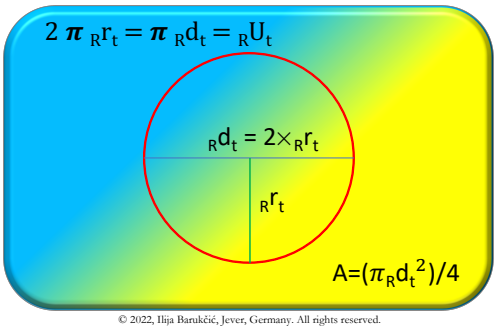


Figure 14. Circumference: circle

As can be seen, the circumference of a right-angled triangle (see figure 13) is given as

$$ {}_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}a_{\mathrm{t}} + {}_{\mathrm{R}}b_{\mathrm{t}} + {}_{\mathrm{R}}c_{\mathrm{t}} \tag{187} $$

However, in nature, under conditions where a circle passes over into a right-angled triangle and vice versa (see figure 13 and figure 14) it is

$$_{\mathrm{R}}U_{\mathrm{t}} \equiv {}_{\mathrm{R}}a_{\mathrm{t}} + {}_{\mathrm{R}}b_{\mathrm{t}} + {}_{\mathrm{R}}c_{\mathrm{t}} \equiv 2 \times \pi \times {}_{\mathrm{R}}r_{\mathrm{t}} \equiv \pi \times {}_{\mathrm{R}}d_{\mathrm{t}} \equiv {}_{\mathrm{R}}U_{\mathrm{t}} \tag{188}$$

It is known that

$$_{\mathrm{R}}c_{\mathrm{t}} \equiv \sqrt[2]{({}_{\mathrm{R}}a_{\mathrm{t}})^{2} + ({}_{\mathrm{R}}b_{\mathrm{t}})^{2}} \tag{189}$$

Equation 188 becomes

$$_{\mathrm{R}}U_{\mathrm{t}} \equiv ({}_{\mathrm{R}}a_{\mathrm{t}}) + ({}_{\mathrm{R}}b_{\mathrm{t}}) + \sqrt[2]{({}_{\mathrm{R}}a_{\mathrm{t}})^{2} + ({}_{\mathrm{R}}b_{\mathrm{t}})^{2}} \equiv \pi \times {}_{\mathrm{R}}d_{\mathrm{t}} \tag{190}$$

Max Karl Ernst Ludwig Planck related mass to frequency and introduced h, the famous Planck's constant (see also Planck, 1901, p. 87). Soon, Dirac adopted Planck's constant h. "In Order that the theory may agree with experiment, we must take $\hbar$ equal to $h/2\pi$, where h is the universal constant that was introduced by Planck, known as Planck's constant." (see also Dirac, 1947, p. 87) or

$$h = 2 \times \pi \times \hbar \tag{191}$$

Archimedes constant π, approximately equal to 3.14159265358979323846264 33..., can be calculated as

$$\pi \equiv \frac{{}_{\mathrm{R}}U_{\mathrm{t}}}{{}_{\mathrm{R}}d_{\mathrm{t}}} \equiv \frac{({}_{\mathrm{R}}a_{\mathrm{t}}) + ({}_{\mathrm{R}}b_{\mathrm{t}}) + \sqrt[2]{({}_{\mathrm{R}}a_{\mathrm{t}})^{2} + ({}_{\mathrm{R}}b_{\mathrm{t}})^{2}}}{{}_{\mathrm{R}}d_{\mathrm{t}}} \equiv \frac{h}{2 \times \hbar} \tag{192}$$

while the diameter of a circle, ${}_{\mathrm{R}}d_{\mathrm{t}}$, is given as

$${}_{\mathrm{R}}d_{\mathrm{t}} \equiv \frac{{}_{\mathrm{R}}U_{\mathrm{t}}}{\pi} \equiv \frac{({}_{\mathrm{R}}a_{\mathrm{t}}) + ({}_{\mathrm{R}}b_{\mathrm{t}}) + \sqrt[2]{({}_{\mathrm{R}}a_{\mathrm{t}})^{2} + ({}_{\mathrm{R}}b_{\mathrm{t}})^{2}}}{\pi} \equiv \frac{({}_{\mathrm{R}}a_{\mathrm{t}}) + ({}_{\mathrm{R}}b_{\mathrm{t}}) + \sqrt[2]{({}_{\mathrm{R}}a_{\mathrm{t}})^{2} + ({}_{\mathrm{R}}b_{\mathrm{t}})^{2}}}{\frac{h}{2 \times \hbar}} \tag{193}$$

Planck's constant h (see equation 192) appears to be very dynamical and can be calculated as

$$h \equiv 2 \times \frac{({}_{\mathrm{R}}a_{\mathrm{t}}) + ({}_{\mathrm{R}}b_{\mathrm{t}}) + \sqrt[2]{({}_{\mathrm{R}}a_{\mathrm{t}})^{2} + ({}_{\mathrm{R}}b_{\mathrm{t}})^{2}}}{{}_{\mathrm{R}}d_{\mathrm{t}}} \times \hbar \tag{194}$$

Especially under conditions where $({}_{\mathrm{R}}a_{\mathrm{t}}) \equiv \sqrt[2]{p\,({}_{\mathrm{R}}X_{\mathrm{t}})}$ and where $({}_{\mathrm{R}}b_{\mathrm{t}}) \equiv \sqrt[2]{p\,({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}$ Planck's constant becomes

$$h \equiv 2 \times \frac{\sqrt[2]{p\,({}_{\mathrm{R}}X_{\mathrm{t}})} + \sqrt[2]{p\,({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})} + 1}{{}_{\mathrm{R}}d_{\mathrm{t}}} \times \hbar \tag{195}$$

2.2.4.2. π and trigonometry In Euclidean geometry, the number π = 3.14159 ..., also referred to as Archimedes's constant, is defined as the ratio of a circle's circumference ${}_{\mathrm{R}}U_{\mathrm{t}}$ to its diameter ${}_{\mathrm{R}}d_{\mathrm{t}}$. The first (see Jones, William, 1706, p. 263) known use of the Greek letter π to represent the ratio of a circle's circumference ${}_{\mathrm{R}}U_{\mathrm{t}}$ to its diameter ${}_{\mathrm{R}}d_{\mathrm{t}}$ is ascribed to the Welsh mathematician William Jones (1675 – 1749) in 1706.

Theorem 2.11. *In general, it is*

$$ {}_{R}U_{t} \equiv ((sin\ \alpha) + (cos\ \alpha) + 1) \times {}_{R}c_{t} \tag{196} $$

Proof by direct proof. The premise

$$ +1 \equiv +1 \tag{197} $$

is true. In the following, we rearrange the premise. We obtain (see equation 189, p. 46)

$$ \begin{aligned} {}_{\mathrm{R}}U_{\mathrm{t}} &\equiv {}_{\mathrm{R}}a_{\mathrm{t}} + {}_{\mathrm{R}}b_{\mathrm{t}} + {}_{\mathrm{R}}c_{\mathrm{t}} \\ &\equiv 2 \times \pi \times {}_{\mathrm{R}}r_{\mathrm{t}} \\ &\equiv \pi \times {}_{\mathrm{R}}d_{\mathrm{t}} \\ &\equiv \pi \times X \times {}_{\mathrm{R}}c_{\mathrm{t}} \\ &\equiv {}_{\mathrm{R}}U_{\mathrm{t}} \end{aligned} \tag{198} $$

Equation 198 is rearranged as

$$ {}_{\mathrm{R}}a_{\mathrm{t}} + {}_{\mathrm{R}}b_{\mathrm{t}} + {}_{\mathrm{R}}c_{\mathrm{t}} \equiv \pi \times X \times {}_{\mathrm{R}}c_{\mathrm{t}} \tag{199} $$

and changes to

$$ \frac{{}_{\mathrm{R}}a_{\mathrm{t}}}{{}_{\mathrm{R}}c_{\mathrm{t}}} + \frac{{}_{\mathrm{R}}b_{\mathrm{t}}}{{}_{\mathrm{R}}c_{\mathrm{t}}} + \frac{{}_{\mathrm{R}}c_{\mathrm{t}}}{{}_{\mathrm{R}}c_{\mathrm{t}}} \equiv \pi \times X \tag{200} $$

It is $sin\ \alpha \equiv \frac{{}_{\mathrm{R}}a_{\mathrm{t}}}{{}_{\mathrm{R}}c_{\mathrm{t}}}$ and $cos\ \alpha \equiv \frac{{}_{\mathrm{R}}b_{\mathrm{t}}}{{}_{\mathrm{R}}c_{\mathrm{t}}}$ and $\frac{{}_{\mathrm{R}}c_{\mathrm{t}}}{{}_{\mathrm{R}}c_{\mathrm{t}}} \equiv +1$. Equation 200 simplifies as

$$ ((sin\ \alpha) + (cos\ \alpha) + 1) \equiv \pi \times X \tag{201} $$

The unknown value of X follows as

$$ X \equiv \frac{sin\ \alpha}{\pi} + \frac{cos\ \alpha}{\pi} + \frac{1}{\pi} \tag{202} $$

The circle's circumference ${}_{\mathrm{R}}U_{\mathrm{t}}$ i s given as (see equation 198, p. 47)

$$ \begin{aligned} {}_{\mathrm{R}}U_{\mathrm{t}} &\equiv \pi \times {}_{\mathrm{R}}d_{\mathrm{t}} \\ &\equiv 2 \times \pi \times {}_{\mathrm{R}}r_{\mathrm{t}} \\ &\equiv \pi \times X \times {}_{\mathrm{R}}c_{\mathrm{t}} \\ &\equiv \pi \times \left(\frac{sin\ \alpha}{\pi} + \frac{cos\ \alpha}{\pi} + \frac{1}{\pi} \right) \times {}_{\mathrm{R}}c_{\mathrm{t}} \\ &\equiv \pi \times \left(\frac{(sin\ \alpha) + (cos\ \alpha) + 1}{\pi} \right) \times {}_{\mathrm{R}}c_{\mathrm{t}} \end{aligned} \tag{203} $$

and finally as

$$ {}_{\mathrm{R}}U_{\mathrm{t}} \equiv ((sin\ \alpha) + (cos\ \alpha) + 1) \times {}_{\mathrm{R}}c_{\mathrm{t}} \tag{204} $$

□

Based on equation 204, π is given as the relationship

$$\pi \equiv \frac{{}_{\mathrm{R}}U_{\mathrm{t}}}{{}_{\mathrm{R}}d_{\mathrm{t}}} \equiv \frac{((sin\ \alpha)+(cos\ \alpha)+1)\times {}_{\mathrm{R}}c_{\mathrm{t}}}{{}_{\mathrm{R}}d_{\mathrm{t}}} \tag{205}$$

2.2.5. Bernoulli distribution

A single event distribution is more or less a discrete probability distribution of any random variable X which takes a certain (observer independent) single value X_t at a **Bernoulli trial** (Uspensky, 1937, p. 45) (period of time) t with the probability $p(X_t)$. The same random variable X takes a certain single anti value $\underline{X}_t$ at a Bernoulli trial (period of time) t with the probability 1-$p(X_t)$. There are conditions in nature where a random variable X can take only the values either +0 or +1 (see Birnbaum, 1961). Under these conditions, the random variable X takes the value 1 with probability $p(X_t = +1)$ and the value 0 with probability $q(X_t = +0) = 1 - p(X_t = +1)$ while the single event distribution passes over into the **Bernoulli distribution**, named after Swiss mathematician Jacob Bernoulli (Bernoulli, 1713). Less formally, many times, the Bernoulli distribution is represented by a (possibly not biased) coin toss where 1 and 0 would represent 'heads'and 'tails'(or vice versa), respectively. However, the relationship between random variables (Gosset, 1914) can be investigated by many (Gosset, 1908) methods, including the tools of probability theory, too.

Definition 2.11 (**Two by two table of single event random variables**).

The two by two or contingency table which has been introduced by Karl Pearson (Pearson, 1904b) in 1904 harbours still a large variety of topics and debates. Central to this is the problem to apply the laws of classical logic on data sets, which concerns the justification of inferences which extrapolate from sample data to general facts. Nevertheless, a contingency table is still an appropriate theoretical model too for studying the relationships between random variables, including *Bernoulli (Bernoulli, 1713) (i.e. +0/+1) distributed random variables* existing or occurring at the same *Bernoulli trial* (Uspensky, 1937) (period of time) t.

In this context, let a random variable A at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by A_t, indicate a risk factor, a condition, a cause et cetera and occur or exist with the probability $p(A_t)$ at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let $E(A_t)$ denote the expectation value of A_t. In general it is

$$p(A_t) \equiv p(a_t) + p(b_t) \tag{206}$$

The expectation value $E(A_t)$ follows as

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv A_t \times (p(a_t) + p(b_t)) \\ &\equiv (A_t \times p(a_t)) + (A_t \times p(b_t)) \\ &\equiv E(a_t) + E(b_t) \end{aligned} \tag{207}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv (+0+1) \times p(A_t) \\ &\equiv p(A_t) \\ &\equiv p(a_t) + p(b_t) \end{aligned} \tag{208}$$

Furthermore, it is

$$p(\underline{A}_t) \equiv p(c_t) + p(d_t) \equiv (1 - p(A_t)) \tag{209}$$

The expectation value E($\underline{A}_t$) is given as

$$\begin{aligned} E(\underline{A}_t) &\equiv A_t \times (1 - p(A_t)) \\ &\equiv A_t \times (p(c_t) + p(d_t)) \\ &\equiv (A_t \times p(c_t)) + (A_t \times p(d_t)) \\ &\equiv E(c_t) + E(d_t) \end{aligned} \tag{210}$$

Under conditions of +0/+1 distributed Bernoulli random variables we obtain

$$\begin{aligned} E(\underline{A}_t) &\equiv A_t \times (1 - p(A_t)) \\ &\equiv (+0+1) \times (1 - p(A_t)) \\ &\equiv (1 - p(A_t)) \\ &\equiv p(c_t) + p(d_t) \end{aligned} \tag{211}$$

Let a random variable B at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by B_t, indicate an outcome, a conditioned, an effect et cetera and occur or exist with the probability p(B_t) at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let E(B_t) denote the expectation value of B_t. In general it is

$$p(B_t) \equiv p(a_t) + p(c_t) \tag{212}$$

The expectation value E(B_t) is given by the equation

$$\begin{aligned} E(B_t) &\equiv B_t \times p(B_t) \\ &\equiv B_t \times (p(a_t) + p(c_t)) \\ &\equiv (B_t \times p(a_t)) + (B_t \times p(c_t)) \\ &\equiv E(a_t) + E(c_t) \end{aligned} \tag{213}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(B_t) &\equiv B_t \times p(B_t) \\ &\equiv (+0+1) \times p(B_t) \\ &\equiv p(B_t) \\ &\equiv p(a_t) + p(c_t) \end{aligned} \tag{214}$$

Furthermore, it is

$$p(\underline{B}_t) \equiv p(b_t) + p(d_t) \equiv (1 - p(B_t)) \tag{215}$$

The expectation value E($\underline{B}_t$) is given by the equation

$$\begin{aligned} E(\underline{B}_t) &\equiv B_t \times (1 - p(B_t)) \\ &\equiv B_t \times (p(b_t) + p(d_t)) \\ &\equiv (B_t \times p(b_t)) + (B_t \times p(d_t)) \\ &\equiv E(b_t) + E(d_t) \end{aligned} \tag{216}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(\underline{B}_t) &\equiv B_t \times (1 - p(B_t)) \\ &\equiv (+0+1) \times (1 - p(B_t)) \\ &\equiv (1 - p(B_t)) \\ &\equiv p(b_t) + p(d_t) \end{aligned} \tag{217}$$

Let $p(a_t) = p(A_t \wedge B_t)$ denote the joint probability distribution of A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(a_t) &\equiv E(A_t \wedge B_t) \\ &\equiv (A_t \times B_t) \times p(A_t \wedge B_t) \\ &\equiv (A_t \times B_t) \times p(a_t) \end{aligned} \tag{218}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(a_t) &\equiv E(A_t \wedge B_t) \\ &\equiv (A_t \times B_t) \times p(A_t \wedge B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(A_t \wedge B_t) \\ &\equiv p(A_t \wedge B_t) \\ &\equiv p(a_t) \end{aligned} \tag{219}$$

Let $p(b_t) = p(A_t \wedge \neg B_t)$ denote the joint probability distribution of A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(b_t) &\equiv E(A_t \wedge \neg B_t) \\ &\equiv (A_t \times \neg B_t) \times p(A_t \wedge \neg B_t) \\ &\equiv (A_t \times \neg B_t) \times p(b_t) \end{aligned} \tag{220}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(b_t) &\equiv E(A_t \wedge \neg B_t) \\ &\equiv (A_t \times \neg B_t) \times p(A_t \wedge \neg B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(A_t \wedge \neg B_t) \\ &\equiv p(A_t \wedge \neg B_t) \\ &\equiv p(b_t) \end{aligned} \tag{221}$$

Let $p(c_t)= p(\neg A_t \wedge B_t)$ denote the joint probability distribution of not A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(c_t) &\equiv E(\neg A_t \wedge B_t) \\ &\equiv (\neg A_t \wedge B_t) \times p(\neg A_t \wedge B_t) \\ &\equiv (\neg A_t \wedge B_t) \times p(c_t) \end{aligned} \tag{222}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(c_t) &\equiv E(\neg A_t \wedge B_t) \\ &\equiv (\neg A_t \times B_t) \times p(\neg A_t \wedge B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(\neg A_t \wedge B_t) \\ &\equiv p(\neg A_t \wedge B_t) \\ &\equiv p(c_t) \end{aligned} \tag{223}$$

Let $p(d_t)= p(\neg A_t \wedge \neg B_t)$ denote the joint probability distribution of not A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(d_t) &\equiv E(\neg A_t \times \neg B_t) \\ &\equiv (\neg A_t \times \neg B_t) \times p(\neg A_t \wedge \neg B_t) \\ &\equiv (\neg A_t \times \neg B_t) \times p(d_t) \end{aligned} \tag{224}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(d_t) &\equiv E(\neg A_t \wedge \neg B_t) \\ &\equiv (\neg A_t \times \neg B_t) \times p(\neg A_t \wedge \neg B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(\neg A_t \wedge \neg B_t) \\ &\equiv p(\neg A_t \wedge \neg B_t) \\ &\equiv p(d_t) \end{aligned} \tag{225}$$

In general, it is

$$p(a_t) + p(b_t) + p(c_t) + p(d_t) \equiv +1 \tag{226}$$

Table 1 provide us with an overview of the definitions above.

Table 1. The two by two table of Bernoulli random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition A_t	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

In our understanding, it is

$$p(B_t) + p(\Lambda_t) \equiv p(a_t) + p(c_t) + p(\Lambda_t) \equiv p(a_t) + p(b_t) \equiv p(A_t) \tag{227}$$

or

$$p(c_t) + p(\Lambda_t) \equiv p(b_t) \tag{228}$$

Under conditions of Einstein's general theory of relativity, Λ denotes the Einstein cosmological (Einstein, 1917) 'constant '.

2.2.6. Binomial random variables

The binomial (see Pearson, 1895, p. 351) distribution (see Cramér, 1937) with parameters n and p has been developed by the Swiss mathematician Jakob Bernoulli (1655-1705) in a proof published in his 1713 book Ars Conjectandi (see Bernoulli, 1713) Part 1. In probability theory and statistics, the probability of getting exactly k successes in n independent Bernoulli trials is given by the probability mass function as

$$p(X_t = k) \equiv \binom{n}{k} \cdot p^k \cdot q^{n-k} \tag{229}$$

is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ the binomial coefficient while the cumulative distribution function is given as

$$p(X_t \leq k) \equiv 1 - p(X_t > k) \equiv \sum_{t=0}^{k} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{230}$$

or as

$$p(X_t > k) \equiv 1 - p(X_t \leq k) \equiv 1 - \sum_{t=0}^{k} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{231}$$

Furthermore, it is

$$p(X_t < k) \equiv 1 - p(X_t \geq k) \equiv \sum_{t=0}^{k-1} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{232}$$

or

$$p(X_t \geq k) \equiv 1 - p(X_t < k) \equiv 1 - \sum_{t=0}^{k-1} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{233}$$

The binomial distribution is the mathematical foundation of a binomial test. The random variable X_t is counting for different things. The discrete geometric (see Feller, 1950, p. 61) distribution describes under certain circumstances the number of Bernoulli trials needed to get one success. The probability that the first occurrence of success requires k independent trials, each with success probability p, is given by the equation

$$p(X_t = k) \equiv p \cdot q^{k-1} \tag{234}$$

The negative (see Fisher, 1941, Haldane, 1941) binomial probability is a discrete probability distribution which defines the number of successes (k) in a sequence of independent and identically distributed Bernoulli trials (n) before a specified (non-random) number of failures (denoted r) occurs. The probability mass function of the negative binomial distribution is

$$p(X_t = r) \equiv \binom{k+r-1}{k-1} p^k \cdot q^r \tag{235}$$

where k is the number of successes, r is the number of failures, and p is the probability of success.

Definition 2.12 (Expectation value and variance of a binomial random variable).

The variance(see Pearson, 1904a, p. 66) of the binomial distribution with parameters n, the number of independent experiments each asking a yes–no question and p, the probability of a single event, is defined in contrast to Pearson (see Barukčić, 2022c) as

$$\sigma\left(X_{\mathrm{t}}\right)^{2} \equiv N \times N \times p\left(X_{\mathrm{t}}\right) \times \left(1 - p\left(X_{\mathrm{t}}\right)\right) \tag{236}$$

Definition 2.13 (Two by two table of Binomial random variables).

Let a, b, c, d, A, $\underline{\mathrm{A}}$, B, and $\underline{\mathrm{B}}$ denote expectation values. Under conditions where *the probability of an event, an outcome, a success et cetera is* ***constant*** *from Bernoulli trial to Bernoulli trial t*, it is

$$\begin{aligned} A &= N \times E\left(A_{\mathrm{t}}\right) \\ &\equiv N \times \left(A_{\mathrm{t}} \times p\left(A_{\mathrm{t}}\right)\right) \\ &\equiv N \times \left(p\left(A_{\mathrm{t}}\right) + p\left(B_{\mathrm{t}}\right)\right) \\ &\equiv N \times p\left(A_{\mathrm{t}}\right) \end{aligned} \tag{237}$$

and

$$\begin{aligned} B &= N \times E\left(B_{\mathrm{t}}\right) \\ &\equiv N \times \left(B_{\mathrm{t}} \times p\left(B_{\mathrm{t}}\right)\right) \\ &\equiv N \times \left(p\left(A_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right)\right) \\ &\equiv N \times p\left(B_{\mathrm{t}}\right) \end{aligned} \tag{238}$$

where N might denote the population or even the sample size. Furthermore, it is

$$a \equiv N \times \left(E\left(A_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(A_{\mathrm{t}}\right)\right) \tag{239}$$

and

$$b \equiv N \times \left(E\left(B_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(B_{\mathrm{t}}\right)\right) \tag{240}$$

and

$$c \equiv N \times \left(E\left(c_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(c_{\mathrm{t}}\right)\right) \tag{241}$$

and

$$d \equiv N \times \left(E\left(d_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(d_{\mathrm{t}}\right)\right) \tag{242}$$

and

$$a + b + c + d \equiv A + \underline{A} \equiv B + \underline{B} \equiv N \tag{243}$$

Table 2 provide us again an overview of a two by two contingency (see also Pearson, 1904b, p. 33) table of Binomial random variables.

Table 2. The two by two table of Binomial random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	a	b	A
A_t	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

"Such a table is termed a contingency table, and the ultimate scientific statement of description of the relation between two things can always be thrown back upon such a contingency table ⋯ Once the reader realizes the nature of such a table, he will have grasped the essence of the conception of association between cause and effect, and the nature of its ideal limit in causation. "

(see also Pearson, 1911, p. 159)

2.2.7. Independence

Definition 2.14 (Independence).

The philosophical, mathematical(Kolmogoroff, Andreĭ Nikolaevich, 1933) and physical(Einstein, 1948) et cetera concept of independence is of fundamental(Kolmogoroff, Andreĭ Nikolaevich, 1933) importance in (natural) sciences as such. Therefore, it is appropriate to investigate the concept of independence as completely as possible. In fact, de Moivre sums it up in his book The Doctrine of Chances (see also Moivre, 1718). "Two Events are **independent**, when they have no connexion one with the other, and that the happening of one neither forwards nor obstructs the happening of the other. Two events are **dependent**, when they are so connected together as that the Probability of either's happening is alter'd by the happening of the other. "(see also Moivre, 1756, p. 6) We should consider Kolmogorov's position on independence before the mind's eye too. "The concept of mutual independence of two or more experiments holds, in a certain sense, a central position in the theory of probability."(see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 8) Furthermore, it is insightful to recall even Einstein's theoretical approach to the concept of independence. "*Ohne die Annahme einer ⋯ Unabhängigkeit der ⋯ Dinge voneinander ⋯ wäre physikalisches Denken ⋯ nicht möglich.*"(Einstein, 1948). In general, an event A_t at the Bernoulli trial t need not, but can be independent of the existence or of the occurrence, of another event B_t *at the same* Bernoulli trial t. De Moivre brings it to the point. "From what has been said, it follows, that if a Fraction expresses the Probability of an Event, and another Fraction the Probability of another Event, and those two Events are independent ; the Probability that both those Events will Happen, will be the Product of those two Fractions."(see also Moivre, 1718, p. 4). Mathematically, in terms of probability theory, independence (Kolmogoroff, Andreĭ Nikolaevich, 1933) of events at the same (period of) time (i.e. Bernoulli trial) t

is defined as

$$p(A_t \wedge B_t) \equiv p(A_t) \times p(B_t) \equiv p(a_t)$$
$$\equiv \frac{\sum_{t=1}^{N} (A_t \wedge B_t)}{N} \equiv \frac{N \times (p(a_t))}{N} \equiv 1 - p(A_t \mid B_t) \equiv 1 - p(A_t \uparrow B_t) \tag{244}$$

while $p(A_t \cap B_t)$ is the joint probability of the events A_t and B_t at a same Bernoulli trial t, $p(A_t)$ is the probability of an event A_t at a same Bernoulli trial t, and $p(B_t)$ is the probability of an event B_t at a same Bernoulli trial t. With respect to a two-by-two table , **under conditions of independence**, it is

$$p(b_t) \equiv p(A_t) \times p(\underline{B}_t) \tag{245}$$

or

$$p(c_t) \equiv p(\underline{A}_t) \times p(B_t) \tag{246}$$

and

$$p(d_t) \equiv p(\underline{A}_t) \times p(\underline{B}_t) \tag{247}$$

Example. In a narrower sense, the conditio sine qua non relationship concerns itself at the end only with the case whether the presence of an event A_t (condition) enables or guarantees the presence of another event B_t (conditioned). Thus far, as a result of the thoughts before, another question worth asking concerns the relationship between the independence of an event A_t (a condition) and another event B_t (conditioned) and the necessary condition relationship. To be confronted with the danger of bias and equally with the burden of inappropriate conclusions drawn, another fundamental question at this stage is whether is it possible that an event A_t (a condition) is a necessary condition of event B_t (conditioned) even under circumstances where the event A_t (a condition) (a necessary condition) is independent of an event B_t (conditioned)? Meanwhile, this question is more or less already answered to the negative (Barukčić, 2018b). An event A_t which is a necessary condition of another event B_t is equally an event without which another event (B_t) could not be, could not occur, and implies as such already a kind of dependence. However, it is not mandatory that such a kind of dependence is a causal one. It is remarkable that **data which provide evidence of a significant conditio sine qua non relationship between two events like A_t and B_t and equally support the hypothesis that A_t and B_t are independent of each other are more or less self-contradictory and of very restricted or of none value for further analysis**. In fact, if the opposite view would be taken as plausible, contradictions are more or less inescapable.

2.2.8. Dependence

Definition 2.15 (Dependence).

Whilst it may be true that the occurrence of an event A_t does not affect the occurrence of an other event B_t the contrary is of no minor importance. Under these other conditions, events, trials and

random variables et cetera are dependent on each other too. The dependence of events (Barukčić, 1989, p. 57-61) is defined as

$$p\left(\underbrace{A_{\mathrm{t}} \wedge B_{\mathrm{t}} \wedge C_{\mathrm{t}} \wedge \ldots}_{n \text{ random variables}}\right) \equiv \sqrt[n]{\underbrace{p\left(A_{\mathrm{t}}\right) \times p\left(B_{\mathrm{t}}\right) \times p\left(C_{\mathrm{t}}\right) \times \ldots}_{n \text{ random variables}}} \tag{248}$$

2.2.9. Sensitivity and specificity

Definition 2.16 (Sensitivity and specificity)**.**

A (medical) test should measure what is supposed to measure. However, the extent to which a test measures what it is supposed to measure varies and is seldom equal to 100 %. In other words, it is necessary to check once and again the accuracy or the validity of a test, we have to fight it out in detail. In clinical practice, the concept of sensitivity and specificity is commonly used to quantify the diagnostic ability of a (medical) test. Sensitivity and specificity were introduced by the American [37] , [38] , [39] , [40] biostatistician Jacob Yerushalmy (see also Yerushalmy, 1947) in the year 1947. The interior logic of sensitivity and specificity is best illustrated using a conventional two- by-two (2 x 2) table (see table 3).

Table 3. Sensitivity and specificity

		Disease B_t		
		present	absent	
Test	positive	a (true positive)	b (false positive)	A
A_t	negative	c (false negative)	d (true negative)	$\underline{A}$
		B	$\underline{B}$	N

The ability of a positive test (A_t) to correctly classify an individual as diseased (B_t) is defined as the proportion of true positives that are correctly identified by the test (a) divided by the individuals being truly diseased (B_t). In general, sensitivity follows as

$$Sensitivity\,(A \mid B) \equiv p\,(a \mid B) \equiv \frac{a}{B} \tag{249}$$

The specificity of a test is the ability of a negative test ($\underline{A}_t$) to correctly classify an individual as not diseased ($\underline{B}_t$ and is defined as the proportion of true negative that are correctly identified by the test (d) divided by the individuals being truly not diseased ($\underline{B}_t$). In general, specificity is given by the equation

$$Specificity\,(\underline{A},\underline{B}) \equiv p\,(d \mid \underline{B}) \equiv \frac{d}{\underline{B}} \tag{250}$$

The positive predictive value (PPV) is defined as

$$PPV\,(A,B) \equiv \frac{a}{a+b} \tag{251}$$

[37] Yerushalmy Jacob. Statistical problems in assessing methods of medical diagnosis, with special reference to X-ray techniques. Public Health Rep. 1947 Oct 3;62(40):1432-49. PMID: 20340527.

[38] Galen RS, Gambino SR. Beyond normality-the predictive value and efficiency of medical diagnosis. New York: NY:Wiley; 1975.

[39] Altman DG, Bland JM. Diagnostic tests. 1: Sensitivity and specificity. BMJ. 1994 Jun 11;308(6943):1552. doi: 10.1136/bmj.308.6943.1552. PMID: 8019315; PMCID: PMC2540489.

[40] Parikh R, Mathai A, Parikh S, Chandra Sekhar G, Thomas R. Understanding and using sensitivity, specificity and predictive values. Indian J Ophthalmol. 2008 Jan-Feb;56(1):45-50. doi: 10.4103/0301-4738.37595. PMID: 18158403; PMCID: PMC2636062.

The negative predictive value (NPV) is defined as

$$NPV\,(A,B) \equiv \frac{d}{c+d} \tag{252}$$

Example.

The importance of sensitivity and specificity in any research should certainly not be underestimated. However, it is essential not to lose sight of the major advantages and limitations [41] of these measures. In the following, in order to avoid misconceptions about sensitivity, specificity et cetera, let us consider a test with a sensitivity of 95 % and a specificity of 95 %. A two-by-two table is used as an illustration (see table 4).

Table 4. Sensitivity and specificity

		Disease B_t		
		present	absent	
Test	positive	95	5	100
A_t	negative	5	95	100
		100	100	200

Sensitivity is calculated as

$$Sensitivity\,(A \mid B) \equiv p\,(a \mid B) \equiv 100 \times \frac{a}{B} \equiv \frac{95}{100} \equiv 95\% \tag{253}$$

There are at least two kinds of medical tests, diagnostic tests and screening tests. Depending on the type of medical test, there are other logical implications. A screening test should correctly identify all people who suffer from a certain disease or all people with a certain outcome. Therefore, the sensitivity of a screening test should be at best 100 %. Under these conditions, we obtain **without** positive test **no** disease/outcome present. However, confusion should be avoided with regard to the adequacy and usefulness of the sensitivity of a screening test. The sensitivity of a test does not take into account events which are false positive (b) or which are true negative (d), the meaning of these events is ignored completely by sensitivity. Therefore, sensitivity is blind on one eye since its inception and underestimates the extent to which a screening test is able to identify the likely presence of a condition of interest. We calculated a 95 % sensitivity while the true possibility of the test to detect a disease is (see table 4)

$$SINE\,(A,B) \equiv 100 \times \frac{a+b+d}{N} \equiv \frac{95+5+95}{200} \equiv 97.5\% \tag{254}$$

In a way similar to sensitivity, specificity is not much better. Diagnostic tests are able to identify people who do not have a certain condition. Specificity is calculated as

$$Specificity\,(\underline{A} \mid \underline{B}) \equiv p\,(d \mid \underline{B}) \equiv 100 \times \frac{d}{\underline{B}} \equiv \frac{95}{100} \equiv 95\% \tag{255}$$

[41] Trevethan R. Sensitivity, Specificity, and Predictive Values: Foundations, Pliabilities, and Pitfalls in Research and Practice. Front Public Health. 2017 Nov 20;5:307. doi: 10.3389/fpubh.2017.00307. PMID: 29209603; PMCID: PMC5701930.

However, specificity does not take into account any individuals who suffer from a disease, who do have the condition and is well-known for being imperfect because of this fact too. Specificity underestimates the possibility of a diagnostic test to detect a disease. Above, the specificity has been calculated as being 95 %. In point of fact, the ability of the test to detect a disease or the relationship **if** test positive **then** disease present is much better and has to be calculated as (see table 4)

$$IMP(A,B) \equiv \frac{a+c+d}{N} \equiv \frac{95+5+95}{200} \equiv 97.5\% \tag{256}$$

As can be seen, the test detected the disease in 97.5 % while specificity allows only 95 %. How valuable is such a measure epistemologicallly? Measures like sensitivity and specificity are blurring of the issue, do risk leading us astray and disorient us systematically again and again. These measures should be abandoned.

2.2.10. Odds ratio (OR)

Definition 2.17 (Odds ratio (OR))**.**

Odds ratios as an appropriate measure for estimating the relative risk have become widely used in medical reports of case-control studies. The odds ratio(Fisher, 1935, p. 50) is defined(Cox, 1958) as the ratio of the odds of an event occurring in one group with respect to the odds of its occurring in another group. Odds(Yule and Pearson, 1900, p. 273) ratio (OR) is a measure of association which quantifies the relationship between two binomial distributed random variables (exposure vs. outcome) and is related to Yule's (Yule and Pearson, 1900, p. 272) Q(Yule, 1912, p. 585/586). Two events A_t and B_t are regarded as independent if $(A_t, B_t) = 1$. Let

a_t = number of persons exposed to A_t and with disease B_t

b_t = number of persons exposed to A_t but without disease $\underline{B}_t$

c_t = number of persons unexposed $\underline{A}_t$ but with disease B_t

d_t = number of persons unexposed $\underline{A}_t$: and without disease $\underline{B}_t$

a_t+c_t = total number of persons with disease B_t (case-patients)

b_t+d_t = total number of persons without disease $\underline{B}_t$ (controls).

Hereafter, consider the table 5. The odds' ratio (OR) is defined as

Table 5. The two by two table of random variables

		Conditioned/Outcome B_t		
		TRUE	FALSE	
Condition/Exposure	TRUE	a_t	b_t	A_t
A_t	FALSE	c_t	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	N_t

$$\begin{aligned} OR(A_t, B_t) &\equiv \left(\frac{a_t}{b_t}\right) / \left(\frac{c_t}{d_t}\right) \\ &\equiv \left(\frac{a_t \times d_t}{b_t \times c_t}\right) \end{aligned} \tag{257}$$

Remark 2.2. *Odds ratios can support logical fallacies and cause difficulties in drawing logically consistent conclusions. The chorus of voices is growing, which demand the immediate ending(Knol, 2012, Sackett, DL and Deeks, JJ and Altman, DG, 1996) of any use of Odds ratio.*

Under conditions where (b = 0), the measure of association odds ratio will collapse, because we need to divide by zero, as can be seen at eq. 257. However, according to today's rules of mathematics,

a division by zero is neither allowed nor generally accepted as possible. It does no harm to remind ourselves that in the case $b = 0$ the event A_t is a sufficient condition of B_t. In other words, odds ratio is not able to recognize elementary relationships of objective reality. In fact, it would be a failure not to recognize how dangerous and less valuable odds ratio is.

Under conditions where ($c = 0$) odds ratio collapses too, because we need again to divide by zero, as can be seen at eq. 257. However, and again, today's rules of mathematics don't allow us a division by zero. In point of fact, in the case $c = 0$ it is more than necessary to point out that A_t is a necessary condition of B_t. In other words, odds ratio or the cross-product ratio is not able to recognize elementary relationships of nature like necessary conditions. We can and need to overcome all the epistemological obstacles as backed by odds ratio entirety. Sooner rather than later, we should give up this measure of relationship completely.

2.2.11. Relative risk (RR)

2.2.11.1. Relative risk (RR_{nc})

Definition 2.18 (Relative risk (RR_{nc}))**.**

The degree of association between the two binomial variables can be assessed by a number of very different coefficients, the relative (Cornfield, 1951, Sadowsky et al., 1953) risk is one(Barukčić, 2021d) of them. In general, relative risk RR_{nc}, which provides some evidence of a necessary condition, is defined as

$$\begin{aligned} RR(A_t,B_t)_{nc} &\equiv \frac{\frac{p(a_t)}{p(A_t)}}{\frac{p(c_t)}{p(NotA_t)}} \\ &\equiv \frac{p(a_t)\times p(NotA_t)}{p(c_t)\times p(A_t)} \\ &\equiv \frac{N\times p(a_t)\times N\times p(NotA_t)}{N\times p(c_t)\times N\times p(A_t)} \\ &\equiv \frac{a_t\times (NotA_t)}{c_t\times A_t} \\ &\equiv \frac{EER\,(A_t,B_t)}{CER\,(A_t,B_t)} \end{aligned} \tag{258}$$

That what scientist generally understand by relative risk is the ratio of a probability of an event occurring with an exposure versus the probability of an event occurring without an exposure. In other words,

relative risk = (probability(event in exposed group)) / (probability(the same event in not exposed group)).

A $RR(A_t,B_t) = +1$ means that exposure does not affect the outcome or both are independent of each other while $RR(A_t,B_t)$ less than +1 means that the risk of the outcome is decreased by the exposure. In this context, an $RR(A_t,B_t)$ greater than +1 denotes that the risk of the outcome is increased by the exposure. Widely known problems with odds ratio and relative risk are already documented in literature.

2.2.11.2. Relative risk (RR (sc))

Definition 2.19 (Relative risk (RR (sc)))**.**

The relative risk (sc), which provides some evidence of a sufficient condition, is calculated from the point of view of an outcome and is defined as

$$\begin{aligned} RR(A_t,B_t)_{sc} &\equiv \frac{\frac{p(a_t)}{p(B_t)}}{\frac{p(b_t)}{p(NotB_t)}} \\ &\equiv \frac{p(a_t)\times p(NotB_t)}{p(b_t)\times p(B_t)} \\ &\equiv \frac{N\times p(a_t)\times N\times p(NotB_t)}{N\times p(b_t)\times N\times p(B_t)} \\ &\equiv \frac{a_t\times (NotB_t)}{b_t\times B_t} \\ &\equiv \frac{OPR\,(A_t,B_t)}{CPR\,(A_t,B_t)} \end{aligned} \tag{259}$$

2.2.11.3. Relative risk reduction (RRR)

Definition 2.20 (Relative risk reduction (RRR))**.**

$$\begin{aligned} RRR\,(A_t,B_t) &\equiv \frac{CER\,(A_t,B_t)-EER\,(A_t,B_t)}{CER\,(A_t,B_t)} \\ &= 1-RR(A_t,B_t) \end{aligned} \tag{260}$$

2.2.11.4. Vaccine efficacy (VE)

Definition 2.21 (Vaccine efficacy (VE))**.**

Vaccine efficacy is defined as the percentage reduction of a disease in a vaccinated group of people as compared to an unvaccinated group of people.

$$\begin{aligned} VE\,(A_t,B_t) &\equiv 100\times (1-RR\,(A_t,B_t)) \\ &\equiv 100\times \left(\frac{CER\,(A_t,B_t)-EER\,(A_t,B_t)}{CER\,(A_t,B_t)}\right) \end{aligned} \tag{261}$$

Historically, vaccine efficacy has been designed to evaluate the efficacy of a certain vaccine by Greenwood and Yule in 1915 for the cholera and typhoid vaccines(Greenwood and Yule, 1915) and best measured using double-blind, randomized, clinical controlled trials. However, the calculated vaccine efficacy is depending too much on the study design, can lead to erroneous conclusions and is only of very limited value.

2.2.11.5. Experimental event rate (EER)

Definition 2.22 (Experimental event rate (EER))**.**

$$EER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{p(a_{\mathrm{t}})}{p(A_{\mathrm{t}})}=\frac{a_{\mathrm{t}}}{a_{\mathrm{t}}+b_{\mathrm{t}}} \tag{262}$$

Definition 2.23 (Control event rate (CER))**.**

$$CER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{p(c_{\mathrm{t}})}{p(\underline{A}_{\mathrm{t}})}=\frac{c_{\mathrm{t}}}{c_{\mathrm{t}}+d_{\mathrm{t}}} \tag{263}$$

2.2.11.6. Absolute risk reduction (ARR)

Definition 2.24 (Absolute risk reducation (ARR))**.**

$$\begin{aligned} ARR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)&\equiv\frac{p(c_{\mathrm{t}})}{p(\underline{A}_{\mathrm{t}})}-\frac{p(a_{\mathrm{t}})}{p(A_{\mathrm{t}})}\\ &=\frac{c_{\mathrm{t}}}{c_{\mathrm{t}}+d_{\mathrm{t}}}-\frac{a_{\mathrm{t}}}{a_{\mathrm{t}}+b_{\mathrm{t}}}\\ &=CER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-EER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right) \end{aligned} \tag{264}$$

2.2.11.7. Absolute risk increase (ARI)

Definition 2.25 (Absolute risk increase (ARI))**.**

$$\begin{aligned} ARI\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)&\equiv\frac{p(a_{\mathrm{t}})}{p(A_{\mathrm{t}})}-\frac{p(c_{\mathrm{t}})}{p(\underline{A}_{\mathrm{t}})}\\ &=EER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-CER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right) \end{aligned} \tag{265}$$

2.2.11.8. Number needed to treat (NNT)

Definition 2.26 (Number needed to treat (NNT))**.**

$$NNT\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{1}{CER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-EER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)} \tag{266}$$

An ideal number needed to treat(Cook and Sackett, 1995, Laupacis et al., 1988), mathematically the reciprocal of the absolute risk reduction, is NNT = 1. Under these circumstances, everyone improves with a treatment, while no one improves with control. A higher number needed to treat indicates more or less a treatment which is less effective.

2.2.11.9. Number needed to harm (NNH)

Definition 2.27 (Number needed to harm (NNH))**.**

$$NNH\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{1}{EER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-CER\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)} \tag{267}$$

The number needed to harm (Massel and Cruickshank, 2002), mathematically the inverse of the absolute risk increase, indicates at the end how many patients need to be exposed to a certain factor, in order to observe a harm in one patient that would not otherwise have been harmed.

2.2.11.10. Outcome prevalence rate (OPR)

Definition 2.28 (Outcome prevalence rate (OPR))**.**

$$OPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{p(a_{\mathrm{t}})}{p(B_{\mathrm{t}})}=\frac{a_{\mathrm{t}}}{a_{\mathrm{t}}+c_{\mathrm{t}}} \tag{268}$$

2.2.11.11. Control prevalence rate (CPR)

Definition 2.29 (Control prevalence rate (CPR))**.**

$$CPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{p(b_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})}=\frac{b_{\mathrm{t}}}{b_{\mathrm{t}}+d_{\mathrm{t}}} \tag{269}$$

Bias and confounding is present to some degree in all research. In order to assess the relationship of exposure with a disease or an outcome, a fictive control group (i.e. of newborn or of young children et cetera) can be of use too. Under certain circumstances, even a CPR = 0 is imaginable.

2.2.11.12. Absolute prevalence reduction (APR)

Definition 2.30 (Absolute prevalence reduction (APR))**.**

$$APR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv CPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-OPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right) \tag{270}$$

2.2.11.13. Absolute prevalence increase (API)

Definition 2.31 (Absolute prevalence increase (API))**.**

$$API\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv OPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-CPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right) \tag{271}$$

2.2.11.14. Relative prevalence reduction (RPR)

Definition 2.32 (Relative prevalence reduction (RPR))**.**

$$\begin{aligned} RPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right) &\equiv \frac{CPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-OPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)}{CPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)} \\ &= 1-RR(A_{\mathrm{t}},B_{\mathrm{t}})_{\mathrm{sc}} \end{aligned} \tag{272}$$

2.2.11.15. The index NNS

Definition 2.33 (The index NNS)**.**

$$NNS\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{1}{CPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-OPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)} \tag{273}$$

Mathematically, the index NNS is the reciprocal of the absolute prevalence reduction.

2.2.11.16. The index NNI

Definition 2.34 (The index NNI)**.**

$$NNI\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)\equiv\frac{1}{OPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)-CPR\left(A_{\mathrm{t}},B_{\mathrm{t}}\right)} \tag{274}$$

Mathematically, the index NNI is the reciprocal of the absolute prevalence increase.

2.2.12. Index of relationship (IOR)

Definition 2.35 (Index of relationship (IOR))**.**

Due to several reasons, it is not always easy to identify the unique characteristics between two events like A_t and B_t. And more than that, it is difficult to decide what to do, and much more difficult to know in which direction one should think and which decision is right. Sometimes it is helpful to know at least something about the direction of the relationship between two events like A_t and B_t. Under conditions where $p(a_t) = p(A_t \wedge B_t)$, the index of relationship(Barukčić, 2021b), abbreviated as IOR, is defined as

$$\begin{aligned} IOR(A_t, B_t) &\equiv \left(\frac{p(A_t \wedge B_t)}{p(B_t) \times p(A_t)} \right) - 1 \\ &\equiv \left(\frac{p(a_t)}{p(B_t) \times p(A_t)} \right) - 1 \\ &\equiv \left(\left(\frac{N \times N \times p(a_t)}{N \times p(B_t) \times N \times p(A_t)} \right) - 1 \right) \\ &\equiv \left(\left(\frac{N \times a}{A \times B} \right) - 1 \right) \end{aligned} \tag{275}$$

where $p(A_t)$ denotes the probability of an event A_t at the Bernoulli trial t and $p(B_t)$ denotes the probability of another event B_t at the same Bernoulli trial t while $p(a_t)$ denotes the joint probability of $p(A_t$ AND $B_t)$ at the same Bernoulli trial t and a, A and B may denote the expectation values.

Definition 2.36 (Multi dimensional index of relationship (NIOR))**.**

The multi dimensional index of relationship (NIOR) is defined as

$$\begin{aligned} NIOR(A_t, B_t) &\equiv \left(\frac{N^k \times p({}_1A_t \wedge {}_2A_t \cdots {}_kA_t)}{N \times (p({}_1A_t)) N \times (p({}_2A_t)) \cdots N \times (p({}_kA_t))} \right) - 1 \\ &\equiv \left(\frac{N^{k\text{-}1} \times E({}_1A_t \wedge {}_2A_t \cdots {}_kA_t)}{E({}_1A_t) \times E({}_2A_t) \cdots \times E({}_kA_t)} \right) - 1 \end{aligned} \tag{276}$$

where N is the sample size and $p({}_1A_t \wedge {}_2A_t \cdots {}_kA_t)$ is the joint distribution function.

However, there might exist circumstances where a multi dimensional index of relationship might take the form of the following equation.

$$\begin{aligned} NIOR(A_t, B_t) &\equiv \left(\frac{{}_1N \times {}_2N \times \cdots {}_kN \times p({}_1A_t \wedge {}_2A_t \cdots {}_kA_t)}{({}_1N \times p({}_1A_t)) \times ({}_2N \times p({}_2A_t)) \cdots \times ({}_kN \times p({}_kA_t))} \right) - 1 \\ &\equiv \left(\frac{{}_1N \times {}_2N \times \cdots {}_kN \times p({}_1A_t \wedge {}_2A_t \cdots {}_kA_t)}{E({}_1A_t) \times E({}_2A_t) \cdots \times E({}_kA_t)} \right) - 1 \end{aligned} \tag{277}$$

2.3. Conditions

Even if a condition and a cause are deeply related, there are circumstances where a sharp distinction between a cause and a condition is necessary. However, exactly this has been denied by John Stuart Mill's (1806-1873) regularity view of causality (see Mill, 1843b). What might seem to be a theoretical difficulty for many authors is none for Mill. Mill simply reduced a cause to a condition and claimed that "... the real cause of the phenomenon is the assemblage of all its conditions." (see Mill, 1843a, p. 403)

2.3.1. Exclusion relationship

Definition 2.37 (Exclusion relationship [*EXCL*]).

Mathematically, the exclusion(see also Barukčić, 2021a) relationship [42] (EXCL), denoted by $p(A_t \mid B_t)$ in terms of statistics and probability theory, is defined(see also Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p(A_t \mid B_t) &\equiv p(A_t \uparrow B_t) \\ &\equiv p(b_t) + p(c_t) + p(d_t) \\ &\equiv \frac{N \times (p(b_t) + p(c_t) + p(d_t))}{N} \\ &\equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \vee \underline{B}_t)}{N} \equiv \frac{b+c+d}{N} \\ &\equiv \frac{b+\underline{A}}{N} \\ &\equiv \frac{c+\underline{B}}{N} \\ &\equiv +1 \end{aligned} \tag{278}$$

Based on the 1913 Henry Maurice Sheffer (1882-1964) relationship, the Sheffer stroke(Nicod, 1917, Sheffer, 1913) usually denoted by $\uparrow$, it is $p(A_t \wedge B_t) \equiv 1 - p(A_t \mid B_t)$ (see table 6).

Table 6. A_t excludes B_t and vice versa.

		Conditioned (COVID-19) B_t		
		TRUE	FALSE	
Condition (Vaccine)	TRUE	**+0**	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

[42] Barukčić, Ilija. (2021). Mutually exclusive events. Causation, 16(11), 5–57. https://doi.org/10.5281/zenodo.5746415

Example 2.1. *Pfizer Inc. and BioNTech SE announced on Monday, November 09, 2020 - 06:45am results from a Phase 3 COVID-19 vaccine trial with 43.538 participants which provides evidence that their vaccine (BNT162b2) is preventing COVID-19 in participants without evidence of prior SARS-CoV-2 infection. In toto, 170 confirmed cases of COVID-19 were evaluated, with 8 in the vaccine group versus 162 in the placebo group. The exclusion relationship can be calculated as follows.*

$$\begin{aligned} p\left(Vaccine : BNT162b2 \mid COVID-19(infection)\right) &\equiv p\left(b_t\right)+p\left(c_t\right)+p\left(d_t\right) \\ &\equiv 1-p\left(a_t\right) \\ &\equiv 1-\left(\frac{8}{43538}\right) \\ &\equiv +0,99981625 \end{aligned} \tag{279}$$

with a P Value = 0,000184.

Following Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables A_t, B_t et cetera at the point t, we obtain

$$\begin{aligned} p\left(A_t \mid B_t\right) &\equiv p\left(\underline{A}_t \cup \underline{B}_t\right) \\ &\equiv 1-p\left(A_t \cap B_t\right) \\ &\equiv 1-\int_{-\infty}^{A_t}\int_{-\infty}^{B_t} f\left(A_t, B_t\right)\, dA_t\, dB_t \\ &\equiv +1 \end{aligned} \tag{280}$$

while $p\left(A_t \mid B_t\right)$ would denote the cumulative distribution function of random variables and $f\left(A_t, B_t\right)$ is the joint density function.

2.3.2. Observational study and exclusion relationship

Under conditions of an observational study, the exclusion relationship follows approximately(see Barukčić, 2021a) as

$$p\left(A_t \mid B_t\right) \equiv p\left(A_t \uparrow B_t\right) \geq 1-\frac{p\left(a_t\right)}{p\left(B_t\right)} \tag{281}$$

2.3.3. Experimental study and exclusion relationship

Under conditions of an experimental study, the exclusion relationship follows approximately(see Barukčić, 2021a) as

$$p\left(A_t \mid B_t\right) \equiv p\left(A_t \uparrow B_t\right) \geq 1-\frac{p\left(a_t\right)}{p\left(A_t\right)} \tag{282}$$

2.3.4. The goodness of fit test of an exclusion relationship

Definition 2.38 (**The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship**).

Under some well known circumstances, testing hypothesis about an exclusion relationship $p(A_t \mid B_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t \mid B_t) \mid A\right) &\equiv \frac{(b-(a+b))^2}{A}+ \\ &\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\ &\equiv \frac{a^2}{A}+0 \\ &\equiv \frac{a^2}{A}\end{aligned} \tag{283}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t \mid B_t) \mid B\right) &\equiv \frac{(c-(a+c))^2}{B}+ \\ &\quad \frac{((b+d)-\underline{B})^2}{\underline{B}} \\ &\equiv \frac{a^2}{B}+0 \\ &\equiv \frac{a^2}{B}\end{aligned} \tag{284}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of an exclusion relationship/distribution $p(A_t \mid B_t)$, in which case the null hypothesis has to be accepted. Yate's (Yates, 1934) continuity correction was not used under these circumstances.

2.3.5. The left-tailed p Value of an exclusion relationship

Definition 2.39 (**The left-tailed p Value of an exclusion relationship**).

It is known that as a sample size, N, increases, a sampling distribution of a special test statistic approaches the normal distribution (central limit theorem). Under these circumstances, the left-tailed

(lt) p Value (Barukčić, 2019e) of an exclusion relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \mid B_{\mathrm{t}}\right) &\equiv 1-e^{-(1-p(A_{\mathrm{t}}|B_{\mathrm{t}}))} \\ &\equiv 1-e^{-(a/N)} \end{aligned} \tag{285}$$

A low p-value may provide some evidence of statistical significance.

2.3.6. Neither nor conditions

Definition 2.40 (Neither A_t nor B_t conditions [*NOR*]).

Mathematically, a neither A_t nor B_t condition (or rejection according to the French philosopher and logician Jean George Pierre Nicod (1893-1924), i.e. Jean Nicod's statement (Nicod, 1924)) relationship (NOR), denoted by $p(A_t \downarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) &\equiv p\left(d_{\mathrm{t}}\right) \\ &\equiv \frac{N-\sum_{t=1}^{N}\left(A_{\mathrm{t}} \vee B_{\mathrm{t}}\right)}{N} \equiv \frac{\sum_{t=1}^{N}\left(\underline{A}_{\mathrm{t}} \wedge \underline{B}_{\mathrm{t}}\right)}{N} \equiv \frac{N \times\left(p\left(d_{\mathrm{t}}\right)\right)}{N} \\ &\equiv \frac{d}{N} \\ &\equiv +1 \end{aligned} \tag{286}$$

2.3.7. The Chi square goodness of fit test of a neither nor condition relationship

Definition 2.41 (The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship).

A neither A_t nor B_t condition relationship $p(A_t \downarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution). The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship with degree of freedom (d. f.) of d. f. = 1 may be calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}\left(\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) \mid A\right) &\equiv \frac{\left(d-(c+d)\right)^2}{\underline{A}}+ \\ &\quad \frac{\left((a+b)-A\right)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}}+0 \end{aligned} \tag{287}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_\text{t}\downarrow B_\text{t})\mid B\right) &\equiv \frac{\left(d-(b+d)\right)^2}{\underline{B}}+\\ &\quad\frac{\left((a+c)-B\right)^2}{B}\\ &\equiv \frac{b^2}{\underline{B}}+0\end{aligned} \tag{288}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.8. The left-tailed p Value of a neither nor B condition relationship

Definition 2.42 (The left-tailed p Value of a neither A_t nor B_t condition relationship).

The left-tailed (lt) p Value (Barukčić, 2019e) of a neither A_t nor B_t condition relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_\text{t}\downarrow B_\text{t}\right) &\equiv 1-e^{-(1-p(A_\text{t}\downarrow B_\text{t}))}\\ &\equiv 1-e^{-p(A_\text{t}\vee B_\text{t})}\\ &\equiv 1-e^{-((a+b+c)/N)}\end{aligned} \tag{289}$$

where $\vee$ may denote disjunction or logical inclusive or. In this context, a low p-value indicates again a statistical significance. In general, it is $p\left(A_\text{t}\vee B_\text{t}\right)\equiv 1-p\left(A_\text{t}\downarrow B_\text{t}\right)$ (see table 7).

Table 7. Neither A_t nor B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	0	0
	NO	0	1	1
		0	1	1

2.3.9. Necessary condition

Definition 2.43 (**Necessary condition [*Conditio sine qua non*]**).

Despite the most extended efforts, the current state of research on conditions and conditioned is still incomplete and very contradictory. However, even thousands of years ago and independently of any human mind and consciousness, water has been and is still a necessary (see Barukčić, 2022b) condition for (human) life. **Without** water, there has been and there is **no** (human) life[43]. It comes therefore as no surprise that one of the first documented attempts to present a rigorous theory of conditions and causation (see also Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica III 2 997a 10 and 13/14) came from the Greek philosopher and scientist Aristotle (384-322 BCE). Thus far, it is amazing that Aristotle himself made already a strict distinction between conditions and causes. Taking Aristotle very seriously, it is necessary to consider that

> "... everything which has a ... ... potency in question ... ... has the potency ... of acting ... not in all circumstances but on certain conditions ... " (see also Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica IX 5 1048a 14-19)

Before going into details, Aristotle went on to define the necessary condition as follows.

> "... necessary ... means ...
>
> without ... a condition, a thing cannot live ... "
>
> (see also Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica V 2 1015a 20-22)

In point of fact, Aristotle developed a theory of conditions and causality commonly referred to as the doctrine of four causes. Many aspects and general features of Aristotle's logical concept of causality are meanwhile extensively and critically debated in secondary literature. However, even if the Greek philosophers Heraclitus, Plato, Aristotle et cetera numbers among the greatest philosophers of all time, the philosophy has evolved. Scientific knowledge and objective reality are deeply interrelated and cannot be reduced only to Greek philosophers like Aristotle. Among many other issues, the specification of necessary conditions has traditionally been part of the philosopher's investigations of different phenomena. However, behind the need of a detailed evidence, it is justified to consider that philosophy or philosophers as such certainly do not possess **a monopoly on the truth** and other areas such as medicine as well as other sciences and technology may transmit truths as well and may be of help to move beyond one's self enclosed unit. Seemingly, **the law's concept of causation** justifies to say few words on this subject, to put some light on some questions. Are there any criteria in law for deciding whether one action or an event A_t has caused another (generally harmful) event B_t? What are these criteria? May causation in legal contexts differ from causation outside the law, for example, in science

[43] Barukčić, Ilija. (2022). Conditio sine qua non (Version 1). Zenodo. https://doi.org/10.5281/zenodo.5854744

or in our everyday life and to what extent? Under which circumstances is it justified to tolerate such differences as may be found to exist? To understand just what is the law's concept of causation, it is useful to re-consider how the highest court of states is dealing with causation. In the case *Hayes v. Michigan Central R. Co., 111 U.S. 228*, the U.S. Supreme Court defined 1884 conditio sine qua non as follows: "... **causa sine qua non – a cause which, if it had not existed, the injury would not have taken place**". (Justice Matthews, Mr., 1884) The German Bundesgerichtshof für Strafsachen stressed once again the importance of conditio sine qua non relationship in his decision by defining the following: "**Ursache eines strafrechtlich bedeutsamen Erfolges jede Bedingung, die nicht hinweggedacht werden kann, ohne daß der Erfolg entfiele**"(Bundesgerichtshof für Strafsachen, 1951) Another lawyer elaborated on the basic issue of **identity and difference between cause and condition**. Von Bar was writing: "Die erste Voraussetzung, welche erforderlich ist, damit eine Erscheinung als die Ursache einer anderen bezeichnet werden könne, ist, daß jene eine der Bedingungen dieser sein. Würde die zweite Erscheinung auch dann eingetreten sein, wenn die erste nicht vorhanden war, so ist sie in keinem Falle Bedingung und noch weniger Ursache. Wo immer ein Kausalzusammenhang behauptet wird, da muß er wenigstens diese Probe aushalten ... **Jede Ursache ist nothwendig auch eine Bedingung eines Ereignisses; aber nicht jede Bedingung ist Ursache zu nennen**."(Bar, Carl Ludwig von, 1871) Von Bar's position translated into English: *The first requirement, which is required, thus that something could be called as the cause of another, is that the one has to be one of the conditions of the other. If the second something had occurred even if the first one did not exist, so it is by no means a condition and still less a cause. Wherever a causal relationship is claimed, the same must at least withstand this test... Every cause is necessarily also a condition of an event too; but not every condition is cause too.* Thus far, let us consider among other the following in order to specify necessary conditions from another, probabilistic point of view. An event (i.e. A_t) which is a necessary condition of another event or outcome (i.e. B_t) must be given, must be present for a conditioned, for an event or for an outcome B_t to occur. A necessary condition (i.e. A_t) is a requirement which need to be fulfilled **at every single Bernoulli trial t**, in order for a conditioned or an outcome (i.e. B_t) to occur, but it alone does not determine the occurrence of such an event. In other words, if a necessary condition (i.e. A_t) is given, an outcome (i.e. B_t) need not to occur. In contrast to a necessary condition, a 'sufficient'condition is the one condition which 'guarantees'that an outcome will take place or will occur for sure. Under which conditions we may infer about the unobserved and whether observations made are able at all to justify predictions about potential observations which have not yet been made or even general claims which my go even beyond the observed (*the 'problem of induction')* is not the issue of the discussion at this point. Besides of the principal necessity of meeting such a challenge, a necessary condition of an event can but need not be at the same Bernoulli trial t a sufficient condition for an event to occur. However, theoretically, it is possible that an event or an outcome is determined by many necessary conditions. Let us focus to some extent on what this means, or in other words how much importance can we attribute to such a special case. *Example*. A human being cannot live without oxygen. A human being cannot live without water. A human being cannot live without a brain. A human being cannot live without kidneys. A human being cannot live without ... et cetera. Thus far, even if oxygen is given, if a brain is given ... et cetera, without water a human being will not survive on the long run. This example is of use to reach the following conclusion. Although it might seem somewhat paradoxical at first sight, **even under circumstances where a condition or an outcome depends on several different necessary conditions it is particularly important that every single of**

these necessary conditions for itself must be given otherwise the conditioned (i.e. the outcome) will not occur. Mathematically, the necessary condition (SINE) relationship, denoted by $p(A_t \leftarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 15-28) as

$$\begin{aligned} p\left(A_t \leftarrow B_t\right) \equiv p\left(A_t \vee \underline{B}_t\right) &\equiv \frac{\sum_{t=1}^{N}\left(A_t \vee \underline{B}_t\right)}{N} \equiv \frac{\left(A_t \vee \underline{B}_t\right) \times p\left(A_t \vee \underline{B}_t\right)}{\left(A_t \vee \underline{B}_t\right)} \\ &\equiv p\left(a_t\right) + p\left(b_t\right) + p\left(d_t\right) \\ &\equiv \frac{N \times \left(p\left(a_t\right) + p\left(b_t\right) + p\left(d_t\right)\right)}{N} \equiv \frac{E\left(A_t \leftarrow B_t\right)}{N} \\ &\equiv \frac{a+b+d}{N} \equiv \frac{E\left(A_t \vee \underline{B}_t\right)}{N} \\ &\equiv \frac{A+d}{N} \equiv \frac{E\left(A_t \leftarrow B_t\right)}{N} \\ &\equiv \frac{a+\underline{B}}{N} \equiv \frac{E\left(A_t \vee \underline{B}_t\right)}{N} \\ &\equiv +1 \end{aligned} \tag{290}$$

where $E\left(A_t \leftarrow B_t\right) \equiv E\left(A_t \vee \underline{B}_t\right)$ indicates the expectation value of the necessary condition. In general, it is $p\left(A_t \prec B_t\right) \equiv 1 - p\left(A_t \leftarrow B_t\right)$ (see Table 8).

Table 8. Necessary condition.

		Conditioned B_t		
		TRUE	FALSE	
Condition A_t	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
	FALSE	**+0**	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

A necessary condition A_t is characterised itself by the property that another event B_t will not occur if A_t is not given, if A_t did not occur (Barukčić, 1989, 1997, 2005, 2016b, 2017b,c, 2020a,b,c,d, Barukčić and Ufuoma, 2020). Taking into account Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables A_t, B_t et cetera at the (period of) time t, we obtain

$$\begin{aligned} p\left(A_t \leftarrow B_t\right) &\equiv +1 \\ &\equiv +1 - p\left(c_t\right) \\ &\equiv +1 - p\left(\underline{A}_t \cap B_t\right) \\ &\equiv \left(\int_{-\infty}^{A_t}\int_{-\infty}^{B_t} f\left(A_t, B_t\right)\, dA_t\, dB_t\right) + \left(1 - \int_{-\infty}^{B_t} f\left(B_t\right)\, dB_t\right) \end{aligned} \tag{291}$$

while $p\left(A_t \leftarrow B_t\right)$ would denote the cumulative distribution function of random variables of a necessary condition. Another adequate formulation of a necessary condition is possible too. If certain conditions

are met, then necessary conditions and sufficient conditions are one way or another converses of each other, too. It is

$$p\left(A_t \leftarrow B_t\right) \equiv \underbrace{\left(A_t \vee \underline{B}_t\right)}_{\text{(Necessary condition)}} \equiv \underbrace{\left(\underline{B}_t \vee A_t\right)}_{\text{(Sufficient condition)}} \equiv p\left(B_t \rightarrow A_t\right) \tag{292}$$

These relationships are illustrated by the following tables.

Table 9. Without A_t no B_t

		B_t		
		TRUE	FALSE	
A_t	TRUE	a_t	b_t	A_t
	FALSE	$c_t = 0$	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	+1

Table 10. If B_t then A_t

		A_t		
		TRUE	FALSE	
B_t	TRUE	a_t	$c_t = 0$	B_t
	FALSE	b_t	d_t	$\underline{B}_t$
		A_t	$\underline{A}_t$	+1

There are circumstances under which

$$p\left(A_t \leftarrow B_t\right) \equiv \underbrace{\left(A_t \vee \underline{B}_t\right)}_{\text{(Nessessary condition)}} \equiv \underbrace{\left(\underline{A}_t \vee B_t\right)}_{\text{(Sufficient condition)}} \equiv p\left(A_t \rightarrow B_t\right) \tag{293}$$

However, equation 292 does not imply the relationship of equation 293 under any circumstances.

Example I.

A wax candle is characterised by various properties, but is also subject to certain conditions. **Without** sufficient amounts of gaseous oxygen **no** burning wax candle, gaseous oxygen is a necessary condition of a burning candle. However, the converse relationship **if** burning wax candle, **then** sufficient amounts of gaseous oxygen are given is is at the same (period of) time t / Bernoulli trial t true. The following tables are illustrating these relationships.

Table 11. Without gaseous oxygen no burning candle

		Burning candle		
		TRUE	FALSE	
Gaseous	TRUE	a_t	b_t	A_t
oxygen	FALSE	$c_t = 0$	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	+1

Table 12. If burning candle then gaseous oxygen

		Gaseous oxygen		
		TRUE	FALSE	
Burning	TRUE	a_t	$c_t = 0$	B_t
candle	FALSE	b_t	d_t	$\underline{B}_t$
		A_t	$\underline{A}_t$	+1

Example II.

Once again, a human being cannot live without water. A human being cannot live without gaseous oxygen, et cetera. Water itself is a necessary condition for human life. However, gaseous oxygen is a necessary condition for human life too. Thus far, even if water is given and even if water is a necessary condition for human life, without gaseous oxygen there will be no human life. In general, if a conditioned or an outcome B_t depends on the necessary condition A_t and equally on numerous other

necessary conditions, an event B_t will not occur if A_t itself is not given independently of the occurrence of other necessary conditions.

Example III.

Another different aspect of a necessary condition relationship is appropriate to be focused upon here. As a direct consequence of a necessary condition **without** sufficient amounts of gaseous oxygen **no** burning wax candle is a special case of an exclusion relationship. The absence of sufficient amounts of gaseous oxygen A_t excludes (see Barukčić, 2021a) a burning wax candle B_t. Thus far, if we want to stop the burning of a wax candle, we would have to significantly reduce the amounts of gaseous oxygen A_t. Under these conditions, a wax candle will stop burning. The following tables (table 13 and table 14) may illustrate this aspect of a necessary condition in more detail.

Table 13. Without gaseous oxygen no burning candle

		Burning candle		
		TRUE	FALSE	
Gaseous oxygen	TRUE	a_t	b_t	A_t
	FALSE	$c_t = 0$	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	+1

Table 14. Absent gaseous oxygen excludes burning wax candle

		Burning candle		
		TRUE	FALSE	
Gaseous oxygen	FALSE	$c_t = 0$	d_t	B_t
	TRUE	a_t	b_t	$\underline{B}_t$
		A_t	$\underline{A}_t$	+1

The necessary condition relationship follows approximately (see Barukčić, 2022b) as

$$p\left(A_t \leftarrow B_t\right) \geq 1 - \frac{p\left(c_t\right)}{p\left(B_t\right)} \tag{294}$$

and as

$$p\left(A_t \leftarrow B_t\right) \geq 1 - \frac{p\left(c_t\right)}{p\left(\underline{A}_t\right)} \tag{295}$$

2.3.10. The Chi-square goodness of fit test of a necessary condition relationship

Definition 2.44 (The $\tilde{\chi}^2$ goodness of fit test of a necessary condition relationship).

Under some well known circumstances, hypothesis about the conditio sine qua non relationship $p(A_t \leftarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or χ^2-distribution), first described by the German statistician Friedrich Robert Helmert (Helmert, 1876) and later rediscovered by Karl Pearson (Pearson, 1900a) in the context of a goodness of fit test. The $\tilde{\chi}^2$ goodness of fit test of a conditio sine qua non relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \leftarrow B_{\text{t}} \mid B\right) &\equiv \frac{\left(a-(a+c)\right)^2}{B}+ \\ &\quad \frac{\left((b+d)-\underline{B}\right)^2}{\underline{B}} \\ &\equiv \frac{c^2}{B}+0 \\ &\equiv \frac{c^2}{B}\end{aligned} \tag{296}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \leftarrow B_{\text{t}} \mid \underline{A}\right) &\equiv \frac{\left(d-(c+d)\right)^2}{\underline{A}}+ \\ &\quad \frac{\left((a+b)-A\right)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}}+0 \\ &\equiv \frac{c^2}{\underline{A}}\end{aligned} \tag{297}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. It has not yet been finally clarified whether the use of Yate's (Yates, 1934) continuity correction is necessary at all.

2.3.11. The left-tailed p Value of the conditio sine qua non relationship

Definition 2.45 (The left-tailed p Value of the conditio sine qua non relationship).

The left-tailed (lt) p Value (Barukčić, 2019e) of the conditio sine qua non relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_{\text{t}} \leftarrow B_{\text{t}}\right) &\equiv 1-e^{-\left(1-p\left(A_{\text{t}} \leftarrow B_{\text{t}}\right)\right)} \\ &\equiv 1-e^{-(c/N)}\end{aligned} \tag{298}$$

2.3.12. Sufficient condition

Definition 2.46 (**Sufficient condition [*Conditio per quam*]**).

Mathematically, the sufficient (Barukčić, 2021c, p. 68-70) condition (see Barukčić, 2022a) (IMP) relationship, denoted by $p(A_t \rightarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p(A_t \rightarrow B_t) \equiv p(\underline{A}_t \vee B_t) &\equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \vee B_t)}{N} \equiv \frac{(\underline{A}_t \vee B_t) \times p(\underline{A}_t \vee B_t)}{(\underline{A}_t \vee B_t)} \\ &\equiv p(a_t) + p(c_t) + p(d_t) \\ &\frac{N \times (p(a_t) + p(c_t) + p(d_t))}{N} \\ &\equiv \frac{a+c+d}{N} \equiv \frac{E(\underline{A}_t \vee B_t)}{N} \\ &\equiv \frac{B+d}{N} \equiv \frac{E(A_t \rightarrow B_t)}{N} \\ &\equiv \frac{a+\underline{A}}{N} \\ &\equiv +1 \end{aligned} \tag{299}$$

In general, it is $p(A_t \succ\!\!- B_t) \equiv 1 - p(A_t \rightarrow B_t)$ (see Table 15).

2.3.12.1. Mackie's INUS Condition John Leslie Mackie (1917-1981) critically examined the theories of causation of various (see Ducasse, 1926) philosophers such as Hume (Book I, Part III, of the Treatise) (see Mackie, 1974, pp. 3-28), Kant (as well as Kantian approaches offered by Strawson and Bennett), Mill and other. Mackie rightly claims that Hume's regularity theory of causation offer only an incomplete picture of the nature of causation. Mackie writes: "It seems appropriate to begin by examining and criticizing it, so that we can take over from it whatever seems to be defensible but develop an improved account by correcting its errors and deficiencies." (see Mackie, 1974, p. 3). Nonetheless, in his trial to develop an improved account of Hume's theory of causation, Mackie's own account of the nature of causation follows Hume's principles of causation very closely (see Mackie, 1974, pp. 3-28). Mackie himself proposed already in 1965 that "the so-called cause is ... an *insufficient* but *necessary* part of a condition which is itself *unnecessary* but *sufficient* for the result ... let us call such a condition ... an INUS condition." (see Mackie, 1965, p. 245). However Mackie's account needs modification, and can be modified and when it is modified we can explain much more satisfactorily what Mackie ordinarily take to be a cause. Mackie is of the opinion that "... cause is ... part of a condition ... " (see Mackie, 1965, p. 245) and that "... a condition ... is ... *unnecessary* but *sufficient* for the result [i. e. effect, author]. " (see Mackie, 1965, p. 245). To put it very simply one could say that Mackie reduces a cause to a sufficient condition, "... cause is ... a condition which is itself ... *sufficient* ... " (see Mackie, 1965, p. 245). Indeed, there are circumstances, where several

different events [44] might be necessary or sufficient et cetera at the same time in order to determine **a compound/complex sufficient condition relationship**. Thus far, it seems appropriate to take over from Mackie's INUS condition whatever seems to be acceptable but to develop an improved account by correcting its deficiencies and errors in order to do justice to the complexity of affairs. Equation 300 illustrates one real-world example of a compound/complex sufficient condition relationship in more detail.

$$\begin{aligned} p\left(\left(\left({}_1X_\mathrm{t} \wedge {}_2X_\mathrm{t} \wedge {}_3X_\mathrm{t} \wedge \cdots\right) \wedge A_\mathrm{t}\right) \rightarrow B_\mathrm{t}\right) &\equiv p\left(\underline{\left(\left({}_1X_\mathrm{t} \wedge {}_2X_\mathrm{t} \wedge {}_3X_\mathrm{t} \wedge \cdots\right) \wedge A_\mathrm{t}\right)} \vee B_\mathrm{t}\right) \\ &\equiv \frac{\sum_{t=1}^{N}\left(\underline{\left(\left({}_1X_\mathrm{t} \wedge {}_2X_\mathrm{t} \wedge {}_3X_\mathrm{t} \wedge \cdots\right) \wedge A_\mathrm{t}\right)} \vee B_\mathrm{t}\right)}{N} \\ &\equiv +1 \end{aligned} \tag{300}$$

Again, taking into account Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables $\mathrm{A_t}$, $\mathrm{B_t}$ et cetera at the (period of) time t, we obtain

$$\begin{aligned} p\left(A_\mathrm{t} \rightarrow B_\mathrm{t}\right) &\equiv +1 \\ &\equiv +1 - p\left(b_\mathrm{t}\right) \\ &\equiv +1 - p\left(A_\mathrm{t} \cap \underline{B}_\mathrm{t}\right) \\ &\equiv \left(\int_{-\infty}^{A_\mathrm{t}} \int_{-\infty}^{B_\mathrm{t}} f\left(A_\mathrm{t}, B_\mathrm{t}\right)\, dA_\mathrm{t}\, dB_\mathrm{t}\right) + \left(1 - \int_{-\infty}^{A_\mathrm{t}} f\left(A_\mathrm{t}\right)\, dA_\mathrm{t}\right) \end{aligned} \tag{301}$$

while $p\left(A_\mathrm{t} \rightarrow B_\mathrm{t}\right)$ would denote the cumulative distribution function of random variables of a sufficient condition. Another adequate formulation of a sufficient condition is possible too.

Table 15. Sufficient condition.

		Conditioned $\mathrm{B_t}$		
		TRUE	FALSE	
Condition	TRUE	$\mathrm{p(a_t)}$	**+0**	$\mathrm{p(A_t)}$
$\mathrm{A_t}$	FALSE	$\mathrm{p(c_t)}$	$\mathrm{p(d_t)}$	$\mathrm{p(\underline{A}_t)}$
		$\mathrm{p(B_t)}$	$\mathrm{p(\underline{B}_t)}$	+1

Remark 2.3. *A sufficient condition A_t is characterized by the property that another event B_t will occur if A_t is given, if A_t itself occured (Barukčić, 1989, 1997, 2005, 2016b, 2017b,c, 2020a,b,c,d, Barukčić and Ufuoma, 2020).* ***Example****. The ground, the streets, the trees, human beings and many other objects too will become wet during heavy rain. Especially,* ***if*** *it is raining (event A_t),* ***then*** *human beings will become wet (event B_t). However, even if this is a common human wisdom, a human being equipped with an appropriate umbrella (denoted by R_t) need not become wet even during heavy rain. An appropriate umbrella (R_t) is similar to an event with the potential to counteract the occurrence of another event*

[44] Barukčić, Ilija. (2022). Conditio per quam. Causation, 17(3), 5–86. https://doi.org/10.5281/zenodo.6369831

*(B_t) and can be understood something as an **anti-dot** of another event. In other words, an appropriate umbrella is an antidote of the effect of rain on human body, an appropriate umbrella has the potential to protect humans from the effect of rain on their body. It is a good rule of thumb that the following relationship*

$$p\left(A_t \to B_t\right) + p\left(R_t \wedge B_t\right) \equiv +1 \tag{302}$$

indicates that R_t is an antidote of A_t. However, taking a shower, swimming in a lake et cetera may make human hair wet too. More than anything else, however, these events does not affect the final outcome, the effect of raining on human body.

The approximate (see Barukčić, 2022a) value of the material implication is given as

$$p\left(A_{\mathrm{t}} \to B_{\mathrm{t}}\right) \geq 1 - \frac{p\left(b_{\mathrm{t}}\right)}{p\left(A_{\mathrm{t}}\right)} \tag{303}$$

and alternatively as

$$p\left(A_{\mathrm{t}} \to B_{\mathrm{t}}\right) \geq 1 - \frac{p\left(b_{\mathrm{t}}\right)}{p\left(\underline{B}_{\mathrm{t}}\right)} \tag{304}$$

2.3.13. The Chi square goodness of fit test of a sufficient condition relationship

Definition 2.47 (The $\tilde{\chi}^2$ goodness of fit test of a sufficient condition relationship).

Under some well known circumstances, testing hypothesis about the conditio per quam relationship $\mathrm{p(A_t \to B_t)}$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a conditio per quam relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} {\tilde{\chi}^2}_{\text{Calculated}}\left(A_{\mathrm{t}} \to B_{\mathrm{t}} \mid A\right) &\equiv \frac{\left(a-(a+b)\right)^2}{A} + \\ &\quad \frac{\left((c+d)-\underline{A}\right)^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + 0 \\ &\equiv \frac{b^2}{A} \end{aligned} \tag{305}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \rightarrow B_{\text{t}} \mid \underline{B}\right) &\equiv \frac{\left(d-(b+d)\right)^2}{\underline{B}}+ \\ &\quad \frac{\left((a+c)-B\right)^2}{B} \\ &\equiv \frac{b^2}{\underline{B}}+0 \\ &\equiv \frac{b^2}{\underline{B}}\end{aligned} \tag{306}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of the conditio per quam relationship/distribution $\text{p}(\text{A}_\text{t} \rightarrow \text{B}_\text{t})$, in which case the null hypothesis is accepted. Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.14. The left-tailed p Value of the conditio per quam relationship

Definition 2.48 (The left-tailed p Value of the conditio per quam relationship).

The left-tailed (lt) p Value (Barukčić, 2019e) of the conditio per quam relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_{\text{t}} \rightarrow B_{\text{t}}\right) &\equiv 1-e^{-(1-p(A_{\text{t}} \rightarrow B_{\text{t}}))} \\ &\equiv 1-e^{-(b/N)}\end{aligned} \tag{307}$$

Again, a low p-value indicates a statistical significance.

2.3.15. Necessary and sufficient conditions

Definition 2.49 (Necessary and sufficient conditions [*EQV*]).

The necessary and sufficient condition (EQV) relationship, denoted by $\text{p}(\text{A}_\text{t} \leftrightarrow \text{B}_\text{t})$ in terms of

statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}\right) &\equiv \frac{\sum_{t=1}^{N}\left(\left(A_{\mathrm{t}} \vee \underline{B}_{\mathrm{t}}\right) \wedge \left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right)\right)}{N} \\ &\equiv p\left(a_{\mathrm{t}}\right) + p\left(d_{\mathrm{t}}\right) \\ &\equiv \frac{N \times \left(p\left(a_{\mathrm{t}}\right) + p\left(d_{\mathrm{t}}\right)\right)}{N} \\ &\equiv \frac{a+d}{N} \\ &\equiv +1 \end{aligned} \tag{308}$$

2.3.16. The Chi square goodness of fit test of a necessary and sufficient condition relationship

Definition 2.50 (The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship).

Even the necessary and sufficient condition relationship $\mathrm{p}(\mathrm{A_t} \leftrightarrow \mathrm{B_t})$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} {\tilde{\chi}^2}_{\text{Calculated}}\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}} \mid A\right) &\equiv \frac{\left(a-\left(a+b\right)\right)^2}{A} + \\ &\quad \frac{d-\left(\left(c+d\right)\right)^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + \frac{c^2}{\underline{A}} \end{aligned} \tag{309}$$

or equally as

$$\begin{aligned} {\tilde{\chi}^2}_{\text{Calculated}}\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}} \mid B\right) &\equiv \frac{\left(a-\left(a+c\right)\right)^2}{B} + \\ &\quad \frac{d-\left(\left(b+d\right)\right)^2}{\underline{B}} \\ &\equiv \frac{c^2}{B} + \frac{b^2}{\underline{B}} \end{aligned} \tag{310}$$

The calculated $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship can be compared with a theoretical chi-square value at a certain level of significance α. Under conditions where the observed values are equal to the expected/theoretical values of a necessary and sufficient condition relationship/distribution $\mathrm{p}(\mathrm{A_t} \leftrightarrow \mathrm{B_t})$, the $\tilde{\chi}^2$-distribution equals zero. It is to be cleared whether Yate's (Yates, 1934) continuity correction should be used at all.

2.3.17. The left-tailed p Value of a necessary and sufficient condition relationship

Definition 2.51 (**The left-tailed p Value of a necessary and sufficient condition relationship**).

The left-tailed (lt) p Value (Barukčić, 2019e) of a necessary and sufficient condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}\right) &\equiv 1-e^{-(1-p(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}))} \\ &\equiv 1-e^{-((b+c)/N)} \end{aligned} \tag{311}$$

In this context, a low p-value indicates again a statistical significance. Table 16 may provide an overview of the theoretical distribution of a necessary and sufficient condition.

Table 16. Necessary and sufficient condition.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	1	0	1
	NO	0	1	1
		1	1	2

2.3.18. Either or conditions

Definition 2.52 (**Either A_t or B_t conditions [*NEQV*]**).

Mathematically, an either A_t or B_t condition relationship (NEQV), denoted by $p(A_t \succ\!\prec B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p\left(A_{\mathrm{t}} \succ\!\prec B_{\mathrm{t}}\right) &\equiv \frac{\sum_{t=1}^{N}\left(\left(A_{\mathrm{t}} \wedge \underline{B}_{\mathrm{t}}\right) \vee \left(\underline{A}_{\mathrm{t}} \wedge B_{\mathrm{t}}\right)\right)}{N} \\ &\equiv p\left(b_{\mathrm{t}}\right)+p\left(c_{\mathrm{t}}\right) \\ &\equiv \frac{N \times \left(p\left(b_{\mathrm{t}}\right)+p\left(c_{\mathrm{t}}\right)\right)}{N} \\ &\equiv \frac{b+c}{N} \\ &\equiv +1 \end{aligned} \tag{312}$$

It is $p\left(A_{\mathrm{t}} \succ\!\prec B_{\mathrm{t}}\right) \equiv 1-p\left(A_{\mathrm{t}} <\!\!> B_{\mathrm{t}}\right)$ (see Table 17).

Table 17. Either A_t or B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	1	1
	NO	1	0	1
		1	1	2

2.3.19. The Chi-square goodness of fit test of an either or condition relationship

Definition 2.53 (**The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship**).

An either or condition relationship $p(A_t \succ\!\prec B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t \succ\!\prec B_t) \mid A\right) &\equiv \frac{(b-(a+b))^2}{A}+ \\ &\quad \frac{c-((c+d))^2}{\underline{A}} \\ &\equiv \frac{a^2}{A}+\frac{d^2}{\underline{A}}\end{aligned} \tag{313}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t \succ\!\prec B_t) \mid B\right) &\equiv \frac{(c-(a+c))^2}{B}+ \\ &\quad \frac{b-((b+d))^2}{\underline{B}} \\ &\equiv \frac{a^2}{B}+\frac{d^2}{\underline{B}}\end{aligned} \tag{314}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.20. The left-tailed p Value of an either or condition relationship

Definition 2.54 (**The left-tailed p Value of an either or condition relationship**).

The left-tailed (lt) p Value (Barukčić, 2019e) of an either or condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \succ\!\prec B_{\mathrm{t}}\right) &\equiv 1 - e^{-(1-p(A_{\mathrm{t}}>-<B_{\mathrm{t}}))} \\ &\equiv 1 - e^{-((a+d)/N)} \end{aligned} \tag{315}$$

In this context, a low p-value indicates again a statistical significance.

2.4. Causation

2.4.1. Causation in general

The history of the denialism of causality in Philosophy, Mathematics, Statistics, Physics et cetera is very long. We only recall David Hume's (1711-1776) account of causation and his inappropriate reduction of the cause-effect relationship to a simple habitual connection in human thinking or Immanuel Kant's (1724-1804) initiated trial to consider causality as nothing more but a '*a priori*'given category (Langsam, 1994) in human reasoning and other similar attempts too.

It is worth noting in this context that especially Karl Pearson (1857 - 1936) himself has been engaged in a long lasting and never-ending crusade against causation too. "**Pearson categorically denies the need for an independent concept of causal relation beyond correlation ... he *exterminated* causation from statistics before it had a chance to take root** "(see Pearl, 2000, p. 340).

At the beginning of the 20th century notable proponents of **conditionalism** like the German anatomist and pathologist David Paul von Hansemann (Hansemann, David Paul von, 1912) (1858 - 1920) and the biologist and physiologist Max Richard Constantin Verworn(Verworn, 1912) (1863 - 1921) started a new attack(Kröber, 1961) on the principle of causality. In his essay "Kausale und konditionale Weltanschauung"Verworn(Verworn, 1912) presented "an exposition of 'conditionism'as contrasted with 'causalism,'(Unknown, 1913) while ignoring cause and effect relationships completely. "**Das Ding ist also identisch mit der Gesamtheit seiner Bedingungen**."(Verworn, 1912) However, Verworn's goal to exterminate causality completely out of science was hindered by the further development of research.

The history of futile attempts to refute **the principle of causality** culminated in a publication by the German born physicist Werner Karl Heisenberg (1901 - 1976). Heisenberg put forward an illogical, inconsistent and confusing uncertainty principle which opened the door to wishful thinking and logical fallacies in physics and in science as such. Heisenberg's unjustified reasoning ended in an act of a manifestly unfounded conclusion: "**Weil alle Experimente den Gesetzen der Quantenmechanik und damit der Gleichung (1) unterworfen sind, so wird durch die Quantenmechanik die Ungültigkeit des Kausalgesetzes definitiv festgestellt**."(Heisenberg, Werner Karl, 1927) while 'Gleichung (1)'denotes Heisenberg's uncertainty principle. Einstein's himself, a major contributor to quantum theory and in the same respect a major critic of quantum theory, disliked Heisenberg's uncertainty principle fundamentally while Einstein's opponents used Heisenberg's Uncertainty Principle against Einstein. After the End of the German Nazi initiated Second World War with unimaginable brutality and high human losses and a death toll due to an industrially organised mass killing of people by the German Nazis which did not exist in this way before, Werner Heisenberg visited Einstein in Princeton (New Jersey, USA) in October 1954 (Neffe, 2006). Einstein agreed to meet Heisenberg only for a very short period of time but their encounter lasted longer. However, there where not only a number of differences between Einstein and Heisenberg, these two physicists did not really loved each other. "Einstein remarked that the inventor of the uncertainty principle was a 'big Nazi'... "(Neffe, 2006) Albert Einstein (1879 - 1955) took again the opportunity to refuse to endorse **Heisenberg's uncertainty principle**

as a fundamental law of nature and rightly too. Meanwhile, Heisenberg's uncertainty principle is refuted (see Barukčić, 2011a, 2014, 2016a) for several times but still not exterminated completely out of physics and out of science as such.

In contrast to such extreme anti-causal positions as advocated by Heisenberg and the **Copenhagen interpretation of quantum mechancis**, the search for a (mathematical) solution of *the issue of causal inferences* is as old as human mankind itself ("*i. e. Aristotle's Doctrine of the Four Causes*") (Hennig, 2009) even if there is still little to go on.

It is appropriate to specify especially the position of D'Holbach(Holbach, Paul Henri Thiry Baron de, 1770). D'Holbach (1723-1789) himself linked cause and effect or causality as such to changes. "**Une *cause*, est un être qui e met un autre en mouvement, ou qui produit quelque changement en lui. *L'effet* est le changement qu'un corps produit dans un autre ...**"(Holbach, Paul Henri Thiry Baron de, 1770). D'Holbach infers in the following: "**De l'action et de la réaction continuelle de tous les êtres que la nature renferme, il résulte une suite de causes et d'effets ...**"(Holbach, Paul Henri Thiry Baron de, 1770).

With more or less meaningless or none progress on the matter in hand even in the best possible conditions, it is not surprising that authors are suggesting more and more different approaches and models for causal inference. Indeed, the hope is justified that logically consistent *statistical methods of causal inference* can help scientist to achieve so much with so little.

One of the methods of causal inference in Bio-sciences are based on the known *Henle(Henle, 1840) (1809–1885) - Koch(Koch, 1878) (1843–1910) postulates* (Carter, 1985) which are applied especially for the identification of a causative agent of an (infectious) disease. However, the pathogenesis of most chronic diseases is more or less very complex and potentially involves the interaction of several factors. In practice, from the 'pure culture' requirement of the Henle-Koch postulates insurmountable difficulties may emerge. In light of subsequent developments (PCR methodology, immune antibodies et cetera) it is appropriate to review the full validity of the Henle-Koch postulates in our days.

In 1965, Sir Austin Bradford Hill (Hill, 1965) published nine criteria (the '*Bradford Hill Criteria* ') in order to determine whether observed epidemiological associations are causal. Somewhat worrying, is at least the fact that, Hill's "... fourth characteristic is *the temporal relationship of the association* " and so-to-speak just a reformulation of the '*post hoc ergo propter hoc'(Barukčić, 1989, Woods and Walton, 1977)* logical fallacy through the back-door and much more then this. It is questionable whether association as such can be treated as being identical with causation. Unfortunately, due to several reasons, it seems therefore rather problematic to rely on Bradford Hill Criteria carelessly.

Meanwhile, several other and competing mathematical or statistical approaches for causal inference have been discussed by various modern authors (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c, Bohr, 1937, Chisholm, 1946, Dempster, 1990, Espejo, 2007, Goodman, 1947, Granger, 1969, Hessen, Johannes, 1928, Hesslow, 1976, 1981, Korch, Helmut, 1965, Lewis, David Kellogg, 1973, 1974, Pearl, 2000, Schlick, Friedrich Albert Moritz, 1931, Spohn, 1983, Suppes, 1970, Todd, 1968, Zesar, 2013) or even established (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c). Nevertheless, the question is still

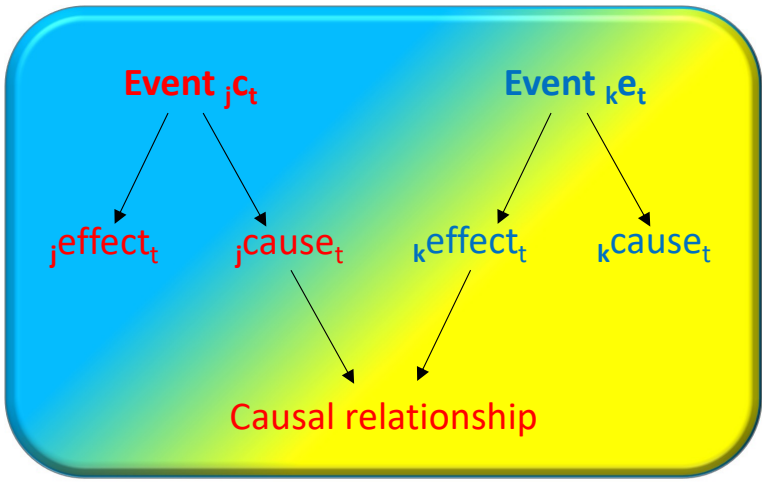
not answered, is it at all possible to establish a cause effect relationship between two factors while applying only certain statistical (Sober, 2001) methods?

2.4.2. Cause and effect

Besides all, there are several further aspects of causation for which our attention so far has not been adequately fixed in this context. In the causal relationship, cause and effect are united, a cause is an effect and an effect is a cause.

> "Thus, in the causal relation, cause and effect are inseparable; a cause which had no effect would not be a cause, just as an effect which had no cause would no longer be an effect. "
>
> (see Hegel, Georg Wilhelm Friedrich, 1991, p. 151)

The unity of cause and effect is a unity of two which are not the same. Cause and effect as inseparable in the causal relation are at the same time mutually related as sheer others; each of both as united in its own self to the other of itself is able to passes over into its own other and vice versa. Yet, to approach from a different point of view, a cause and an effect are separated in the same relation too, a cause is not an effect and an effect is not a cause, both are different in the same relation.



"Therefore, though the cause has an effect and is at the same time itself effect, and the effect not only has a cause but is also itself cause, yet the effect which the cause has, and the effect which it is, are different, as are also the cause which the effect has, and the cause which it is." (see Hegel, Georg Wilhelm Friedrich, 1991, p. 565/566)

2.4.2.1. What is a cause, what is an effect? An important fact to which we must pay attention here is that in a causal relation, under certain circumstances, an individual cause and an individual effect are related to each other in their own particular way. An effect which vanishes in its own cause in the same respect equally becomes again in it and vice versa. A cause which is merely extinguished in its own effect becomes again in the same. In fact, each of these determinations presupposes in its own other its own self and constitutes the intimate tie between an individual cause and its own individual

effect. Thus far, under conditions of a **positive** causal relationship k, an event U_t which is for sure a cause of another event W_t is at the same time t a necessary and sufficient condition of an event W_t. Table 18 may illustrate this relationship. A matter of great theoretical importance is the fundamental

Table 18. What is the cause, what is the effect?

		Effect W_t		
		TRUE	FALSE	
Cause	TRUE	**+1**	**+0**	$p(U_t)$
U_t	FALSE	**+0**	**+1**	$p(\underline{U}_t)$
		$p(W_t)$	$p(\underline{W}_t)$	+1

relationship between a cause and a condition. Are both, cause and condition, at the end identical? As of now, following Mill (see Mill, 1843a, p. 403), Verworn (see Verworn, 1912), Mackie and others, we can give a clear 'Yes'in reply to this question: "... cause is ... a condition which is itself ... *sufficient* ... " (see Mackie, 1965, p. 245). However, this issue is not as simple as it sounds, according to Mackie. Thus far, it is essential to eliminate some errors. Indeed, there are circumstances where a cause and a condition are identical, a cause and a condition are equivalent. However, as outlined in this publication, both, a cause and a condition, are different too and a cause and a condition are not identical either.

> "Jede Ursache ist nothwendig auch eine Bedingung eines Ereignisses;
> aber nicht jede Bedingung ist Ursache zu nennen. "
>
> (see Bar, Carl Ludwig von, 1871, p. 4)

The crux of the matter is that not every condition is a cause too, in German: "... nicht jede Bedingung ist Ursache ... "(see Bar, Carl Ludwig von, 1871, p. 4). However, and in contrast to a condition, every cause as such is indeed a condition too, in German: "Jede Ursache ist ... auch eine Bedingung ... "(see Bar, Carl Ludwig von, 1871, p. 4). In general, a cause U_t is a necessary condition of an effect W_t. In other words, **without** a cause U_t **no** effect W_t. One consequence of the necessary condition relationship between cause and effect is that "... an effect which had no cause would no longer be an effect. " (see Hegel, Georg Wilhelm Friedrich, 1991, p. 151). However, a cause U_t being a necessary condition of an effect W_t is equivalent to an effect W_t being a sufficient condition of the same cause U_t and vice versa too. In our everyday words,

without

U_t

no

W_t

is equivalent with

if

W_t

then

U_t

and vice versa. As can be seen, there is a kind of strange mirroring between U_t and W_t at the same Bernoulli trial t. Lastly, both are converses of each other too. In other words, U_t's being a necessary condition of W_t's is equivalent to W_t's being a sufficient condition of U_t's (and vice versa). In general, it is

$$(U_t \vee \underline{W}_t) \equiv (\underline{W}_t \vee U_t) \equiv ((U_t \vee \underline{W}_t) \wedge (\underline{W}_t \vee U_t)) \equiv +1 \tag{316}$$

		Effect W_t		
		TRUE	FALSE	
Cause	TRUE	a_t	b_t	U_t
U_t	FALSE	$c_t = 0$	d_t	$\underline{U}_t$
		W_t	$\underline{W}_t$	+1

Table 19. Without U_t no W_t

		Cause U_t		
		TRUE	FALSE	
Effect	TRUE	a_t	$c_t = 0$	W_t
W_t	FALSE	b_t	d_t	$\underline{W}_t$
		U_t	$\underline{U}_t$	+1

Table 20. If W_t then U_t

The other side of the causal relation at the same (period of) time / Bernoulli trial t is the fact that a cause U_t is equally a sufficient condition of an effect W_t too or shortly **if** cause U_t **then** effect W_t. One straightforward consequence of this fundamental relationship between a cause and an effect is that "... a cause which had no effect would not be a cause ... " (see Hegel, Georg Wilhelm Friedrich, 1991, p. 151). But even this is not without difficulties, because a cause U_t being a sufficient condition of an effect W_t is equivalent to effect W_t being a necessary condition of the same cause U_t. In different words,

if

U_t

then

W_t

is equivalent with

without

W_t

no

U_t.

		Effect W_t		
		TRUE	FALSE	
Cause	TRUE	a_t	$b_t = 0$	U_t
U_t	FALSE	c_t	d_t	$\underline{U}_t$
		W_t	$\underline{W}_t$	+1

Table 21. If U_t then W_t

		Cause U_t		
		TRUE	FALSE	
Effect	TRUE	a_t	c_t	W_t
W_t	FALSE	$b_t = 0$	d_t	$\underline{W}_t$
		U_t	$\underline{U}_t$	+1

Table 22. Without W_t no U_t

To bring it to the point, necessary and sufficient conditions are at the end converses (see Gomes, Gilberto, 2009) of each other and far more than this. In fact, there is a kind of reciprocity or mirroring between cause and effect. Necessary and sufficient conditions are relationships used to describe the relationship between two events at the same Bernoulli trial t. In more detail, if U_t then W_t is equivalent with W_t is necessary for U_t, because the truth of U_t guarantees the truth of W_t. In general, it is

$$(\underline{U}_t \vee W_t) \equiv (W_t \vee \underline{U}_t) \equiv ((\underline{U}_t \vee W_t) \wedge (W_t \vee \underline{U}_t)) \equiv +1 \tag{317}$$

In other words, it is impossible to have U_t without W_t (Bloch, 2011). Similarly, U_t is sufficient for W_t, because U_t being true always implies that W_t is true, but U_t not being true does not always imply that W_t is not true. And we should use this relationships to make our point. In general, **without** gaseous oxygen (U_t), there is **no** burning wax candle (W_t); hence the relationship **if** burning wax candle (W_t) **then** gaseous oxygen (U_t) is equally true and given. This everyday knowledge is known and secured since centuries and might be illustrated as follows.

		Wax candle B_t		
		burning	not burning	
Gaseous	present	a_t	b_t	A_t
oxygen_t	not present	$c_t = 0$	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	+1

Table 23. Without A_t no B_t

		Gaseous oxygen A_t		
		present	not present	
Wax candle	burning	a_t	$c_t = 0$	B_t
	not burning	b_t	d_t	$\underline{B}_t$
		A_t	$\underline{A}_t$	+1

Table 24. If B_t then A_t

Nonetheless, and independently of this secured everyday knowledge, **a burning wax candle is a sufficient condition of gaseous oxygen but not the cause of gaseous oxygen**.

Given all the circumstances, it is at least this simple **counter-example** which provides us with a convincing evidence that **a sufficient condition alone is not enough to describe a cause completely**. In general, a cause as such cannot be reduced to a simple sufficient condition.

In contrast to this obvious fact, other authors prefer another approach to the definition of a cause. "So that, more explicitly, if a given particular event is regarded as having been sufficient to the occurrence of another, it is said to have been its cause; if regarded as having been necessary to the occurrence of another, it is said to have been a condition of it; ..." (see Ducasse, 1926, p. 58). Therefore, in order

to be a cause of oxygen, additional evidence is necessary that a burning wax candle is a necessary condition of gaseous oxygen too. However, even if the relationship **without** gaseous oxygen **no** burning wax candle is given, this relationship is not given vice versa. The relationship **without** burning wax candle **no** gaseous oxygen is not given. Like other fundamental concepts, the concepts of cause and effect can be associated with difficulties too. Under certain conditions, the causal relationship between U_t and W_t, when correctly defined and recognised, is closely allied with the requirement that a certain study or that at least other, different studies provided evidence of a necessary condition between U_t and W_t and of a sufficient condition between U_t and W_t and if possible of **a necessary and sufficient condition** between U_t and W_t too.

Mathematically, a necessary and sufficient condition between U_t and W_t is defined as

$$(U_t \vee \underline{W}_t) \wedge (\underline{U}_t \vee W_t) \equiv +1 \tag{318}$$

However, I think it necessary to make a clear distinction between a necessary and sufficient condition and the converse relationship (Eq. 316) above.

$$((U_t \vee \underline{W}_t) \wedge (\underline{W}_t \vee U_t)) \neq (U_t \vee \underline{W}_t) \wedge (\underline{U}_t \vee W_t) \tag{319}$$

2.4.2.2. The direction of causation In general, a cause is related to its own effect in its own way and vice versa (see Mackie, 1966, p. 160) too. The effect (see Black, 1956) of this cause is itself related to its own cause in some way in which the cause is not related to its own effect (see Dummett and Flew, 1954). This can be considered as one of the reasons why the relation between cause and effect is taken to be asymmetrical.

2.4.2.3. The priority of cause to effect Contemporary discussions of causation are greatly influenced by the causal relation that 'an effect W_t is causally dependent upon a cause U_t'. However, under certain conditions (mono-causality), to say that 'an effect W_t is causally dependent upon a cause U_t'is to say that 'if a cause U_t had not occurred, then an effect W_t would not have occurred too.' (see Lewis, David Kellogg, 1973, 1974). However, what came first, the hen or the egg, the cause or the effect?

2.4.3. Definition causal relationship k

Definition 2.55 (Causal relationship k).

Nonetheless, mathematically, the causal(Barukčić, 2011a,b, 2012) relationship (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c, 2021c) between a cause U_t (German: Ursache) and an effect W_t (German: Wirkung), denoted by k(U_t, W_t), is defined *at each single(Thompson, 2006) Bernoulli trial t* in terms of statistics and probability theory [45] , [46] , [47] as

$$\begin{aligned} k\left(U_t,W_t\right) &\equiv \frac{\sigma\left(U_t,W_t\right)}{\sigma\left(U_t\right)\times\sigma\left(W_t\right)} \\ &\equiv \frac{p\left(U_t\wedge W_t\right)-p\left(U_t\right)\times p\left(W_t\right)}{\sqrt[2]{\left(p\left(U_t\right)\times\left(1-p\left(U_t\right)\right)\right)\times\left(p\left(W_t\right)\times\left(1-p\left(W_t\right)\right)\right)}} \end{aligned} \tag{320}$$

where σ (U_t , W_t) denotes the co-variance between a cause U_t and an effect W_t *at every single Bernoulli trial t*, σ (U_t) denotes the standard deviation of a cause U_t at the same single Bernoulli trial t, σ (W_t) denotes the standard deviation of an effect W_t at same single Bernoulli trial t. Table 25 illustrates the theoretically possible relationships between a cause and an effect.

Table 25. Sample space and the causal relationship k

		Effect B_t		
		TRUE	FALSE	
Cause A_t	TRUE	p(a_t)	p(b_t)	p(U_t)
	FALSE	p(c_t)	p(d_t)	p($\underline{U}_t$)
		p(W_t)	p($\underline{W}_t$)	+1

However, even if one thinks to recognise the trace of Bravais (Bravais, 1846) (1811-1863) - Pearson's (1857-1936) "*product-moment coefficient of correlation*"(Galton, 1877, Pearson, 1896) inside the causal relationship k (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c) both are completely different. According to Pearson: "*The fundamental theorems of correlation were for the first time and almost exhaustively discussed by B r a v a i s ('Analyse mathematique sur les probabilities des erreurs de situation d'un point.' Memoires par divers Savans, T. IX., Paris, 1846, pp. 255-332) nearly half a century ago.*"(Pearson, 1896) Neither does it make much sense to elaborate once again on the issue causation(Blalock, 1972) and correlation, since both are not identical (Sober, 2001) nor does it make sense to insist on the fact that "*Pearson's philosophy discouraged him from looking too far behind phenomena.*"(Haldane, 1957) Whereas it is essential to consider that the causal relationship k, in contrast to Pearson's product-moment coefficient of correlation(Pearson, 1896) or to Pearson's phi

[45] Ilija Barukčić, "The Mathematical Formula of the Causal Relationship k," International Journal of Applied Physics and Mathematics vol. 6, no. 2, pp. 45-65, 2016. https://doi.org/10.17706/ijapm.2016.6.2.45-65

[46] Barukčić, Ilija. (2015). The Mathematical Formula Of The Causal Relationship k. https://doi.org/10.5281/zenodo.3944666

[47] Ilija Barukčić. The causal relationship k. MATEC Web Conf., 336 (2021) 09032 DOI: https://doi.org/10.1051/matecconf/202133609032

coefficient(Pearson, 1904b), is defined at every single Bernoulli trial t. This might be a very *small* difference. However, even a small difference might determine a difference. However, in this context and in any case, this small difference *makes*(Barukčić, 2018a) the difference.

2.5. *Axioms*

Whether science needs new and obviously generally valid statements (axioms) which are able to assure the truth of theorems proved from them may remain an unanswered question. In order to be accepted, a new axiom candidate (see Easwaran, 2008) should be at least as simple as possible and logically consistent to enable advances in our knowledge of nature. The importance of axioms is particularly emphasized by Albert Einstein. "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**." (see Einstein, 1919, p. 17). In general, lex identitatis, lex contradictionis and lex negationis have the potential to denote the most simple, the most general and the most far-reaching axioms of science, the foundation of our today's and of our future scientific inquiry.

2.5.1. Principium identitatis (Axiom I)

Principium identitatis or **lex identitatis** or axiom I, is closely related to central problems of metaphysics, epistemology and of science as such. It turns out that it is more than rightful to assume that

$$+1 \equiv +1 \tag{321}$$

is true, otherwise there is every good reason to suppose that nothing can be discovered at all.

Identity as the epitome of a self-identical or of self-reference is at the same time different from difference, identity is free from difference, identity is not difference, identity is at the same time the other of itself, identity is non-identity. Identity as simple equality with itself is determined by a non-being, by a non-being of its own other, by a non-being of difference, identity is different from difference. Identity is in its very own nature different and is in its own self the opposite of itself (symmetry). It is equally

$$-1 \equiv -1 \tag{322}$$

In general, +1 and -1 are distinguished, however these distinct are related to one and the same 1. Identity as a vanishing of otherness, therefore, is this distinguishedness in one relation. It is

$$0 \equiv +1-1 \equiv 0 \times 1 \equiv 0 \tag{323}$$

Identity, as the unity of something and its own other is in its own self a separation from difference, and as a moment of separation might pass over into an equivalence relation which itself is reflexive, symmetric and transitive. Nonetheless, backed by thousands of years of often bitter human experience, the scientific development has taught us all that human knowledge is relative too. Even if experiments and other suitable proofs are of help to encourage us more and more in our belief of the correctness of a theory, it is difficult to prove the correctness of a theorem or of a theory et cetera once and for all. The challenge for all the science is the need to comply with Einstein's position: **"Niemals aber kann die Wahrheit einer Theorie erwiesen werden. Denn niemals weiß man, daß auch in Zukunft eine Erfahrung bekannt werden wird, die Ihren Folgerungen widerspricht..."** (Einstein, 1919).

Albert Einstein's position translated into English: 'But the truth of a theory can never be proven. For one never knows if future experience will contradict its conclusion; and furthermore, there are always other conceptual systems imaginable which might coordinate the very same facts.'Our human experience tells us that everything in life is more or less transitory, and that nothing lasts. As a result of our knowledge and experience, several scientific theories have a glorious past to look back on, but all the glory of such scientific theories might remain in the past if scientist don't continue to innovate. In a word, theories can be refuted by time.

> "No amount of experimentation can ever prove me right;
> **a single experiment can prove me wrong**."
>
> (Albert Einstein according to: Robertson, 1998, p. 114)

In the light of the foregoing, it is clear that appropriate axioms and conclusions derived from the same are a main logical foundation of any 'theory'.

> "**Grundgesetz (Axiome)** und **Folgerungen** zusammen bilden das was man **eine 'Theorie'** nennt.
> "
>
> (Einstein, 1919)

However, another point is worth being considered again. One single experiment can be enough to refute a whole theory. Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, an axiom as a free creation of the human mind which precedes all science should be like all other axioms, as simple as possible and as self-evident as possible. Historically, the earliest documented use of **the law of identity** can be found in Plato's dialogue Theaetetus (185a) as "... each of the two is different from the other and the same as itself "[48] . However, Aristotle (384–322 B.C.E.), Plato's pupil and equally one of the greatest philosophers of all time, elaborated on the law of identity too. In Metaphysica, Aristotle wrote:

> "... all things ... have some unity and identity. "
>
> (see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica, Chapter IV, 999a, 25-30, p. 66)

[48] Plato's dialogue Theaetetus (185a), p. 104.

In Prior Analytics, [49] , [50] Aristotle, a tutor of Alexander, the thirteen-year-old son of Philip, the king of Macedon, is writing: "When A applies to the whole of B and of C, and is other predicated of nothing else, and B also applies to all C, A and B must be convertible. For since A is stated only of B and C, and B is predicated both of itself and of C, it is evident that B will also be stated of all subjects of which A is stated, except A itself. "[51] , [52] For the sake of completeness, it should be noted at the outset that Aristotle himself preferred **the law of contradiction** and **the law of excluded middle** as examples of fundamental axioms. Nonetheless, it is worth noting that **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally the most simple axiom of science but has been treated by Aristotle in an inadequate manner without having any clear and determined meaning for Aristotle himself. Nonetheless, something which is really just itself is equally different from everything else. In point of fact, is such an equivalence (Degen, 1741) which everything has to itself inherent or must the same be constructed by human mind and consciousness. Can and how can something be **identical with itself** (Förster and Melamed, 2012, Hegel, Georg Wilhelm Friedrich, 1812a, Koch, 1999, Newstadt, 2015) and in the same respect different from itself. An increasingly popular view on identity is the one advocated by Gottfried Wilhelm Leibniz (1646-1716):

"**Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra**
A est A,
B est B. "
(Leibniz, 1765, p. 327)

or **A = A**, **B = B** or **+1 = +1**. In other words, a thing is what it is (Leibniz, 1765, p. 327). Leibniz' **principium identitatis indiscernibilium** (p.i.i.), the principle of the indistinguishable, occupies a central position in Leibniz' logic and metaphysics and was formulated by Leibniz himself in different ways in different passages (1663, 1686, 1704, 1715/16). All in all, Leibniz writes:

"C'est
le principe des indiscernables,
en vertu duquel
il ne saurait exister dans la nature deux êtres identiques.
...
Il n'y a point deux individus indiscernables. "
(see Leibniz, Gottfried Wilhelm, 1886, p. 45)

Exactly in complete compliance with Leibniz, Johann Gottlieb Fichte (1762 - 1814) elaborates on this subject as follows:

[49] Aristotle, Prior Analytics, Book II, Part 22, 68a
[50] Kenneth T. Barnes. Aristotle on Identity and Its Problems. Phronesis. Vol. 22, No. 1 (1977), pp. 48-62 (15 pages)
[51] Aristotle, Prior Analytics, Book II, Part 22, 68a, p. 511.
[52] Ivo Thomas. On a passage of Aristotle. Notre Dame J. Formal Logic 15(2): 347-348 (April 1974). DOI: 10.1305/ndjfl/1093891315

"**Each thing is what it is ;**
it has those realities which are posited when it is posited,
(A = A.) "
(Fichte, 1889)

Georg Wilhelm Friedrich Hegel (1770 – 1831) himself objected the Law of Identity by claiming that "A = A is ... an empty tautology. "(see Hegel, Georg Wilhelm Friedrich, 1991, p. 413) provided an example of his own mechanical understanding of the Law of Identity. "the empty tautology: nothing is nothing; ... from nothing only nothing becomes ... nothing remains nothing. "(see Hegel, Georg Wilhelm Friedrich, 1991, p. 84). Nonetheless, Hegel preferred to reformulate an own version of Leibniz principium identitatis indiscernibilium in his own way by writing that "All things are different, or: there are no two things like each other. "(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422). Much of the debate about identity is still a matter of controversy. This issue has attracted the attention of many authors and has been discussed by Hegel too. In this context, it is worth to consider Hegel's radical position on identity.

"The other expression of the law of identity: A cannot at the same time be A and not-A, has a negative form; it is called
the law of contradiction. "
(Hegel, Georg Wilhelm Friedrich, 1991, p. 416)

We may, usefully (see Barukčić, 2019a), state Russell's position with respect to the identity law as mentioned in his book 'The problems of philosophy ' (see Russell, 1912). In particular, according to Russell,

"...principles have been singled out by tradition under the name of 'Laws of Thought.' They are as follows:
(1) **The law of identity**: 'Whatever is,is.
(2)**The law of contradiction**: 'Nothing can both be and not be.'
(3) The law of excluded middle: 'Everything must either be or not be.'
These three laws are samples of self-evident logical principles, but are not really more fundamental or more self-evident than various other similar principles: for instance, the one we considered just now, which states that what follows from a true premise is true. The name 'laws of thought' is also misleading, for what is important is not the fact that we think in accordance with these laws, but the fact that **things behave in accordance with them**; "

(see Russell, 1912, p. 113)

Russell's critique, that we tend too much to focus only on the formal aspects of the 'Laws of Thoughts' with the consequence that "... we thing in accordance with these laws" (see Russell, 1912, p. 113) is

justified. Judged solely in terms of this aspect, it is of course necessary to think in accordance with the 'Laws of Thoughts'. But this is not the only aspect of the 'Laws of Thoughts'. The other and may be much more important aspect of these 'Laws of Thoughts'is the fact that quantum mechanical objects or that "... things behave in accordance with them" (see Russell, 1912, p. 113).

2.5.2. Principium contradictionis (Axiom II)

Principium contradictionis or **lex contradictionis** [53] , [54] , [55] or axiom II, the other of lex identitatis, the negative of lex identitatis, the opposite of lex identitatis, a complementary of lex identitatis, can be expressed mathematically as

$$+0 \equiv 0 \times 1 \equiv +1 \tag{324}$$

In addition to the above, from the point of view of mathematics, axiom II (equation 324) is equally the most simple mathematical expression and formulation of a contradiction. However, there is too much practical and theoretical evidence that a lot of 'secured'mathematical knowledge and rules differ too generously from real world processes, and the question may be asked whether mathematical truths can be treated as absolute truths at all. Many of the basic principle of today's mathematics allow every single author defining the real world events and processes et cetera in a way as everyone likes it for himself. Consequentially, a resulting dogmatic epistemological subjectivism and at the end agnosticism too, after all, is one of the reasons why we should rightly heed the following words of wisdom of Albert Einstein.

"I don't believe in mathematics."

(Albert Einstein cited according to Brian, 1996, p. 76)

In the long term, however, the above attitude of mathematics is not sustainable. History has taught us time and time again that objective reality has the potential to correct wrong human thinking slowly but surely, and many more than this. Objective reality has demonstrably corrected wrong human thinking again and again in the past.

[53] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

[54] Barukčić I. Aristotle's law of contradiction and Einstein's special theory of relativity. Journal of Drug Delivery and Therapeutics (JDDT). 15Mar.2019;9(2):125-43. https://jddtonline.info/index.php/jddt/article/view/2389

[55] Barukčić, Ilija. (2020, December 28). The contradiction is exsiting objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

Despite all the adversities, it is necessary and crucial to consider that a self-identical as the opposite of itself is no longer only self-identity but a difference of itself from itself within itself. In other words, "All things are different, or: there are no two things like each other ... is, in fact, opposed to the law of identity ..."(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422) Each on its own and without any respect to the other is distinctive within itself and from itself and not only from another. As the opposite of its own something, is no longer only self-identity, but also a negation of itself out of itself and therefore a difference of itself from itself within itself. In other words, in opposition, a self-identical is able to return into simple unity with itself, with the consequence that even as a self-identical the same self-identical is inherently self-contradictory. A question of fundamental theoretical importance is, however, why should something be itself and at the same time the other of itself, the opposite of itself, not itself? Is something like this even possible at all and if so, why and how? These and similar questions have occupied many thinkers, including Hegel.

"Something is therefore
alive only in so far as it contains contradiction within it,
and moreover is this power to
hold and endure the contradiction within it. "

(see Hegel, Georg Wilhelm Friedrich, 1991, p. 440)

However, as directed against identity, contradiction itself is also at the same time a source of self-changes of a self-identical out of itself.

"... contradiction
is the root of all movement and vitality;
it is only in so far as something has a contradiction within it
that it moves, has an urge and activity. "

(see Hegel, Georg Wilhelm Friedrich, 1991, p. 439)

The further advance of science will throw any contribution to scientific progress of each of us back into scientific insignificance, as long as principium contradictionis is not given enough and the right attention. **The contradiction [56] is existing objectively and real and is the heartbeat of every self-identical.** We have reason to be delighted by the fact that very different aspects of principium contradictionis have been examined since centuries from different angles by various authors. According to Aristotle, principium contradictionis applies to everything that is, it is the first and the firmest of all principles of philosophy.

[56] Barukčić, Ilija. (2020, December 28). The contradiction is existing objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

"... the same ... cannot at the same time belong and not belong to the same
... in the same respect ... This, then, is
the most certain of all principles "

(see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaph., IV, 3, 1005b, 16–22)

Principium contradictionis or axiom II has many facets. As long as we follow Leibniz in this regard, we should consider that "**Le principe de contradiction est en general ...** "(Leibniz, 1765, p. 327). Scientist inevitably have false beliefs and make mistakes. In order to prevent scientific results from falling into logical inconsistency or logical absurdity, it is necessary to posses among other the methodological possibility to start a reasoning with a (logical) contradiction too. However and in contrast to the way of reasoning with inconsistent premises as proposed by para-consistent (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) and other logic, in the absence of technical and other errors of reasoning, the contradiction itself need to be preserved. In other words, **from a contradiction does not anything follows but the contradiction itself** while the theoretical question is indeed justified "What is so Bad about Contradictions? " (Priest, 1998). Historically, **the principle of (deductive) explosion** (Carnielli and Marcos, 2001, Priest, 1998, Priest et al., 1989), coined by 12th-century French philosopher William of Soissons, demand us to accept that anything, including its own negation, can be proven or can be inferred from a contradiction. In short, according to **ex falso sequitur quodlibet**, a (logical) contradiction implies anything. Respecting the principle of explosion, the existence of a contradiction (or the existence of logical inconsistency) in a scientific theorem, rule et cetera is disastrous. However, the historical development of science shows that scientist inevitably revise the theories, false positions and claims are identified once and again, and we all make different kind of mistakes. In order to avert disproportionately great damage to science and to prevent reducing science into pure subjective belief, a negation of the principle of explosion is required. Nonetheless, a justified negation of the **ex contradictione quodlibet principle** (Carnielli and Marcos, 2001) does not imply the correctness of para consistent logic (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) as such as advocated especially by the Peruvian philosopher Francisco Miró Quesada (Quesada, 1977) and other (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989). In general, scientific theories appear to progress from lower and simpler to higher and more complex levels. However, high level theories cannot be taken for granted because high level theories are grounded on a lot of assumptions, definitions and other procedures and may rest upon too much erroneous stuff even if still not identified. Therefore, it should be considered to check at lower at simpler levels like with like.

2.5.2.1. Zero power zero

Theorem 2.12. *In general, it is*

$$+0^{+2} \equiv +0 \tag{325}$$

is false.

Proof by direct proof. The premise

$$+0 \equiv +1 \tag{326}$$

is false. In the following, any rearrangement of the premise which is free of (technical) errors, need to end up at a contradiction. In other words, the contradiction will be preserved. We obtain

$$+0 \times +0 \equiv +1 \times +0 \tag{327}$$

Equation 327 becomes

$$+0^{+2} \equiv +0 \tag{328}$$

□

2.5.2.2. Zero divided by zero

Theorem 2.13. *In general,*

$$\frac{1}{0} \equiv \frac{0}{0} \tag{329}$$

is false.

Proof by direct proof. If the premise

$$+1 \equiv +0 \tag{330}$$

is false, then the relationship

$$\frac{1}{0} \equiv \frac{0}{0} \tag{331}$$

is also false.

□

2.5.3. Principium negationis (Axiom III)

Lex negationis or axiom III, is often mismatched with simple opposition. However, from the point of view of philosophy and other sciences, identity, contradiction, negation and similar notions are equally mathematical descriptions of the most simple laws of objective reality. What sort of natural process is negation at the end? Mathematically, we define principium negationis or lex negationis or axiom III as

$$\text{Negation}(0) \ \times 0 \equiv \neg\,(0) \times 0 \equiv +1 \tag{332}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, Fritz H., 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990b). In this context, there is some evidence that

$$\text{Negation}(1) \ \times 1 \equiv \neg\,(1) \times 1 = 0 \tag{333}$$

Logically, it follows that

$$\text{Negation}(1) \ \equiv 0 \tag{334}$$

In the following we assume that axiom I is universal. Under this assumption, the following theorem follows inevitably.

Theorem 2.14 (Zero divided by zero)**.** *According to classical logic, it is*

$$\frac{0}{0} \equiv 1 \tag{335}$$

Proof by direct proof. The premise

$$1 \equiv 1 \tag{336}$$

is true. It follows that

$$\begin{aligned} 0 &\equiv 0 \\ &\equiv 0 \times 1 \end{aligned} \tag{337}$$

In the following, we rearrange the premise (see equation 332, p. 104). We obtain

$$0 \times (\text{Negation}(0) \ \times 0) \equiv 0 \tag{338}$$

Equation 338 changes slightly (see equation 333, p. 104). It is

$$(\text{Negation}(1) \ \times 1) \times (\text{Negation}(0) \ \times 0) \equiv 0 \tag{339}$$

Equation 339 demands that

$$(\text{Negation}(1) \ ) \times (\text{Negation}(0) \ ) \times 0 \equiv 0 \tag{340}$$

Equation 340 is logically possible (see equation 323, p. 96) only if

$$(\text{Negation}(1)\ ) \times (\text{Negation}(0)\ ) \equiv 1 \tag{341}$$

(see theorem 2.12, equation 325) whatever the meaning of Negation(1) or of Negation(0) might be, equation 341 demands that

$$\text{Negation}(0) \equiv \frac{1}{\text{Negation}(1)} \tag{342}$$

and that

$$\text{Negation}(1) \equiv \frac{1}{\text{Negation}(0)} \tag{343}$$

Equation 342 simplifies as (see equation 334, p. 104)

$$\begin{aligned} \text{Negation}(0) &\equiv \frac{+1}{\text{Negation}(1)} \\ &\equiv \frac{+1}{+0} \end{aligned} \tag{344}$$

It follows that

$$\neg\,(0) \times 0 \equiv \frac{1}{0} \times 0 \equiv \frac{0}{0} \equiv 1 \tag{345}$$

To bring it to the point. Classical logic, assumed as generally valid, demands that

$$\frac{0}{0} \equiv 1 \tag{346}$$

□

Concepts like identity, difference, negation, opposition et cetera engaged the attention of scholars at least over the last twenty-three centuries (see also Horn, 1989, Speranza and Horn, 2010). As long as we first and foremost follow Josiah Royce, negatio or negation "is one of the simplest and most fundamental relations known to the human mind. For the study of logic, no more important and fruitful relation is known." (see also Royce, 1917, p. 265) But, do we really know what, for sure, what negation is? Based on what we know about negation, Aristotle (see also Wedin, 1990a) has been one of the first to present a theory of negation, which can be found in discontinuous chunks in his works the Metaphysics, the Categories, De Interpretatione, and the Prior Analytics (see also Horn, 1989, p. 1). Negation (see also Newstadt, 2015) as a fundamental philosophical concept found its own very special melting point especially in Hegel's dialectic and is more than just a formal logical process or operation which converts true to false or false to true. Negation as such is a natural process too and equally '**an engine of changes of objective reality** " (see also Barukčić, 2019a). However, it remains an open question to establish a generally accepted link between this fundamental philosophical concept and an adequate counterpart in physics, mathematics and mathematical statistics et cetera. Especially the relationship between creation and conservation or creatio ex nihilio (see

also Donnelly, 1970, Ehrhardt, 1950, Ford, 1983), determination and negation (see also Ayer, 1952, Hedwig, 1980, Heinemann, Fritz H., 1943, Kunen, 1987) has been discussed in science since ancient (see also Horn, 1989, Speranza and Horn, 2010) times too. Why and how does an event occur or why and how is an event created (creation), why and how does an event maintain its own existence over time (conservation)? The development of the notion of negation leads from Aristotle to Meister Eckhart (see also Eckhart, 1986) von Hochheim (1260-1328), commonly known as Meister Eckhart (see also Tsopurashvili, 2012) or Eckehart, to Spinoza (1632 – 1677), to Immanuel Kant (1724-1804) and finally to Georg Wilhelm Friedrich Hegel (1770-1831) and other authors too. One point is worth being noted, even if it does not come as a surprise, it was especially Benedict de Spinoza (1632 – 1677) as one of the philosophical founding fathers of the Age of Enlightenment who addressed the relationship between determination and negation in his lost letter of June 2, 1674 to his friend Jarig Jelles (see also Förster and Melamed, 2012) by the discovery of his fundamental insight that " **determinatio negatio est**" (see also Spinoza, 1674, p. 634). Hegel went even so far as to extended the slogan raised by Spinoza into to "Omnis determinatio est negatio" (see also Hegel, Georg Wilhelm Friedrich, 1812b, 2010, p. 87). Finally, it did not take too long, and the notion of negation entered the world of mathematics and mathematical logic at least with Boole's (see also Boole, 1854) publication in the year 1854. "Let us, for simplicity of conception, give to the symbol x the particular interpretation of men, then 1 - x will represent the class of 'not-men'." (see also Boole, 1854, p. 49). Finally, the philosophical notion negation found its own way into physics by the contributions of authors like Woldemar Voigt (see Voigt, 1887), George Francis FitzGerald (see FitzGerald, 1889), Hendrik Antoon Lorentz (see Lorentz, 1892, 1899), Joseph Larmor (see Larmor, 1897), Jules Henri Poincaré (see Poincaré, 1905) and Albert Einstein (see Einstein, 1905) by contributions to the physical notion "Lorentz factor".

3. Results

3.1. Oral BCG vaccine excludes Crohn's disease relapse (very unfair study design)

Burnham et al. [57] conducted a randomised double-blind trial over one year with oral BCG treatment of chronic Crohn's disease compared with a control. "Of the 50 patients ⋯ 24 were allocated to BCG and 26 to placebo ⋯ The disease relapsed in three patients taking BCG and seven taking the placebo ". Burnham et al. published to have demonstrated no significant benefit for CD patients from oral BCG treatment. We obtain the following data and statistical analysis (see table 26).

Table 26. Oral BCG vaccination and CD Relapse (Study Burnham et al. , 1979).

		CD Relapse		
		YES	NO	
Oral BCG vaccination	YES	3	21	24
	NO	7	19	26
		10	40	50

STATISTICAL ANALYSIS.	
Causal relationship k =	-0,1801441730
p Value left tailed (HGD) =	0,1794029
p (EXCL) =	0,9400000000
p (EXCL) approx.=	0,7000000000
$\tilde{\chi}^2$ (EXCL— A_t) =	0,3750
$\tilde{\chi}^2$ (EXCL— B_t) =	0,9000
p Value (EXCL) =	0,0582354664
RELATIVE RISK (RR).	
RR (nc) =	0,4643
RR (sc) =	0,5714
ADDITIONAL MEASURES.	
OR =	0,4400
IOR =	-0,3750
STUDY DESIGN.	
p(IOU)=	0,32
p(IOI)=	0,28

The study design of the study of Burnham et al. with p(IOI) = 0,28 (Barukčić, 2019c) was very unfair . The data of the study of Burnham et al. are potentially biased. Therefore, the data are just of some restricted value. However, even under these conditions, Burnham et al. provided evidence that an oral BCG vaccination excludes CD relapse (reason: $\tilde{\chi}^2$ calculated are very low).

[57] Burnham WR, Lennard-Jones JE, Hecketsweiler P, Colin R, Geffroy Y. Oral BCG vaccine in Crohn's disease. Gut. 1979 Mar;20(3):229-33. doi: 10.1136/gut.20.3.229. PMID: 374194; PMCID: PMC1412307.

3.2. *Oral BCG vaccine excludes Crohn's disease relapse (fair study design with fictive placebo group)*

In the following, we assume fair experimental conditions. Under fair conditions (p(IOI) = 0) we obtain the following two by two table (see table 27).

Table 27. Oral BCG vaccination and CD Relapse (Study Burnham et al. , 1979).

		CD Relapse		
		YES	NO	
Oral BCG vaccination	YES	3	21	24
	NO	21	57	78
		24	78	102

STATISTICAL ANALYSIS.	
Causal relationship k =	-0,1442307692
p Value left tailed (HGD) =	0,1161054
p (EXCL) =	0,9705882353
p (EXCL) approx.=	0,8750000000
$\tilde{\chi}^2$ (EXCL— A_t) =	0,3750
$\tilde{\chi}^2$ (EXCL— B_t) =	0,3750
p Value (EXCL) =	0,0289834482
RELATIVE RISK (RR).	
RR (nc) =	0,4643
RR (sc) =	0,4643
ADDITIONAL MEASURES.	
OR =	0,5882
IOR =	-0,4688
STUDY DESIGN.	
p(IOU)=	0,529411765
p(IOI)=	0

As already reported, the study of Burnham et al. with p(IOI) = 0,28 (Barukčić, 2019c) was very unfair . In the following we assume fair examination conditions, i.e. conditions which ensure that experimental studies as well as case-control studies enable us to reach the same conclusions. The probability of (not oral BCG vaccination and CD relapse) according to Burnham et al. under condition of (not oral BCG vaccination) is $p\left(\text{not oral BCG vaccination and CD relapse}\backslash\text{not oral BCG vaccination}\right) \equiv \frac{7}{26} = 0,269230769$. It is in no way certain that such a relationship exists in the population. Nonetheless, we presume for preliminary reason that this is so. The study design has to ensure that

b = c.

We obtain $c \equiv \left(7 \times \frac{21}{7}\right) = 21$, while $d \equiv \left(19 \times \frac{21}{7}\right) = 57$ and Not Oral BCG vaccination $\equiv \left(26 \times \frac{21}{7}\right) = 78$. Under conditions of fair study design, an oral BCG vaccination excludes CD relapse (p Value (EXCL) = 0,0289834482). Nonetheless, the question is justified why an intestine already damaged by Crohn's disease had to be exposed to an additional load of an oral BCG vaccination. Despite these and other shortcomings and contrary to Burnham et al. conclusion, a BCG vaccination prevents CD relapse. These results in connection with the publication of Behr et al. [58] (see figure 1 and figure 2) and the proof that **Mycobacterium avium subspecies paratuberculosis (MAP) is the cause of Crohn's disease** (see Barukčić, 2018) requires from us to rethink immediately the treatment of Crohn's disease in a timely manner. Mycobacterium avium subspecies paratuberculosis is an extremely slow-growing, acid-fast, mycobactin-dependent multispecies [59] obligate intracellular [60] pathogen which is unable to synthesize mycobactin. MAP requires mycobactin to obtain iron from environmental sources. This is one of the reasons why MAP cannot replicate outside of a host. Unfortunately, MAP can colonise the host for years without causing disease and might escape the human immune surveillance.

3.3. Chemotherapy of Crohn's disease

Independent of any unfortunate turn of events, we may justifiably hope that our possibilities and capabilities are better today than the same were yesterday, but still that the same are worse than they will be tomorrow. Following this path of thoughts and given the evidence presented in this publication, we should ask whether the principle of going from the known to the unknown is of any help for the development of new therapeutic options against Crohn's disease. CD is a mycobacterial disease, while MAP itself is a mycobacterium which lack its cell wall in humans. [61] For this reason, it could be potentially inappropriate to use antibiotic drugs for the treatment of CD which target cell wall [62] of MAP mostly. This underlines the urgent need for new strategies for the antibacterial treatment of CD. In general and in addition to the BCG vaccination, we also have drugs which are reasonably effective against tuberculosis, even if the resistance situation in this regard is becoming more and more problematic. It is known that Delamanid, Bedaquilin, Ethambutol, Isoniazid and Cycloserin are targeting more or less the cell wall of a Mycobacterium. Moxifloxacin (DNA replication), Rifampicin (RNA synthesis), Rifapentine (Protein synthesis ihibitor), Streptomycin (Protein synthesis ihibitor), Linezolid (Protein synthesis ihibitor), Sutezolid (Protein synthesis ihibitor), pyrazinamide (Protein synthesis ihibitor) are not target-

[58] Behr MA, Bruere P, Oxlade O. Global rates of Crohn's disease. Inflamm Bowel Dis. 2008 Aug;14(8):1170-2. doi: 10.1002/ibd.20435. PMID: 18340644.

[59] Lingling Li, John P. Bannantine, Qing Zhang, +5 , Alongkorn Amonsin, Barbara J. May, David Alt, Nilanjana Banerji, Sagarika Kanjilal, and Vivek Kapur, Proceedings of the National Academy of Sciences (PNAS), 102 (35) 12344-12349. https://doi.org/10.1073/pnas.0505662102

[60] Naser SA, Sagramsingh SR, Naser AS, Thanigachalam S. Mycobacterium avium subspecies paratuberculosis causes Crohn's disease in some inflammatory bowel disease patients. World J Gastroenterol. 2014 Jun 21;20(23):7403-15. doi: 10.3748/wjg.v20.i23.7403. PMID: 24966610; PMCID: PMC4064085.

[61] Naser SA, Ghobrial G, Romero C, Valentine JF. Culture of Mycobacterium avium subspecies paratuberculosis from the blood of patients with Crohn's disease. Lancet. 2004 Sep 18-24;364(9439):1039-44. doi: 10.1016/S0140-6736(04)17058-X. PMID: 15380962.

[62] Bhat ZS, Rather MA, Maqbool M, Lah HU, Yousuf SK, Ahmad Z. Cell wall: A versatile fountain of drug targets in Mycobacterium tuberculosis. Biomed Pharmacother. 2017 Nov;95:1520-1534. doi: 10.1016/j.biopha.2017.09.036. Epub 2017 Sep 21. PMID: 28946393.

ing the mycobaterial cell wall primarily. Today, there are several TB treatment regimen recommended for TB disease which can take 4, 6, or 9 months. [63] Until better studies or drugs to treat CD with anti TB drugs are available, a shorter regimen should be preferred to help patients to complete treatment faster. Based on Carr et al. [64] 4-month treatment regimen of tuberculosis with high-dose daily rifapentine (RPT) with moxifloxacin (MOX), isoniazid (INH), and pyrazinamide (PZA), a simillar 4-month treatment regimen of Crohn's disease should consists of high-dose daily with clarithromycin (CLA), moxifloxacin (MOX), rifapentine (RPT) and pyrazinamide (PZA). Additionally, Prednisolone should be given daily from Monday to Friday. Such a 4-month rifapentine-moxifloxacin regimen has an intensive phase of 2 months, followed by a continuation phase of 2 months and 1 week (total 17 weeks for treatment). View months after successful CD therapy, BCG vaccination in order to prevent CD relapse should be considered. An appropriate treatment of CD with effective antibiotics for sufficient duration in order to kill MAP organism and able to induce remission of Crohn's disease would provide one of the most convincing contributions to the evidence of a causal role of MAP in Crohn's disease. Meanwhile, some MAP antibiotic regimen including clarithromycin (CLA), rifabutin (RIF) and Clofazimine (CLO) or RHB-104 [65] (63.3 % CLA, 30 % RIF, and 6.7 % CLO) have been investigated with contradictory results. [66] , [67] , [68] , [69] , [70] , [71] , [72] , [73] , [74]

[63] Treatment for TB Disease. Centers for Disease Control and Prevention (CDC). 1600 Clifton Road Atlanta, GA 30329-4027, USA.

[64] Carr W, Kurbatova E, Starks A, Goswami N, Allen L, Winston C. Interim Guidance: 4-Month Rifapentine-Moxifloxacin Regimen for the Treatment of Drug-Susceptible Pulmonary Tuberculosis - United States, 2022. MMWR Morb Mortal Wkly Rep. 2022 Feb 25;71(8):285-289. doi: 10.15585/mmwr.mm7108a1. PMID: 35202353.

[65] Alcedo KP, Thanigachalam S, Naser SA. RHB-104 triple antibiotics combination in culture is bactericidal and should be effective for treatment of Crohn's disease associated with Mycobacterium paratuberculosis. Gut Pathog. 2016 Jun 14;8:32. doi: 10.1186/s13099-016-0115-3. PMID: 27307791; PMCID: PMC4908774.

[66] Grant IR, Ball HJ, Neill SD, Rowe MT. Inactivation of Mycobacterium paratuberculosis in cows' milk at pasteurization temperatures. Appl Environ Microbiol. 1996 Feb;62(2):631-6. doi: 10.1128/aem.62.2.631-636.1996. PMID: 8593064; PMCID: PMC167829.

[67] Borody TJ, Leis S, Warren EF, Surace R. Treatment of severe Crohn's disease using antimycobacterial triple therapy–approaching a cure? Dig Liver Dis. 2002 Jan;34(1):29-38. doi: 10.1016/s1590-8658(02)80056-1. PMID: 11926571.

[68] Eltholth MM, Marsh VR, Van Winden S, Guitian FJ. Contamination of food products with Mycobacterium avium paratuberculosis: a systematic review. J Appl Microbiol. 2009 Oct;107(4):1061-71. doi: 10.1111/j.1365-2672.2009.04286.x. Epub 2009 Mar 30. PMID: 19486426.

[69] Faria AC, Schwarz DG, Carvalho IA, Rocha BB, De Carvalho Castro KN, Silva MR, Moreira MA. Short communication: Viable Mycobacterium avium subspecies paratuberculosis in retail artisanal Coalho cheese from Northeastern Brazil. J Dairy Sci. 2014 Jul;97(7):4111-4. doi: 10.3168/jds.2013-7835. Epub 2014 May 3. PMID: 24797534.

[70] Ricchi M, De Cicco C, Kralik P, Babak V, Boniotti MB, Savi R, Cerutti G, Cammi G, Garbarino C, Arrigoni N. Evaluation of viable Mycobacterium avium subsp. paratuberculosis in milk using peptide-mediated separation and Propidium Monoazide qPCR. FEMS Microbiol Lett. 2014 Jul;356(1):127-33. doi: 10.1111/1574-6968.12480. Epub 2014 Jun 9. PMID: 24860938.

[71] Akiyoshi T, Ito M, Murase S, Miyazaki M, Guengerich FP, Nakamura K, Yamamoto K, Ohtani H. Mechanism-based inhibition profiles of erythromycin and clarithromycin with cytochrome P450 3A4 genetic variants. Drug Metab Pharmacokinet. 2013;28(5):411-5. doi: 10.2133/dmpk.dmpk-12-rg-134. Epub 2013 Mar 19. PMID: 23514827.

[72] Chamberlin W, Ghobrial G, Chehtane M, Naser SA. Successful treatment of a Crohn's disease patient infected with bacteremic Mycobacterium paratuberculosis. Am J Gastroenterol. 2007 Mar;102(3):689-91. doi: 10.1111/j.1572-0241.2007.01040_7.x. PMID: 17335456.

[73] Selby W, Pavli P, Crotty B, Florin T, Radford-Smith G, Gibson P, Mitchell B, Connell W, Read R, Merrett M, Ee H, Hetzel D; Antibiotics in Crohn's Disease Study Group. Two-year combination antibiotic therapy with clarithromycin, rifabutin, and clofazimine for Crohn's disease. Gastroenterology. 2007 Jun;132(7):2313-9. doi: 10.1053/j.gastro.2007.03.031. Epub 2007 Mar 21. PMID: 17570206.

[74] https://www.ncbi.nlm.nih.gov/pmc/articles/PMC8989010/

3.4. BCG vaccination and Crohn's disease

BCG Vaccine induces a cell-mediated immune response and protects to some extent against an infection with M. tuberculosis. Unfortunately, the duration of immunity against an infection with M. tuberculosis after BCG vaccination is still not known. Nonetheless, there are some indications of a waning immunity after 10 years. In about 6 weeks after vaccination, vaccinated persons normally become tuberculin positive. Nonetheless, the relationship between the post vaccination tuberculin skin test reaction and the degree of protection afforded by BCG vaccination remains unclear even if a positive tuberculin skin test indicates a response of the immune system. Tuberculin testing prior to administration of BCG Vaccine appears not to be necessary.

Method of injection

Today, the dose of live attenuated Mycobacterium bovis BCG (Bacillus Calmette-Guerin) vaccine (i.e. BCG Vaccine AJV, Copenhagen, Denmark) is applied on the basis of national official recommendations and injected strictly by the intradermal route. Antiseptics (such as alcohol) applied to swab the skin should evaporate completely before the injection is made. The injection site of BCG Vaccine should be clean and dry. BCG Vaccine is injected strictly intradermally in the arm, over the distal insertion of the deltoid muscle onto the humerus (approx. one third down the upper arm) by personnel trained in the intradermal technique. The skin of the injection site is stretched between thumb and forefinger. The needle of the injection should be almost parallel with the skin surface and slowly inserted (bevel upwards), approximately 2 mm into the superficial layers of the dermis. The needle of the injection should be visible through the epidermis during insertion. The injection of the BCG Vaccine is given slowly while a blanched, raised bleb is a sign of correct injection. Finally, the injection site is best left uncovered in order to facilitate healing.

Undesirable effects

BCG vaccine might induce undesirable effects. Gross overdosage increases the risk of undesirable BCG complications including systemic infections or persistent local infections. The management of systemic infections or persistent local infections following vaccination with BCG Vaccine might include Isoniazid [75] , [76] , Streptomycin [77] , Ethambutol [78] and Rifampicin et cetera.

Dosage

Half of the recommended dose of the reconstituted BCG vaccine is injected strictly by the intradermal route in the right arm, over the distal insertion of the deltoid muscle onto the humerus.

[75] Griffith DE, Aksamit T, Brown-Elliott BA, Catanzaro A, Daley C, Gordin F, Holland SM, Horsburgh R, Huitt G, Iademarco MF, Iseman M, Olivier K, Ruoss S, von Reyn CF, Wallace RJ Jr, Winthrop K; ATS Mycobacterial Diseases Subcommittee; American Thoracic Society; Infectious Disease Society of America. An official ATS/IDSA statement: diagnosis, treatment, and prevention of nontuberculous mycobacterial diseases. Am J Respir Crit Care Med. 2007 Feb 15;175(4):367-416. doi: 10.1164/rccm.200604-571ST. Erratum in: Am J Respir Crit Care Med. 2007 Apr 1;175(7):744-5. Dosage error in article text. PMID: 17277290.

[76] Isoniazid at Drugs.com

[77] Streptomycin at Drugs.com

[78] Ethambutol at Drugs.com

Two months later, another half of the recommended dose of the reconstituted BCG vaccine is injected strictly by the intradermal route in the left arm, over the distal insertion of the deltoid muscle onto the humerus.

Finally, 12 months after the first BCG vaccine, a full dose of the reconstituted BCG vaccine is injected strictly by the intradermal route in the right arm, over the distal insertion of the deltoid muscle onto the humerus.

(Institution)

(Name of patient)

		CD Intensive Phase Of Therapy														CD Continuation Phase Of Therapy								
Start:		**Days of a week**							**Week number**								Week number							
Action: What to do?		Mo	Tu	We	Th	Fr	Sa	Su	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	Supportive measures:																								
	2 l heated water/day	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	Diät: no lactose-prod.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	Laxantive measures						x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	Iron	x							x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	B12, Folic acid			x					x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	25 OH Vitamine D3							x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	Laboratory investig.																								
	Calprotectin								x			x				x				x				x	
	ALT, AST, AP, Bilirubin								x			x				x				x				x	
	CRP, Creatinin, Platlate								x			x				x				x				x	
	Gastro./Coloscopy								x			x				x				x				x	
	Chest radiograph			x					x							x								x	
	Contact the physician			x					x				x				x				x				x
	Therapy.																								
	Prednisolon 40 mg/day	x	x	x	x	x			x		x		x		x		x		x		x		x		x
	Clarithromycin (CLA)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	Moxifloxacin (MOX)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	Rifapentine (RPT)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
	Pyrazinamide (PZA)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x									
	BCG Vaccination																								
	Ask the physician.			x																					x

(Validated by medical doctor)

Figure 15. Crohn's disease chemotherapy.

4. Discussion

Several studies [79] , [80] , [81] , [82] , [83] , [84] , [85] reported contradictory results on the relationship, BCG vaccination and CD. However, following Villumsen et al. [86] BCG vaccination do not cause CD later in life. Nonetheless, BCG vaccination as an attenuated live vaccine has several aspects in common with mycobacteria other than Mycobacterium tuberculosis — so called non-tuberculous mycobacteria (NTM) and is able to provide a kind of cross-protection against NTM infections too. [87] The protective effect of BCG vaccination against NTM should be taken into consideration when recommending the usage of BCG vaccination for CD treatment. In this context, we have to take into account that **Mycobacterium avium subspecies paratuberculosis (MAP) as the cause of Crohn's disease** (see Barukčić, 2018) is lacking its cell wall in humans. [88] For this reason, BCG vaccination dosage, application route and time schedule may need to be adjusted in order to provide protection, especially against MAP. Whatever, we can no longer ignore the results of the study of Burnham et al. Despite all the adversities discussed before, Burnham [89] et al. were able to provide evidence that BCG vaccination is effective against CD relapse.

5. Conclusion

There is convincing evidence that BCG vaccination is protective against CD relapse.

[79] Geffroy Y, Colin R, Segrestin M, Hecketsweiler P. B.C.G. treatment in Crohn's disease. Lancet. 1970 Sep 12;2(7672):571-2. doi: 10.1016/s0140-6736(70)91381-4. PMID: 4195236.

[80] Rahban S, Sherman JH, Opelz G, Conley DR, Panish JF, Marks JW, Terasaki PI, Schoenfield LJ. BCG treatment of Crohn's disease. Am J Gastroenterol. 1979 Feb;71(2):196-201. PMID: 373425.

[81] Burnham WR, Lennard-Jones JE, Hecketsweiler P, Colin R, Geffroy Y. Oral BCG vaccine in Crohn's disease. Gut. 1979 Mar;20(3):229-33. doi: 10.1136/gut.20.3.229. PMID: 374194; PMCID: PMC1412307.

[82] Leigh RJ, Turnberg LA. BCG vaccination and Crohn's disease. Dig Dis Sci. 1980 Dec;25(12):972. doi: 10.1007/BF01308050. PMID: 7449594.

[83] Rua EC, Blacarce N, Drut R. Early bacille de Calmette-Guérin vaccination and Crohn's disease: possible relationship? Pediatr Dev Pathol. 2000 Sep-Oct;3(5):501-3. doi: 10.1007/s100240010098. PMID: 10890937.

[84] El-Matary W, Yap J, Gilmour S. Bacille Calmette-Guerin (BCG) vaccine for induction of remission in Crohn's disease. Cochrane Database Syst Rev. 2019 Jun 20;2019(6):CD008029. doi: 10.1002/14651858.CD008029.pub2. PMCID: PMC6587008.

[85] Dow CT. Proposing BCG Vaccination for Mycobacterium avium ss. paratuberculosis (MAP) Associated Autoimmune Diseases. Microorganisms. 2020 Feb 5;8(2):212. doi: 10.3390/microorganisms8020212. PMID: 32033287; PMCID: PMC7074941.

[86] Villumsen M, Jess T, Sørup S, Ravn H, Sturegrd E, Benn CS, Aaby P, Roth A. Risk of inflammatory bowel disease following Bacille Calmette-Guérin and smallpox vaccination: a population-based Danish case-cohort study. Inflamm Bowel Dis. 2013 Jul;19(8):1717-24. doi: 10.1097/MIB.0b013e318281f34e. PMID: 23624886.

[87] Zimmermann P, Finn A, Curtis N. Does BCG Vaccination Protect Against Nontuberculous Mycobacterial Infection? A Systematic Review and Meta-Analysis. J Infect Dis. 2018 Jul 24;218(5):679-687. doi: 10.1093/infdis/jiy207. PMID: 29635431.

[88] Naser SA, Ghobrial G, Romero C, Valentine JF. Culture of Mycobacterium avium subspecies paratuberculosis from the blood of patients with Crohn's disease. Lancet. 2004 Sep 18-24;364(9439):1039-44. doi: 10.1016/S0140-6736(04)17058-X. PMID: 15380962.

[89] Burnham WR, Lennard-Jones JE, Hecketsweiler P, Colin R, Geffroy Y. Oral BCG vaccine in Crohn's disease. Gut. 1979 Mar;20(3):229-33. doi: 10.1136/gut.20.3.229. PMID: 374194; PMCID: PMC1412307.

Acknowledgments

No funding or any financial support by a third party was received. Thanks to Overleaf, Zotero, Gnuplot, Zenodo, TeX Shop and other.

6. Patient consent for publication

Not required.

Conflict of interest statement

No conflict of interest to declare.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

References

Arbuthnot, John. An Argument for Divine Providence, Taken from the Constant Regularity Observ'd in the Births of Both Sexes. *Philosophical Transactions*, 27:186–190, 1710. URL https://www.jstor.org/stable/103111. The Royal Society.

Aristotle, of Stageira (384-322 B.C.E). *Metaphysica. Volume VIII. Translated by William David Ross and John Alexander Smith*. The works of Aristotle. At The Clarendon Press, Oxford, 1908. URL http://archive.org/details/worksofaristotle12arisuoft. Archive.org Zenodo.

A. J. Ayer. Negation. *The Journal of Philosophy*, 49(26):797–815, January 1952. doi: 10.2307/2020959. JSTOR.

AL Bailey. The analysis of covariance. *Journal of the American Statistical Association*, 26(176):424–435, 1931.

Bar, Carl Ludwig von. *Die Lehre vom Kausalzusammenhang im Recht, besonders im Strafrecht*. Verlag von Bernhard Tauchnitz, Leipzig, 1871. URL http://dlib-pr.mpier.mpg.de/m/kleioc/0010/exec/bigpage/%22101657_00000012.gif%22. Zenodo.

Ilija Barukčić. Mycobacterium avium subspecies paratuberculosis. *Modern Health Science*, 1(1):19–34, 2018. URL https://doi.org/10.30560/mhs.v1n1p19. MHS.

Ilija Barukčić. *Die Kausalität*. Wiss.-Verl., Hamburg, 1. aufl. edition, January 1989. ISBN 3-9802216-0-1. ISBN: 978-3-9802216-0-1.

Ilija Barukčić. *Die Kausalität*. Scientia, Wilhelmshaven, 2., völlig überarb. aufl. edition, January 1997. ISBN 3-9802216-4-4. ISBN: 978-3-9802216-4-1.

Ilija Barukčić. *Causality: New statistical methods*. Books on Demand GmbH, Hamburg-Norderstedt, January 2005. ISBN 978-3-8334-3645-1.

Ilija Barukčić. *Causality: New statistical methods*. Books on Demand GmbH, Hamburg-Norderstedt, January 2006a. ISBN 3-8334-3645-X. Deutsche Nationalbibliothek Frankfurt.

Ilija Barukčić. Local hidden variable theorem. *Causation*, 1(1):11–17, Dec. 2006b. URL https://www.causation.eu/index.php/causation/article/view/3.

Ilija Barukčić. Anti Heisenberg-Refutation Of Heisenberg's Uncertainty Relation. In *American Institute of Physics - Conference Proceedings*, volume 1327, pages 322–325, Linnaeus University, Växjö, Sweden, June 14 - 17, 2010, January 2011a. doi: 10.1063/1.3567453. AQT. Web of Science. Full text: American Institute of Physics.

Ilija Barukčić. The Equivalence of Time and Gravitational Field. *Physics Procedia*, 22:56–62, January 2011b. ISSN 18753892. doi: 10.1016/j.phpro.2011.11.008. ICPST 2011. Web of Science. Free full text: Elsevier.

Ilija Barukčić. Anti-Bell - Refutation of Bell's theorem: In: Quantum Theory: Reconsideration of Foundations-6 (QTRF6), Växjö, (Sweden), 11-14 June 2012. In *American Institute of Physics - Conference Proceedings*, volume 1508, pages 354–358, Växjo, Sweden, 2012. American Institute of Physics - Conference Proceedings. doi: 10.1063/1.4773147. QTRF 6. Web of Science. Full text: American Institute of Physics.

Ilija Barukčić. The relativistic wave equation. *International Journal of Applied Physics and Mathematics*, 3(6):387–391, 2013. ISSN 2010362X. doi: 10.7763/IJAPM.2013.V3.242.

Ilija Barukčić. Anti Heisenberg – Refutation of Heisenberg's Uncertainty Principle. *International Journal of Applied Physics and Mathematics*, 4(4):244–250, 2014. IJAPM DOI: 10.7763/IJAPM.2014.V4.292.

Ilija Barukčić. Anti einstein – refutation of einstein's general theory of relativity. *International Journal of Applied Physics and Mathematics*, 5(1):18–28, 2015. ISSN 2010362X. doi: 10.17706/ijapm.2015.5.1.18-28.

Ilija Barukčić. Anti Heisenberg—The End of Heisenberg's Uncertainty Principle. *Journal of Applied Mathematics and Physics*, 04(05): 881–887, 2016a. ISSN 2327-4352. JAMP DOI: 10.4236/jamp.2016.45096.

Ilija Barukčić. The Mathematical Formula of the Causal Relationship k. *International Journal of Applied Physics and Mathematics*, 6 (2):45–65, January 2016b. doi: 10.17706/ijapm.2016.6.2.45-65. IJAPM. Free full text: IAP.

Ilija Barukčić. Unified field theory. *Journal of Applied Mathematics and Physics*, 4(88):1379–1438, 8 2016c. doi: 10.4236/jamp.2016.48147.

Ilija Barukčić. Anti Bohr — Quantum Theory and Causality. *International Journal of Applied Physics and Mathematics*, 7(2):93–111, 2017a. doi: 10.17706/ijapm.2017.7.2.93-111. IJAPM DOI: 10.17706/ijapm.2017.7.2.93-111.

Ilija Barukčić. *Die Kausalität (1989)*. Books on Demand, Hamburg-Norderstedt, reprint edition, 2017b. ISBN 978-3-7448-1595-6. ISBN-13: 9783744815956.

Ilija Barukčić. *Theoriae causalitatis principia mathematica*. Books on Demand, Hamburg-Norderstedt, 2017c. ISBN 978-3-7448-1593-2. ISBN-13: 9783754331347.

Ilija Barukčić. Fusobacterium nucleatum —The Cause of Human Colorectal Cancer. *Journal of Biosciences and Medicines*, 06(03):31–69, 2018a. ISSN 2327-5081. doi: 10.4236/jbm.2018.63004.

Ilija Barukčić. Human Papillomavirus—The Cause of Human Cervical Cancer. *Journal of Biosciences and Medicines*, 06(04):106–125, 2018b. ISSN 2327-5081. doi: 10.4236/jbm.2018.64009.

Ilija Barukčić. Aristotle's law of contradiction and einstein's special theory of relativity. *Journal of Drug Delivery and Therapeutics*, 9 (22):125–143, 3 2019a. ISSN 2250-1177. doi: 10.22270/jddt.v9i2.2389.

Ilija Barukčić. Classical Logic And The Division By Zero. *International Journal of Mathematics Trends and Technology IJMTT*, 65(7):31–73, 2019b. doi: 10.14445/22315373/IJMTT-V65I8P506. URL http://www.ijmttjournal.org/archive/ijmtt-v65i8p506. Free full text: IJMTT.

Ilija Barukčić. Index of Independence. *Modern Health Science*, 2(2):1–25, 10 2019c. ISSN 2576-7305. doi: 10.30560/mhs.v2n2p1. URL https://j.ideasspread.org/index.php/mhs/article/view/331. Modern Health Science.

Ilija Barukčić. Index of Unfairness. *Modern Health Science*, 2(1):22, 4 2019d. ISSN 2576-7305, 2576-7291. doi: 10.30560/mhs.v2n1p22. Modern Health Science.

Ilija Barukčić. The P Value of likely extreme events. *International Journal of Current Science Research*, 5(11):1841–1861, 2019e. IJCSR . Free full text: ZENODO.

Ilija Barukčić. Causal relationship k. *International Journal of Mathematics Trends and Technology IJMTT*, 66(10):76–115, 2020a. URL http://www.ijmttjournal.org/archive/ijmtt-v66i10p512. IJMTT.

Ilija Barukčić. Locality and Non locality. *European Journal of Applied Physics*, 2(5):1–13, October 2020b. ISSN 2684-4451. doi: 10.24018/ejphysics.2020.2.5.22. URL https://ej-physics.org/index.php/ejphysics/article/view/22. Free full text: EJAP.

Ilija Barukčić. *N-th index D-dimensional Einstein gravitational field equations. Geometry unchained.*, volume 1. Books on Demand GmbH, Hamburg-Norderstedt, 1 edition, 2020c. ISBN 978-3-7526-4490-6. ISBN-13: 9783752644906 . Free full text (preprint): ZENODO.

Ilija Barukčić. *Zero and infinity. Mathematics without frontiers*. Books on Demand GmbH, Hamburg-Norderstedt, 1 edition, 2020d. ISBN 978-3-7519-1873-2. ISBN-13: 9783751918732.

Ilija Barukčić. Mutually exclusive events. *Causation*, 16(11):5–57, November 2021a. doi: 10.5281/zenodo.5746415. URL https://doi.org/10.5281/zenodo.5746415. Zenodo.

Ilija Barukčić. Index of relationship. *Causation*, 16(8):5–37, April 2021b. doi: 10.5281/zenodo.5163179. URL https://doi.org/10.5281/zenodo.5163179. Zenodo.

Ilija Barukčić. The causal relationship k. *MATEC Web of Conferences*, 336:09032, 2021c. ISSN 2261-236X. doi: 10.1051/matecconf/202133609032. CSCNS2020. Web of Science. Free full text: EDP Sciences.

Ilija Barukčić. The logical content of the risk ratio. *Causation*, 16(4):5–41, February 2021d. doi: 10.5281/zenodo.4679509. Free full text: Zenodo.

Ilija Barukčić. Conditio per quam. *Causation*, 17(3):5–86, March 2022a. doi: 10.5281/zenodo.6369831. URL https://doi.org/10.5281/zenodo.6369831. Zenodo.

Ilija Barukčić. Conditio sine qua non. *Causation*, 17(jan):23–102, 2022b. URL https://doi.org/10.5281/zenodo.5854744. Zenodo.

Ilija Barukčić. Variance of binomial distribution. *Causation*, 17(1):5–22, 1 2022c. URL https://www.causation.eu/index.php/causation/article/view/7.

Ilija Barukčić and Okoh Ufuoma. *Analysis of Switching Resistive Circuits A Method Based on the Unification of Boolean and Ordinary Algebras*. Books on Demand, Hamburg-Norderstedt, first edition edition, 2020. ISBN 978-3-7519-8474-4. ISBN-13: 9783751984744.

Thomas Bayes. LII. An essay towards solving a problem in the doctrine of chances. By the late Rev. Mr. Bayes, FRS communicated by Mr. Price, in a letter to John Canton, AMFR S. *Philosophical transactions of the Royal Society of London*, 53(1):370–418, 1763. The Royal Society London.

Jacobi Bernoulli. *Ars conjectandi, Opus posthumus: Accedit Tractatus de seriebus infinitis ; et epistola Gallice scripta De Ludo Pilae Reticularis*. Impensis Thurnisiorum [Tournes], fratrum, Basileae (Basel, Suisse), January 1713. doi: 10.3931/e-rara-9001. Free full text: e-rara, Zurich, CH.

Alvin K. Bettinger and John A. Englund. *Algebra And Trigonometry*. International Textbook Company., 1960.

Alan Birnbaum. On the foundations of statistical inference: Binary experiments. *Annals of Mathematical Statistics*, 32(1):340–341, 1961. jstor.

Max Black. Why cannot an effect precede its cause? *Analysis*, 16(3):49–58, 1956.

Hubert M. Blalock. *Causal inferences in nonexperimental research*. Univ. of North Carolina Press, Chapel Hill, NC, 6. printing edition, 1972. ISBN 978-0-8078-0917-4.

Ethan D. Bloch. *Proofs and fundamentals: a first course in abstract mathematics*. Springer, 2nd ed edition, 2011. ISBN 978-1-4419-7126-5.

David Bohm. A suggested interpretation of the quantum theory in terms of hidden variables. i. *Physical review*, 85(2):166, 1952. APS.

Niels Bohr. Causality and Complementarity. *Philosophy of Science*, 4(3):289–298, July 1937. ISSN 0031-8248, 1539-767X. doi: 10.1086/286465. URL http://www.informationphilosopher.com/solutions/scientists/bohr/Causality_and_Complementarity.pdf. Informationphilosopher.

George Boole. *An investigation of the laws of thought, on which are founded mathematical theories of logic and probabilities*. New York, Dover, 1854. Free full text: archive.org, San Francisco, CA 94118, USA.

Max Born. Zur Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, 37(12):863–867, December 1926. ISSN 0044-3328. doi: 10.1007/BF01397477. URL https://doi.org/10.1007/BF01397477.

Auguste Bravais. Analyse mathématique sur les probabilités d es erreurs de situation d'un point. *Mémoires Présentées Par Divers Savants À L'Académie Royale Des Sciences De L'Institut De France*, 9:255–332, January 1846.

Denis Brian. *Einstein: a life*. J. Wiley, New York, N.Y, 1996. ISBN 978-0-471-11459-8.

Bundesanwaltschaft Bundesgerichtshof für Strafsachen. *Entscheidungen des Bundesgerichtshofes*, volume 1 of *Entscheidungen des Bundesgerichtshofes*. Carl Heymanns Verlag, Detmold, 1951. URL https://juris.bundesgerichtshof.de/cgi-bin/rechtsprechung/document.py?Gericht=bgh&Art=en&Datum=2008&Seite=99&nr=43553&pos=2985&anz=3634.

Walter A. Carnielli and João Marcos. Ex contradictione non sequitur quodlibet. *Bulletin of Advanced Reasoning and Knowledge*, 7 (1):89–109, 2001. URL https://www.researchgate.net/publication/236647971_Ex_contradictione_non_sequitur_quodlibet.

K. C. Carter. Koch's postulates in relation to the work of Jacob Henle and Edwin Klebs. *Medical History*, 29(4):353–374, October 1985. ISSN 0025-7273. doi: 10.1017/s0025727300044689.

Roderick M Chisholm. The contrary-to-fact conditional. *Mind*, 55(220):289–307, 1946.

R. J. Cook and D. L. Sackett. The number needed to treat: a clinically useful measure of treatment effect. *BMJ (Clinical research ed.)*, 310(6977):452–454, 2 1995. ISSN 0959-8138. doi: 10.1136/bmj.310.6977.452. PMID: 7873954. Free full text: PMCID: PMC2548824.

J. Cornfield. A method of estimating comparative rates from clinical data; applications to cancer of the lung, breast, and cervix. *Journal of the National Cancer Institute*, 11(6):1269–1275, 6 1951. ISSN 0027-8874. PMID: 14861651.

D. R. Cox. The regression analysis of binary sequences. *Journal of the Royal Statistical Society. Series B (Methodological)*, 20(2): 215–242, 1958. ISSN 0035-9246. JSTOR.

Harald Cramér. *Random variables and probability distributions*. Cambridge University Press, 1937.

Burrill Bernard Crohn, Leon Ginsberg, and Gordon D. Oppenheimer. Regional enteritis. a pathologic and clinical history. *Journal of the American Medical Association*, 109:1323–1329, 1932. URL https://doi.org/10.1001/jama.1932.02740680019005. JAMA.

Newton C. A. da Costa. On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic*, 15(4):497–510, October 1974. ISSN 0029-4527. doi: 10.1305/ndjfl/1093891487.

Newton Carneiro Alfonso da Costa. Nota sobre o conceito de contradição. *Anuário da Sociedade Paranaense de Matemática*, 1(2):6–8, 1958. URL Portuguese.

De Broglie, Louis. La mécanique ondulatoire et la structure atomique de la matière et du rayonnement. *Journal de Physique et le Radium*, 8(5):225–241, 1927. JPR.

Augustus De Morgan. *Formal logic: or, the calculus of inference, necessary and probable*. Taylor and Walton, 1847.

Bernhard Jacob Degen. *Principium identitatis indiscernibilium*. Meyer, 1741.

Morris H. DeGroot and Mark J. Schervish. *Probability and Statistics*. Ministry of Education of the People's Republic of China - Higher Education Press, third edition edition, 2005. ISBN 978-7-04-016765-8.

A. P. Dempster. Causality and statistics. *Journal of Statistical Planning and Inference*, 25(3):261–278, July 1990. ISSN 0378-3758. doi: 10.1016/0378-3758(90)90076-7. URL http://www.sciencedirect.com/science/article/pii/0378375890900767.

Paul Adrien Maurice Dirac. *The Principles of Quantum Mechanics. Third Edition*. At the Calenderon Press, 1947. archive.org, San Francisco, CA 94118, USA.

John Donnelly. Creation ex nihilo. In *Proceedings of the American Catholic Philosophical Association*, volume 44, pages 172–184, 1970. DOI: 10.5840/acpaproc19704425 .

Curt John Ducasse. On the nature and the observability of the causal relation. *The Journal of Philosophy*, 23(3):57–68, 1926. JSTOR.

AE Dummett and Antony Flew. Symposium: Can an effect precede its cause? *Proceedings of the Aristotelian Society, Supplementary Volumes*, pages 27–62, 1954.

Kenny Easwaran. The role of axioms in mathematics. *Erkenntnis*, 68(3):381–391, 2008. DOI: 10.1007/s10670-008-9106-1.

Meister Eckhart. *Meister Eckhart: Die deutschen Werke, Band 1: Predigten. Editor Josef Quint*, volume 2. W.Kohlhammer Verlag, 1986. ISBN: 978-3-17-061210-5.

Arnold Ehrhardt. Creatio ex nihilo. *Studia Theologica - Nordic Journal of Theology*, 4(1):13–43, 1950. doi: 10.1080/00393385008599697. URL https://doi.org/10.1080/00393385008599697. DOI: 10.1080/00393385008599697 .

A. Einstein. Quanten-Mechanik Und Wirklichkeit. *Dialectica*, 2(3-4):320–324, 1948. ISSN 1746-8361. doi: 10.1111/j.1746-8361.1948.tb00704.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1746-8361.1948.tb00704.x. Dialectica.

Albert Einstein. Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, 322(10):891–921, 1905. ISSN 1521-3889. doi: https://doi.org/10.1002/andp.19053221004. Wiley Online Library.

Albert Einstein. Die grundlage der allgemeinen relativitätstheorie. *Annalen der Physik*, 354(7):769–822, 1916. ISSN 1521-3889. doi: https://doi.org/10.1002/andp.19163540702.

Albert Einstein. Kosmologische betrachtungen zur allgemeinen relativitätstheorie. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin)*, page 142–152, 1917.

Albert Einstein. Induktion and Deduktion in der Physik. *Berliner Tageblatt and Handelszeitung*, page Suppl. 4, December 1919. URL https://einsteinpapers.press.princeton.edu/vol7-trans/124. Berliner Tageblatt and Handelszeitung.

Albert Einstein. *Geometrie und Erfahrung. Erweiterte Fassung des Festvortrages Gehalten an der Preussischen Akademie der Wissenschaften zu Berlin am 27. Januar 1921*. Julius Springer, 1921. ISBN 978-3-642-49903-6. URL https://doi.org/10.1007/978-3-642-49903-6. archive.org MacTutor.

Albert Einstein, Boris Podolsky, and Nathan Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47(10):777–780, 5 1935. doi: 10.1103/PhysRev.47.777.

Ruiz Espejo. Review of Causality: New Statistical Methods, 2nd edn (by Ilija Barukcic; Books on Demand, Norderstedt DE, 2006): 34:1013-1014. *Journal of Applied Statistics*, 34(8):1011–1017, October 2007. doi: 10.1080/02664760701590707. URL http://www.tandfonline.com/doi/abs/10.1080/02664760701590707.

Euclid, of Alexandria (300 B. C. E.). *The Thirteen Books of the Elements. Volume 2. Books 3-9. Translated from the text of Heiberg. By Thomas L. Heath*. Cambridge University Press, Cambridge, December 1908. archive.org, San Francisco, CA 94118, USA Zenodo.

William Feller. *Introduction to Probability Theory and its Applications. Volume I & II.*, volume 1. John Wiley and Sons Inc., 1st edition edition, 1 1950. John Wiley and Sons Inc.

Johann Gottlieb Fichte. *Science of knowledge*. The english and foreign philosophical library. Trübner & Co., London, 1889.

Paul Finsler. *Über Kurven und Flächen in allgemeinen Räumen*. PhD thesis, Georg-August Universität, Göttingen, 1918.

Ronald Aylmer Fisher. On the Interpretation of Chi square from Contingency Tables, and the Calculation of P. *Journal of the Royal Statistical Society*, 85(1):87–94, 1922. ISSN 0952-8385. doi: 10.2307/2340521. JSTOR.

Ronald Aylmer Fisher. The logic of inductive inference. *Journal of the Royal Statistical Society*, 98(1):39–82, 1935. ISSN 0952-8385. doi: 10.2307/2342435. JSTOR.

Ronald Aylmer Fisher. The negative binomial distribution. *Annals of Eugenics*, 11(1):182–187, 1941. ISSN 2050-1439. doi: 10.1111/j.1469-1809.1941.tb02284.x. Wiley Online Library.

Fisher, Ronald Aylmer. On an absolute criterion for fitting frequency curves. *Messenger of mathematics*, 41:155–156, 1912.

Fisher, Ronald Aylmer. XV.—The Correlation between Relatives on the Supposition of Mendelian Inheritance. *Earth and Environmental Science Transactions of The Royal Society of Edinburgh*, 52(2):399–433, 1919. ISSN 2053-5945. doi: 10.1017/S0080456800012163. Royal Society of Edinburgh.

Fisher, Ronald Aylmer. A mathematical examination of the methods of determining the accuracy of an observation by the mean error, and by the mean square error. *Monthly Notices of the Royal Astronomical Society*, 80:758–770, 1920.

Fisher, Ronald Aylmer. *The genetical theory of natural selection*. At the University Press, 1930. Archive.org.

George Francis FitzGerald. The ether and the earth's atmosphere. *Science (New York, N.Y.)*, 13(328):390, 5 1889. ISSN 0036-8075. doi: 10.1126/science.ns-13.328.390.

Lewis S Ford. An alternative to creatio ex nihilo. *Religious Studies*, 19(2):205–213, 1983. DOI: 10.1017/S0034412500015031 .

Jöran Friberg. Methods and traditions of babylonian mathematics: Plimpton 322, pythagorean triples, and the babylonian triangle parameter equations. *Historia Mathematica*, 8(3):277–318, 1981.

Eckart Förster and Yitzhak Y Melamed. *"Omnis determinatio est negatio" – Determination, Negation and Self-Negation in Spinoza, Kant, and Hegel. In: Spinoza and German idealism. Eckart Forster & Yitzhak Y. Melamed (eds.)*. Cambridge University Press, Cambridge [England]; New York, 2012. ISBN 978-1-283-71468-6. URL https://doi.org/10.1017/CBO9781139135139.

Francis Galton. Typical Laws of Heredity. *Nature*, 15(388):492–495, April 1877. ISSN 0028-0836, 1476-4687. doi: 10.1038/015492a0. URL http://www.nature.com/articles/015492a0.

Gauß, Carl Friedrich. *Theoria motus corporum coelestium in sectionibus conicis solem ambientium (Theory of motion of the celestial bodies moving in conic sections around the sun)*. sumtibus Frid. Perthes et I. H. Besser, 1809. e-rara, Zurich, CH.

Gomes, Gilberto. Are necessary and sufficient conditions converse relations? *Australasian Journal of Philosophy*, 87(3):375–387, 2009. Taylor & Francis.

H. T. Gonin. XIV. The use of factorial moments in the treatment of the hypergeometric distribution and in tests for regression. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 21(139):215–226, January 1936. ISSN 1941-5982. doi: 10.1080/14786443608561573. Taylor and Francis.

Nelson Goodman. The problem of counterfactual conditionals. *The Journal of Philosophy*, 44(5):113–128, 1947.

William Sealy Gosset. The probable error of a mean. *Biometrika*, 6(1):1–25, 1908. ISSN 0006-3444. doi: 1. JSTOR.

William Sealy Gosset. The elimination of spurious correlation due to position in time or space. *Biometrika*, 10(1):179–180, 1914. ISSN 0006-3444. doi: 10.2307/2331746. JSTOR.

C. W. J. Granger. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, 37(3):424—-438, 1969. JSTOR.

Major Greenwood and G. Udny Yule. The statistics of anti-typhoid and anti-cholera inoculations, and the interpretation of such statistics in general. *Proceedings of the Royal Society of Medicine*, 8:113–194, 6 1915. ISSN 0035-9157. doi: 10.1177/003591571500801433. PMID: 19978918. Free full text: PMCID: PMC2004181.

J. B. S. Haldane. Karl Pearson, 1857-1957. Being a Centenary Lecture. *Biometrika*, 44(3/4):303–313, 1957. ISSN 0006-3444. doi: 10.2307/2332863. URL https://www.jstor.org/stable/2332863. Biometrika Trust.

John Burdon Sanderson Haldane. The fitting of binomial distributions. *Annals of Eugenics*, 11(1):179–181, 1941. ISSN 2050-1439. doi: 10.1111/j.1469-1809.1941.tb02283.x. Wiley Online Library.

Hansemann, David Paul von. *Ueber das konditionale Denken in der Medizin und seine Bedeutung für die Praxis*. A. Hirschwald, Berlin, 1912. URL https://catalogue.bnf.fr/ark:/12148/cb30574869t.public.

Klaus Hedwig. Negatio negationis: Problemgeschichtliche Aspekte einer Denkstruktur. *Archiv für Begriffsgeschichte*, 24(1):7–33, 1980. ISSN 0003-8946. URL www.jstor.org/stable/24359358.

Hegel, Georg Wilhelm Friedrich. *Wissenschaft der Logik. Erster Band. Erstes Buch*. Johann Leonhard Schrag, Nürnberg, December 1812a. doi: 10.5281/zenodo.5917182. URL https://doi.org/10.5281/zenodo.5917182. Online at: Archive.org Zenodo.

Hegel, Georg Wilhelm Friedrich. *Wissenschaft der Logik. Erster Band. Erstes Buch*. Johann Leonhard Schrag, Nürnberg, December 1812b. doi: 10.5281/zenodo.5917182. URL https://doi.org/10.5281/zenodo.5917182. Online at: Archive.org Zenodo.

Hegel, Georg Wilhelm Friedrich. *Wissenschaft der Logik. Erster Band. Zweites Buch*. Johann Leonhard Schrag, Nürnberg, December 1813. doi: 10.5281/zenodo.5919885. URL https://doi.org/10.5281/zenodo.5919885. Online at: Archive.org Zenodo.

Hegel, Georg Wilhelm Friedrich. *Wissenschaft der Logik. Zweiter Band*. Johann Leonhard Schrag, Nürnberg, December 1816. doi: 10.5281/zenodo.5920022. URL https://doi.org/10.5281/zenodo.5920022. Online at: Archive.org Zenodo.

Hegel, Georg Wilhelm Friedrich. *Hegels Science of Logic*. Prometheus Books, New York, USA, 1991. ISBN 13: 9781573922807.

Hegel, Georg Wilhelm Friedrich. *The Science of Logic. Translated and edited by George Di Giovanni*. Cambridge University Press, Cambridge, USA, 2010. ISBN-13: 978-0-511-78978-6.

Heinemann, Fritz H. The Meaning of Negation. *Proceedings of the Aristotelian Society*, 44:127–152, 1943. ISSN 0066-7374. Oxford University Press.

Heisenberg, Werner Karl. Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Zeitschrift für Physik*, 43 (3):172–198, March 1927. ISSN 0044-3328. doi: 10.1007/BF01397280. URL https://doi.org/10.1007/BF01397280.

Friedrich Robert Helmert. Über die Wahrscheinlichkeit der Potenzsummen der Beobachtungsfehler und über einige damit im Zusammenhange stehende Fragen. *Zeitschrift für Mathematik und Physik*, 21(3):102–219, 1876.

Friedrich Gustav Jacob Henle. *Von den Miasmen und Contagien und von den miasmatisch-contagiösen Krankheiten*. Verlag von August Hirschwald, Berlin, 1840. URL https://doi.org/10.11588/diglit.15175. University Heidelberg urn:nbn:de:bsz:16-diglit-151756.

Boris Hennig. The Four Causes. *The Journal of Philosophy*, 106(3):137–160, March 2009. doi: 10.5840/jphil200910634.

Hessen, Johannes. *Das Kausalprinzip*. Verlag: Filser, Augsburg, 1928.

Germund Hesslow. Two Notes on the Probabilistic Approach to Causality. *Philosophy of Science*, 43(2):290–292, June 1976. ISSN 0031-8248, 1539-767X. doi: 10.1086/288684. URL https://www.journals.uchicago.edu/doi/10.1086/288684.

Germund Hesslow. Causality and Determinism. *Philosophy of Science*, 48(4):591–605, 1981. ISSN 0031-8248. URL https://www.jstor.org/stable/186838.

Austin Bradford Hill. The environment and disease: association or causation? *Proceedings of the Royal Society of Medicine*, 58: 295–300, January 1965.

Holbach, Paul Henri Thiry Baron de. *Système de la nature, ou des loix du monde physique et du monde moral. Première partie*. Par Jean Baptiste de Mirabaud, Londres, 1770. URL https://doi.org/10.3931/e-rara-14756. Zenodo.

Laurence R. Horn. *A natural history of negation*. University of Chicago Press, Chicago, 1989. ISBN 978-0-226-35337-1. ISBN: 978-0-226-35337-1.

David Hume. *A Treatise of Human Nature: Being an Attempt to Introduce the Experimental Method of Reasoning into Moral Subjects*, volume Volume 1. John Noon, 1739. URL https://ia802605.us.archive.org/14/items/treatiseofhumann00hume_0/treatiseofhumann00hume_0.pdf.

Christiaan Huygens and Frans van Schooten. *De ratiociniis in ludo alae: In: Exercitationum mathematicarum liber primus [- quintus]*. ex officina Johannis Elsevirii, Lugdunum Batavorum (Leiden, The Netherlands), January 1657. doi: 10.3931/e-rara-8813. Free full text: e-rara, Zurich, CH.

Jones, William. *Synopsis Palmariorum Matheseos: Or, a new Introduction to the Mathematics: Containing the Principles of Arithmetic & Geometry Demonstrated, in a Short and Easie Method.* Printed by J. Matthews, London, 1706. URL https://doi.org/10.3931/e-rara-14741. Archive.org, USA e-rara, CH.

Justice Matthews, Mr. Hayes, by next Friend, v. Michigan Central R. Co., 111 U.S. 228, 1884. Argued March 19, 1884. Decided April 7. *U. S. Supreme Court*, 1884. U. S. Supreme Court, 1884.

Truman Lee Kelley. *Statistical Method*, volume 1. Macmillan, 1924.

Daniel A. Klain and Gian-Carlo Rota. *Introduction to geometric probability*. Lezioni lincee. Cambridge University Press, 1997. ISBN 978-0-521-59362-5.

Mirjam J. Knol. Down with odds ratios: risk ratios in cohort studies and randomised clinical trials (article in dutch). *Nederlands Tijdschrift Voor Geneeskunde*, 156(28):A4775, 2012. ISSN 1876-8784. PMID: 22805792.

Anton Friedrich Koch. Die Selbstbeziehung der Negation in Hegels Logik. *Zeitschrift für philosophische Forschung*, 53(1):1–29, 1999. ISSN 0044-3301. URL www.jstor.org/stable/20484868.

Robert Koch. Neue Untersuchungen über die Mikroorganismen bei infektiösen Wundkrankheiten. *Deutsche Medizinische Wochenschrift*, 4(43):531–533, 1878. Zenodo.

Kolmogoroff, Andreĭ Nikolaevich. *Grundbegriffe der Wahrscheinlichkeitsrechnung*. Springer Berlin Heidelberg, Berlin, Heidelberg, January 1933. ISBN 978-3-642-49596-0. Springer.

Kolmogorov, Andreĭ Nikolaevich. *Foundations of the theory of probability. Translated by Nathan Morrison*. Chelsea Publishing Company, 1950. ISBN 978-0-486-82159-7. archive.org, San Francisco, CA 94118, USA.

Korch, Helmut. *Das Problem der Kausalität*. Dt. Verlag der Wissenschaften, Berlin, 1965.

Günter Kröber. Der Konditionalismus und seine Kritik in der sowjetischen Wissenschaft. *Wissenschaftliche Zeitschrift der Karl-Marx Universität Leipzig*, 10(2):137–153, 1961.

Kenneth Kunen. Negation in logic programming. *The Journal of Logic Programming*, 4(4):289–308, December 1987. ISSN 0743-1066. doi: 10.1016/0743-1066(87)90007-0. URL http://www.sciencedirect.com/science/article/pii/0743106687900070.

Harold Langsam. Kant, Hume, and Our Ordinary Concept of Causation. *Philosophy and Phenomenological Research*, 54(3):625, September 1994. ISSN 00318205. doi: 10.2307/2108584. URL https://www.jstor.org/stable/2108584?origin=crossref.

Pierre Simon de LaPlace. *Théorie analytique des probabilités*. Courcier, 1 1812. e-rara, Zurich, CH.

Joseph Larmor. IX. A dynamical theory of the electric and luminiferous medium.— Part III. Relations with material media. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, 190:205–300, 1 1897. doi: 10.1098/rsta.1897.0020.

Andreas Laupacis, David L. Sackett, and Robin S. Roberts. An assessment of clinically useful measures of the consequences of treatment. *New England Journal of Medicine*, 318(26):1728–1733, 1988. doi: 10.1056/NEJM198806303182605. PMID: 3374545. NEJM.

Gottfried Wilhelm Leibniz. *Oeuvres philosophiques latines & françoises de feu Mr. de Leibnitz*. Chez Jean Schreuder, Amsterdam (NL), 1765. URL https://archive.org/details/oeuvresphilosoph00leibuoft/page/n9.

Leibniz, Gottfried Wilhelm. *La monadologie (Nouvelle édition) / Leibniz ; nouvelle édition, avec une introduction, des sommaires, un commentaire perpétuel extrait des autres ouvrages de Leibniz, des exercices et un lexique de la terminologie leibnizienne. Bertrand, Alexis (1850-1923). Éditeur scientifique: Alexis Bertrand*. Vve E. Belin et fils, Paris, 1886. URL https://gallica.bnf.fr/http://catalogue.bnf.fr/ark:/12148/cb30781197z. Bibliothèque nationale de France.

Leon Brillouin. *Wave Propagation In Periodic Structures Electric Filters And Crystal Lattices First Edition*. Mcgraw-hil Book Company, Inc., New York (USA), 1946. URL http://archive.org/details/in.ernet.dli.2015.166889.

Lewis, David Kellogg. *Counterfactuals*. Blackwell, 1973. Blackwell.

Lewis, David Kellogg. Causation. *The journal of philosophy*, 70(17):556–567, 1974.

Hendrik Anton Lorentz. De relatieve beweging van de aarde en den aether. *Verslagen der Afdeeling Natuurkunde van de Koninklijke Akademie van Wetenschappen*, 1:74–79, 1892.

Hendrik Antoon Lorentz. Simplified theory of electrical and optical phenomena in moving systems. *Verhandelingen der Koninklijke Akademie van Wetenschappen*, 1:427–442, 1899.

John Leslie Mackie. Causes and conditions. *American philosophical quarterly*, 2(4):245–264, 1965. JSTOR.

John Leslie Mackie. The direction of causation. *The Philosophical Review*, 75(4):441–466, 1966. JSTOR.

John Leslie Mackie. *The cement of the universe: a study of causation*. Clarendon Press, 1974. ISBN-13: 9780198246428.

Eli Maor. *The Pythagorean Theorem: A 4,000-Year History*. Princeton University Press, 2007. ISBN 978-0-691-19688-6. URL https://www.jstor.org/stable/j.ctvh9w0ks.

D. Massel and M. K. Cruickshank. The number remaining at risk: an adjunct to the number needed to treat. *The Canadian Journal of Cardiology*, 18(3):254–258, 3 2002. ISSN 0828-282X. PMID: 11907613.

Karl Menger. Probabilistic geometry. *Proceedings of the National Academy of Sciences of the United States of America*, 37(4):226, 1951.

Karl Menger. *Probabilistic Geometry. Editors: Menger, Karl and Schweizer, Bert and Sklar, Abe and Sigmund, Karl and Gruber, Peter and Hlawka, Edmund and Reich, Ludwig and Schmetterer, Leopold*, page 441–444. Springer, 2003. ISBN 978-3-7091-6045-9. doi: 10.1007/978-3-7091-6045-9_37. URL https://doi.org/10.1007/978-3-7091-6045-9_37.

John Stuart Mill. *A system of logic, ratiocinative and inductive: Being a connected view of the 646 principles of evidence and the methods of scientific investigation. In two volumes. Volume 1*. John W. Parker, London, 1843a. archive.org, San Francisco, CA 94118, USA.

John Stuart Mill. *A system of logic, ratiocinative and inductive: Being a connected view of the 646 principles of evidence and the methods of scientific investigation. In two volumes. Volume 2*. John W. Parker, London, 1843b. archive.org, San Francisco, CA 94118, USA.

Milman, Vitali D. *Geometrization of Probability. Editors: Kapranov, Mikhail and Manin, Yuri Ivanovich and Moree, Pieter and Kolyada, Sergiy and Potyagailo, Leonid*, page 647–667. Progress in Mathematics. Birkhäuser, 2008. ISBN 978-3-7643-8608-5. doi: 10.1007/978-3-7643-8608-5_15. URL https://doi.org/10.1007/978-3-7643-8608-5_15.

Abraham de Moivre. *The Doctrine of Chances or a Method of Calculating the Probability of Events in Play. First Edition*. W. Pearson, for the author, London, January 1718. doi: 10.3931/e-rara-10420. Archive.org, USA e-rara, Zurich, CH.

Abraham de Moivre. *The Doctrine of Chances or a Method of Calculating the Probability of Events in Play. Third Edition*. A. Miller, in the Strand, London, January 1756. doi: 10.5281/zenodo.5758598. e-rara, Zurich, CH Zenodo.

Jürgen Neffe. *Einstein: A Biography*. Farrar, Straus and Giroux, New York (USA), 2006.

Russell Newstadt. *Omnis Determinatio est Negatio: A Genealogy and Defense of the Hegelian Conception of Negation*. Loyola University Chicago, Chicago (IL), dissertation edition, 2015. Free full text: Loyola University Chicago, USA.

Isaac Newton. *Philosophiae naturalis principia mathematica*. Jussu Societatis Regiae ac Typis Josephi Streater. Prostat apud plures bibliopolas, Londini, 1687. URL https://doi.org/10.5479/sil.52126.39088015628399. E-Rara archive.org Zenodo.

Jerzy Neyman and Egon Sharpe Pearson. IX. On the problem of the most efficient tests of statistical hypotheses. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, 231(694–706): 289–337, 2 1933. ISSN 0264-3952, 2053-9258. doi: 10.1098/rsta.1933.0009. The Royal Society, London, GB.

Jean George Pierre Nicod. A reduction in the number of primitive propositions of logic. *Proceedings of the Cambridge Philosophical Society*, 19:32–41, 1917.

Jean George Pierre Nicod. Les relations des valeurs et les relations de sens en logique formelle. *Revue de métaphysique et de morale*, 31:467–480, 1924.

Judea Pearl. *Causality: models, reasoning, and inference*. Cambridge University Press, Cambridge, U.K. ; New York, 2000. ISBN 978-0-521-89560-6.

Karl Pearson. III. Contributions to the Mathematical Theory of Evolution. On the Dissection of Asymmetrical-Frequency Curves. *Philosophical Transactions of the Royal Society of London, Series A*, 185:71–85, 1894. The Royal Society.

Karl Pearson. X. Contributions to the mathematical theory of evolution. — II. Skew variation in homogeneous material. *Philosophical Transactions of the Royal Society of London. (A.)*, 186:343–414, Jan 1895. doi: 10.1098/rsta.1895.0010. Publisher: Royal Society.

Karl Pearson. VII. Mathematical contributions to the theory of evolution.—III. Regression, heredity, and panmixia. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, 187:253–318, January 1896. doi: 10.1098/rsta.1896.0007. URL https://royalsocietypublishing.org/doi/abs/10.1098/rsta.1896.0007.

Karl Pearson. XV. On certain properties of the hypergeometrical series, and on the fitting of such series to observation polygons in the theory of chance. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 47(285):236–246, January 1899. ISSN 1941-5982. doi: 10.1080/14786449908621253. Taylor and Francis.

Karl Pearson. X. On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 50(302):157–175, July 1900a. ISSN 1941-5982. doi: 10.1080/14786440009463897. Taylor and Francis.

Karl Pearson. X. On the Criterion that a Given System of Deviations from the Probable in the Case of a Correlated System of Variables is such that it can be Reasonably Supposed to have Arisen from Random Sampling. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 50(302):157–175, 1900b.

Karl Pearson. III. Mathematical contributions to the theory of evolution. — XII. On a generalised Theory of alternative Inheritance, with special reference to Mendel's laws. *p 66: Standard dev binomial distribution*, 203(359–371):53–86, 1 1904a. doi: 10.1098/rsta.1904.0015. Royal Society.

Karl Pearson. *Mathematical contributions to the theory of evolution. XIII. On the theory of contingency and its relation to association and normal correlation*. Biometric Series I. Dulau and Co., London, January 1904b. Free full text: archive.org, San Francisco, CA 94118, USA.

Karl Pearson. *The Grammar of Science. Third Edition*. Adam and Charles Black, London (GB), 1911. ISBN 978-1-108-07711-8. doi: 10.1017/CBO9781139878548. Free full text: archive.org, San Francisco, CA 94118, USA.

Max Karl Ernst Ludwig Planck. Ueber das Gesetz der Energieverteilung im Normalspectrum. *Annalen der Physik*, 309(3):553–563, 1901. ISSN 1521-3889. doi: 10.1002/andp.19013090310. Wiley Online Library.

Jules Henri Poincaré. Sur la dynamique de l'électron. *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, 140: 1504–1508, 1905.

Graham Priest. What is so Bad about Contradictions? *The Journal of Philosophy*, 95(8):410–426, 1998. ISSN 0022-362X. doi: 10.2307/2564636. URL https://www.jstor.org/stable/2564636.

Graham Priest, Richard Sylvan, Jean Norman, and A. I. Arruda, editors. *Paraconsistent logic: essays on the inconsistent*. Analytica. Philosophia, München ; Hamden [Conn.], 1989. ISBN 978-3-88405-058-3.

Francisco Miró Quesada, editor. *Heterodox logics and the problem of the unity of logic. In: Non-Classical Logics, Model Theory, and Computability: Proceedings of the Third Latin-American symposium on Mathematical Logic, Campinas, Brazil, July 11-17, 1976. Arruda, A. I., Costa, N. C. A. da, Chuaqui, R. (Eds.).*, volume 89 of *Studies In Logics And The Foundations Of Mathematics*. North-Holland, Amsterdam ; New York : New York, February 1977. ISBN 978-0-7204-0752-5.

Connie Robertson. *The Wordsworth Dictionary of Quotations. Edited by Connie Robertson*. Wordsworth, Ware, Hertfordshire, 1998. ISBN 978-1-85326-751-2. ISBN: 1-85326-489-X.

Josiah Royce. *Negation*, volume 9 of *Encyclopaedia of Religion and Ethics. J. Hastings (ed.)*. Charles Scribner's Sons, New York (USA), 1917. Free full text: archive.org, San Francisco, CA 94118, USA.

Bertrand Russell. *The problems of philosophy*. H. Holt, 1912. archive.org.

Sackett, DL and Deeks, JJ and Altman, DG. Down with odds ratios! *Evidence-Based Med.*, 1:164–166, 1996. *.pdf file DOI: 10.1136/ebm.1996.1.164.

D. A. Sadowsky, A. G. Gilliam, and J. Cornfield. The statistical association between smoking and carcinoma of the lung. *Journal of the National Cancer Institute*, 13(5):1237–1258, 4 1953. ISSN 0027-8874. PMID: 13035448.

Schlick, Friedrich Albert Moritz. Die Kausalität in der gegenwärtigen Physik. *Naturwissenschaften*, 19:145–162, 2 1931. ISSN 0028-1042. doi: 10.1007/BF01516406. Springer.

Schrödinger, Erwin Rudolf Josef Alexander. An undulatory theory of the mechanics of atoms and molecules. *Physical Review*, 28: 1049–1070, Dec 1926. doi: 10.1103/PhysRev.28.1049. URL https://link.aps.org/doi/10.1103/PhysRev.28.1049.

Schrödinger, Erwin Rudolf Josef Alexander. Was ist ein Naturgesetz? *Naturwissenschaften*, 17(1):9–11, January 1929. ISSN 1432-1904. doi: 10.1007/BF01505758. URL https://doi.org/10.1007/BF01505758.

Schrödinger, Erwin Rudolf Josef Alexander. Are there quantum jumps? *British Journal for the Philosophy of Science*, 3(11):233–242, 1952. doi: 10.1093/bjps/III.11.233.

Henry Maurice Sheffer. A set of five independent postulates for boolean algebras, with application to logical constants. *Transactions of the American Mathematical Society*, 14(4):481–488, 1913. ISSN 0002-9947, 1088-6850. doi: 10.1090/S0002-9947-1913-1500960-1. Free full text: archive.org, San Francisco, CA 94118, USA.

E. Sober. Venetian Sea Levels, British Bread Prices, and the Principle of the Common Cause. *The British Journal for the Philosophy of Science*, 52(2):331–346, January 2001. ISSN 0007-0882. DOI: 10.1093/bjps/52.2.331.

Herbert Solomon. *Geometric probability*. CBMS-NSF regional conference series in applied mathematics. Society for Industrial and Applied Mathematics, 1978. ISBN 978-0-89871-025-0. URL https://epubs.siam.org/doi/pdf/10.1137/1.9781611970418.fm.

Antonín Špaček. Note on k. menger's probabilistic geometry. *Czechoslovak Mathematical Journal*, 6(1):72–74, 1956.

J. L. Speranza and Laurence R. Horn. A brief history of negation. *Journal of Applied Logic*, 8(3):277–301, September 2010. ISSN 1570-8683. DOI: 10.1016/j.jal.2010.04.001 ScienceDirect.

Benedictus de Spinoza. *Opera quae supersunt omnia / iterum edenda curavit, praefationes, vitam auctoris, nec non notitias, quae ad historiam scriptorum pertinent*. in bibliopolio academico, June 1674. doi: 10.5281/zenodo.5651174. URL https://doi.org/10.5281/zenodo.5651174. Zenodo.

Wolfgang Spohn. *Eine Theorie der Kausalität*. Fakultät für Philosophie, Wissenschaftstheorie und Statistik. Ludwig-Maximilians-Universität München, 1983. Uni Konstanz.

Stenonis, Nicolai. *De solido intra solidum naturaliter contento dissertationis prodromus*. ex typographia sub signo stellae, 1669. e-rara.

Patrick Suppes. *A probabilistic theory of causality*. Number Fasc. 24 in Acta philosophica Fennica. North-Holland Pub. Co, Amsterdam, 1970. ISBN 978-0-7204-2404-1.

M. E. Thompson. Ilija Barukčić. Causality. New Statistical Methods. A Book Review. *International Statistical Institute - Short Book Review*, 26(01):6, January 2006. ISI - Short Book Reviews, p. 6.

William Todd. Causes and counterfactuals. In *Analytical Solipsism*, pages 127–149. Springer, 1968.

Tamar Tsopurashvili. Negatio negationis als Paradigma in der Eckhartschen Dialektik. In *Universalità della Ragione. A. Musco (ed.)*, volume II.1, pages 595–602, Palermo, 17-22 settembre 2007, 2012. Luglio.

author Unknown. Kausale und konditionale Weltanschauung. *Nature*, 90(2261):698–699, February 1913. ISSN 1476-4687. doi: 10.1038/090698a0. URL https://www.nature.com/articles/090698a0. Number: 2261 Publisher: Nature Publishing Group.

J. v. Uspensky. *Introduction To Mathematical Probability*. McGraw-Hill Company, New York (USA), 1937. Free full text: archive.org, San Francisco, CA 94118, USA.

John Venn. *The logic of chance: an essay on the foundations and province of the theory of probability, with especial reference to its logical bearings and its application to moral and social science, and to statistics*. Macmillan And Company, 1888. archive.org.

Max Verworn. *Kausale und konditionale Weltanschauung*. Verlag von Gustav Fischer, Jena, 1912.

Woldemar Voigt. *Über das Doppler'sche Princip*, volume 8 of *Nachrichten von der Köönigl. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität zu Göttingen*. Dieterichsche Verlags-Buchhandlung, 1887. URL http://archive.org/details/nachrichtenvond04gtgoog. archive.org.

Michael V Wedin. Negation and quantification in aristotle. *History and Philosophy of Logic*, 11(2):131–150, 1990a. Taylor & Francis.

Michael V. Wedin. Negation and quantification in aristotle. *History and Philosophy of Logic*, 11(2):131–150, January 1990b. ISSN 0144-5340. doi: 10.1080/01445349008837163. DOI: 10.1080/01445349008837163.

William Allen Whitworth. *Choice and Chance. With 1000 Exercises*. Deighton, Bell & Co., fifth eidtion edition, 1901. archive.org, San Francisco, CA 94118, USA .

John Woods and Douglas Walton. Post Hoc, Ergo Propter Hoc. *The Review of Metaphysics*, 30(4):569–593, 1977. ISSN 0034-6632. URL https://www.jstor.org/stable/20126985.

Frank Yates. Contingency Tables Involving Small Numbers and the Chi square Test. *Supplement to the Journal of the Royal Statistical Society*, 1(2):217–235, 1934. ISSN 1466-6162. doi: 10.2307/2983604. JSTOR.

Shwu Yeng, T. Lin, and You-Feng Lin. The n-dimensional pythagorean theorem. *Linear and Multilinear Algebra*, April 2008. Publisher: Gordon and Breach Science Publishers.

Jacob Yerushalmy. Statistical problems in assessing methods of medical diagnosis, with special reference to x-ray techniques. *Public Health Reports (1896-1970)*, pages 1432–1449, 1947. doi: 10.1016/S0140-6736(21)02316-3. JSTOR PMID: 20340527.

George Udny Yule. On the methods of measuring association between two attributes. *Journal of the Royal Statistical Society*, 75(6): 579–652, 1912. ISSN 0952-8385. doi: 10.2307/2340126. JSTOR.

George Udny Yule and Karl Pearson. VII. On the association of attributes in statistics: with illustrations from the material of the childhood society. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, 194(252–261):257–319, 1 1900. doi: 10.1098/rsta.1900.0019. The Royal Society, London, GB.

Patrick Manuel Zesar. *nihil fit sine causa - Die Kausalität im Spanischen und Portugiesischen: DIPLOMARBEIT. Magister der Philosophie*. Universität Wien, Wien, January 2013. URL http://othes.univie.ac.at/25095/1/2013-01-22_0506065.pdf.

I was born October, 1st 1961 in Novo Selo, Bosnia and Herzegovina, former Yogoslavia. I am of Croatian origin. From 1982-1989 C.E., I studied human medicine at the University of Hamburg, Germany. Meanwhile, I am working as a specialist of internal medicine. My basic field of research since my high school days at the Wirtschaftsgymnasium Bruchsal, Baden Württemberg, Germany is the mathematization of the relationship between a cause and an effect valid without any restriction under any circumstances including the conditions of classical logic, probability theory, quantum mechanics, special and general theory of relativity, human medicine et cetera. I endeavour to investigate positions of quantum mechanics, relativity theory, mathematics et cetera, only insofar as these positions put into question or endanger **the general validity of the principle of causality**.

[a] https://orcid.org/0000-0002-6988-2780
[b] https://cel.webofknowledge.com/InboundService.do?app=wos&product=CEL&Func=Frame&SrcApp=Publons&SrcAuth=Publons_CEL&locale=en-US&SID=F4r5Tsr3OcrmFbYrqiF&customersID=Publons_CEL&smartRedirect=yes&mode=FullRecord&IsProductCode=Yes&Init=Yes&action=retrieve&UT=WOS%3A000298855300006
[c] https://publons.com/researcher/3501739/ilija-barukcic/
[d] https://www.scopus.com/authid/detail.uri?authorId=37099674500
[e] https://www.scopus.com/authid/detail.uri?authorId=54974181600
[f] https://www.mendeley.com/search/?authorFullName=Ilija%20Baruk%C4%8Di%C4%87&page=1&query=Barukcic&sortBy=relevance
[g] https://www.researchgate.net/profile/Ilija-Barukcic-2
[h] https://zenodo.org/search?page=1&size=20&q=keywords:%22Baruk%C4%8Di%C4%87%22&sort=mostviewed
[i] https://zenodo.org/search?page=1&size=20&q=keywords:%22Baruk%C4%8Di%C4%87,%20Conference%22
[j] https://twitter.com/ilijabarukcic?lang=de
[k] https://twitter.com/Causation_Journ
[l] https://vixra.org/author/ilija_barukcic
[m] https://www.youtube.com/channel/UCwf3w1IngcukIOOjpw8HTwg
[n] https://portal.dnb.de/opac/showNextResultSite?currentResultId=%22Barukcic%22%26any&currentPosition=30