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Modus inversus

Research article

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Abstract

Background:

A true conclusion follows deductively from true premises. However, what if the premises are not true? Therefore, certain aspects of the relation between premises and conclusions in valid arguments are re-investigated again.

Methods:

Methods of logical inference (modus ponens, modus tollens et cetera) were analysed again.

Results:

Reliable scientific proof methods are of use in order to identify as soon as possible non-scientific claims and to help authors not to suffer too long from self-contradictory and sometimes highly abstract, especially mathematical stuff.

Conclusion:

Modus inversus has the potential to prevent us from accepting seemingly contradictory theorems or rules in science.

Keywords: Premises; Conclusions; Cause; Effect; Causal relationship k; Causality; Causation

1. Introduction

Under usual circumstances, a true conclusion follow from true premises (i.e axioms) and an argument is treated as a true or good argument. Unfortunately, we have to note besides of recognisable progress, that errors in human reasoning (fallacies) occur, either unintentionally or intentionally i. e. in order to deceive other people. Nonetheless, are errors as such and especially errors in human reasoning really inevitable, unavoidable and justified per se? In particular, what kind of a conclusions follow from false premises? What is it for a conclusion to be a consequence of false premises? Does from a contradiction or from falsehood, anything follow, as claimed by the **Ex Contradictione Non Sequitur Quodlibet Principle** (ECQ) [1] (principle of explosion (Carnielli and Marcos, 2001))? Today's standard logical view is that we cannot coherently reason about logical inconsistency. Nontheless, the most basic requirement of any orthodox logic is still the notion of logical coherence: no theory can include any contradictions. However, we cannot ignore any longer that contradictions (Quesada,

[1] Priest, Graham, Koji Tanaka, and Zach Weber, "Paraconsistent Logic", The Stanford Encyclopedia of Philosophy (Spring 2022 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2022/entries/logic-paraconsistent/.

1977) are existing (Barukčić, 2019a) objectively [2] and real. Well, what next, what remains to be done? Such and similar questions are in many respects one important part of logic and play a central role in contemporary accounts of logical consequence. Historically, the development of the methods of logical inference [3] by humans remains mostly in the dark. In point of fact, there is some evidence of documented human trials of systematic logical inquiry, which can be traced back to Greek and Roman antiquity. Especially Aristotle (384–322 BCE) himself developed a system of deductive inference (Aristotle's theory syllogism (see Aristotle, Prior Analytics)) [4]) by systematising much of the work of his predecessors. Much of Aristotle's logic is about **deduction** (sullogismos), the thing supposed is a **premise** (protasis) while what results of necessity determines the **conclusion** (sumperasma). [5] In point of fact and independent of Aristotle's and other achievements, in view of the many and often each other contracting, excluding and competing scientific results, position, theorems, theories et cetera of objective reality and the nature and of our world, a theoretical appreciation of scientific proof methods becomes pressing. Especially some questions forces themselves upon us with all its might, what is true, what is false and is there a logically correct model of scientific argumentation and reasoning et cetera? Furthermore, what is the natural basis of human logic and of human thinking? Logically consistent proof methods thereby constitute our grounds of scientific evidence, which itself might help us to refute or to confirm arguments, positions, theorems and scientific theories. For these reasons and others, scientific proof methods are equally necessary for scientific knowledge and the demarcation line between ((justified) personal) belief and exceedingly clear and well-verified scientific knowledge and at the end between ideology and science. For these reasons and others, it is appropriate to explore the nature of modus inversus [6] once again. In what follows, we will touch on few central aspects of the relationship between premises and conclusions. [7] There are many different ways to attempt to analyse the relationship between premises or axioms and conclusions or theories. As is well known, human cognition is also historically determined. As an example, the ancient Romans could not objectively prove the existence of Mycobacterium tuberculosis, but they were certainly exposed to to the same. Theoretically, we cannot exclude that there are natural processes, events et cetera, whose existence we have not proven at the present time or even which cannot be proved with our methodological possibilities today. However, does this immediately mean that what is not possible today will also be impossible tomorrow? Nonetheless, as long as we only rely on Kurt Friedrich Gödel's (1906-1978) insights and his own two incompleteness (see Gödel, 1931) theorems (i. e. Satz VI), [8] , [9] the basic question whether there are claims, statements, conclusions, et cetera which can neither be proved nor disproved does not remain open and is no longer an unresolved issue. According to Gödel, there are not

[2] Barukčić, Ilija. (2020, December 28). The contradiction is exsiting objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

[3] Bobzien, Susanne, "Ancient Logic", The Stanford Encyclopedia of Philosophy (Summer 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/sum2020/entries/logic-ancient/.

[4] Lagerlund, Henrik, "Medieval Theories of the Syllogism", The Stanford Encyclopedia of Philosophy (Summer 2022 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/sum2022/entries/medieval-syllogism/.

[5] Smith, Robin, "Aristotle's Logic", The Stanford Encyclopedia of Philosophy (Fall 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2020/entries/aristotle-logic/. See also: Aristotle, Prior Analytics I.2, 24b18–20

[6] Barukčić, Ilija. (2019). Modus Inversus – If (Premise is False) Then (Conclusion is False) (Version 1). Zenodo. https://doi.org/10.5281/zenodo.3986654

[7] Beall, Jc, Greg Restall, and Gil Sagi, "Logical Consequence", The Stanford Encyclopedia of Philosophy (Spring 2019 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2019/entries/logical-consequence/.

[8] Gödel, Kurt. "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I." Monatshefte für mathematik und physik 38.1 (1931): 173-198.

[9] Gödel, Kurt. "Zum intuitionistischen aussagenkalkül." Anzeiger der Akademie der Wissenschaften in Wien 69 (1932): 65–66 .

only historical limits of provability. It is much more serious that there are already theoretical limits of provability. In other words, according to Gödel, there are claims, statements, theories et cetera which formally can neither be proved nor disproved. In contrast to Gödel and after highlighting some formal aspects, in anticipating and emphasizing the importance of real-world examples, we'll work through alternative possibilities.

2. Material and methods

Scientific knowledge and objective reality are more than only interrelated. It cannot be repeated often enough that objective reality or processes of objective reality is the foundation of any scientific knowledge. Our human experience teaches us however that seen by light, grey is never merely simply grey, and looked at from different angles, many paths may lead to climb up a certain mountain. In general, it is appropriate to ensure as much as possible a broader consideration of a research question and to take into account the different facets and viewpoints of an issue investigated in order to reach a goal. Today's science has become to a very great extent ideological. Rightly or wrongly, science is and has been misused since ever to support the ideologies of its practitioners or of certain ideologies as such and vice versa. Ideologies are meanwhile an unjustified part of the nature of scientific inquiry. Unfortunately, science is not hermetically sealed off from today's dominant, very aggressive, inhuman and leading ideology "In maximizing profit we believe". Even if not all scientist are equally susceptible to appropriation or ideological influence, there is documented [10] evidence that to many times the one who pays commands even the result obtained by scientific investigations. In particular, the all-encompassing dictatorship of maximizing profit is on the way to make science purely trivial, just one and sometimes meaningless views among many others. In order to solve real-world challenges, science taken more seriously should decrease the influence of non-science on science to a maximum. Scientific proof methods are of use to distinguish between scientific knowledge and false, even if popular belief or deceptively bad arguments.

[10] Bombardier C, Laine L, Reicin A, Shapiro D, Burgos-Vargas R, Davis B, Day R, Ferraz MB, Hawkey CJ, Hochberg MC, Kvien TK, Schnitzer TJ; VIGOR Study Group. Comparison of upper gastrointestinal toxicity of rofecoxib and naproxen in patients with rheumatoid arthritis. VIGOR Study Group. N Engl J Med. 2000 Nov 23;343(21):1520-8, 2 p following 1528. doi: 10.1056/NEJM200011233432103. PMID: 11087881.

2.1. Material

2.2. Methods

Definitions should help us to provide and assure a systematic approach to a scientific issue. It also goes without the need of further saying that a definition as such need to be logically consistent and correct.

2.2.1. Bernoulli distribution

A single event distribution is more or less a discrete probability distribution of any random variable X which takes a certain (observer independent) single value X_t at a **Bernoulli trial** (Uspensky, 1937, p. 45) (period of time) t with the probability $p(X_t)$. The same random variable X takes a certain single anti value $\underline{X}_t$ at a Bernoulli trial (period of time) t with the probability $1-p(X_t)$. There are conditions in nature where a random variable X can take only the values either +0 or +1 (see Birnbaum, 1961). Under these conditions, the random variable X takes the value 1 with probability $p(X_t = +1)$ and the value 0 with probability $q(X_t = +0) = 1 - p(X_t = +1)$ while the single event distribution passes over into the **Bernoulli distribution**, named after Swiss mathematician Jacob Bernoulli (Bernoulli, 1713). Less formally, many times, the Bernoulli distribution is represented by a (possibly not biased) coin toss where 1 and 0 would represent 'heads'and 'tails'(or vice versa), respectively. However, the relationship between random variables (Gosset, 1914) can be investigated by many (Gosset, 1908) methods, including the tools of probability theory, too.

Definition 2.1 (**Two by two table of single event random variables**).

The two by two or contingency table which has been introduced by Karl Pearson (Pearson, 1904b) in 1904 harbours still a large variety of topics and debates. Central to this is the problem to apply the laws of classical logic on data sets, which concerns the justification of inferences which extrapolate from sample data to general facts. Nevertheless, a contingency table is still an appropriate theoretical model too for studying the relationships between random variables, including *Bernoulli (Bernoulli, 1713) (i.e. +0/+1) distributed random variables* existing or occurring at the same *Bernoulli trial* (Uspensky, 1937) (period of time) t.

In this context, let a random variable A at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by A_t, indicate a risk factor, a condition, a cause et cetera and occur or exist with the probability $p(A_t)$ at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let $E(A_t)$ denote the expectation value of A_t. In general it is

$$p(A_t) \equiv p(a_t) + p(b_t) \tag{1}$$

The expectation value $E(A_t)$ follows as

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv A_t \times (p(a_t) + p(b_t)) \\ &\equiv (A_t \times p(a_t)) + (A_t \times p(b_t)) \\ &\equiv E(a_t) + E(b_t) \end{aligned} \tag{2}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv (+0+1) \times p(A_t) \\ &\equiv p(A_t) \\ &\equiv p(a_t) + p(b_t) \end{aligned} \tag{3}$$

Furthermore, it is

$$p(\underline{A}_t) \equiv p(c_t) + p(d_t) \equiv (1 - p(A_t)) \tag{4}$$

The expectation value $E(\underline{A}_t)$ is given as

$$\begin{aligned} E(\underline{A}_t) &\equiv A_t \times (1 - p(A_t)) \\ &\equiv A_t \times (p(c_t) + p(d_t)) \\ &\equiv (A_t \times p(c_t)) + (A_t \times p(d_t)) \\ &\equiv E(c_t) + E(d_t) \end{aligned} \tag{5}$$

Under conditions of +0/+1 distributed Bernoulli random variables we obtain

$$\begin{aligned} E(\underline{A}_t) &\equiv A_t \times (1 - p(A_t)) \\ &\equiv (+0+1) \times (1 - p(A_t)) \\ &\equiv (1 - p(A_t)) \\ &\equiv p(c_t) + p(d_t) \end{aligned} \tag{6}$$

Let a random variable B at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by B_t, indicate an outcome, a conditioned, an effect et cetera and occur or exist with the probability $p(B_t)$ at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let $E(B_t)$ denote the expectation value of B_t. In general it is

$$p(B_t) \equiv p(a_t) + p(c_t) \tag{7}$$

The expectation value $E(B_t)$ is given by the equation

$$\begin{aligned} E(B_t) &\equiv B_t \times p(B_t) \\ &\equiv B_t \times (p(a_t) + p(c_t)) \\ &\equiv (B_t \times p(a_t)) + (B_t \times p(c_t)) \\ &\equiv E(a_t) + E(c_t) \end{aligned} \tag{8}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E\left(B_{t}\right) &\equiv B_{t} \times p\left(B_{t}\right) \\ &\equiv (+0+1) \times p\left(B_{t}\right) \\ &\equiv p\left(B_{t}\right) \\ &\equiv p\left(a_{t}\right) + p\left(c_{t}\right) \end{aligned} \tag{9}$$

Furthermore, it is

$$p\left(\underline{B}_{t}\right) \equiv p\left(b_{t}\right) + p\left(d_{t}\right) \equiv \left(1 - p\left(B_{t}\right)\right) \tag{10}$$

The expectation value $E(\underline{B}_t)$ is given by the equation

$$\begin{aligned} E\left(\underline{B}_{t}\right) &\equiv B_{t} \times \left(1 - p\left(B_{t}\right)\right) \\ &\equiv B_{t} \times \left(p\left(b_{t}\right) + p\left(d_{t}\right)\right) \\ &\equiv \left(B_{t} \times p\left(b_{t}\right)\right) + \left(B_{t} \times p\left(d_{t}\right)\right) \\ &\equiv E\left(b_{t}\right) + E\left(d_{t}\right) \end{aligned} \tag{11}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E\left(\underline{B}_{t}\right) &\equiv B_{t} \times \left(1 - p\left(B_{t}\right)\right) \\ &\equiv (+0+1) \times \left(1 - p\left(B_{t}\right)\right) \\ &\equiv \left(1 - p\left(B_{t}\right)\right) \\ &\equiv p\left(b_{t}\right) + p\left(d_{t}\right) \end{aligned} \tag{12}$$

Let $p(a_t)= p(A_t \wedge B_t)$ denote the joint probability distribution of A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E\left(a_{t}\right) &\equiv E\left(A_{t} \wedge B_{t}\right) \\ &\equiv \left(A_{t} \times B_{t}\right) \times p(A_{t} \wedge B_{t}) \\ &\equiv \left(A_{t} \times B_{t}\right) \times p(a_{t}) \end{aligned} \tag{13}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E\left(a_{t}\right) &\equiv E\left(A_{t} \wedge B_{t}\right) \\ &\equiv \left(A_{t} \times B_{t}\right) \times p(A_{t} \wedge B_{t}) \\ &\equiv ((+0+1) \times (+0+1)) \times p(A_{t} \wedge B_{t}) \\ &\equiv p(A_{t} \wedge B_{t}) \\ &\equiv p\left(a_{t}\right) \end{aligned} \tag{14}$$

Let $p(b_t)= p(A_t \wedge \neg B_t)$ denote the joint probability distribution of A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E\left(b_{t}\right) &\equiv E\left(A_{t} \wedge \neg B_{t}\right) \\ &\equiv \left(A_{t} \times \neg B_{t}\right) \times p(A_{t} \wedge \neg B_{t}) \\ &\equiv \left(A_{t} \times \neg B_{t}\right) \times p(b_{t}) \end{aligned} \tag{15}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E\left(b_{\mathrm{t}}\right) &\equiv E\left(A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}\right) \\ &\equiv \left(A_{\mathrm{t}} \times \neg B_{\mathrm{t}}\right) \times p(A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}) \\ &\equiv \left(\left(+0+1\right) \times \left(+0+1\right)\right) \times p(A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}) \\ &\equiv p(A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}) \\ &\equiv p\left(b_{\mathrm{t}}\right) \end{aligned} \tag{16}$$

Let $p(c_t) = p(\neg A_t \wedge B_t)$ denote the joint probability distribution of not A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E\left(c_{\mathrm{t}}\right) &\equiv E\left(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}\right) \\ &\equiv \left(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}\right) \times p\left(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}\right) \\ &\equiv \left(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}\right) \times p(c_{\mathrm{t}}) \end{aligned} \tag{17}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E\left(c_{\mathrm{t}}\right) &\equiv E\left(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}\right) \\ &\equiv \left(\neg A_{\mathrm{t}} \times B_{\mathrm{t}}\right) \times p(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}) \\ &\equiv \left(\left(+0+1\right) \times \left(+0+1\right)\right) \times p\left(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}\right) \\ &\equiv p(\neg A_{\mathrm{t}} \wedge B_{\mathrm{t}}) \\ &\equiv p\left(c_{\mathrm{t}}\right) \end{aligned} \tag{18}$$

Let $p(d_t) = p(\neg A_t \wedge \neg B_t)$ denote the joint probability distribution of not A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E\left(d_{\mathrm{t}}\right) &\equiv E\left(\neg A_{\mathrm{t}} \times \neg B_{\mathrm{t}}\right) \\ &\equiv \left(\neg A_{\mathrm{t}} \times \neg B_{\mathrm{t}}\right) \times p\left(\neg A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}\right) \\ &\equiv \left(\neg A_{\mathrm{t}} \times \neg B_{\mathrm{t}}\right) \times p(d_{\mathrm{t}}) \end{aligned} \tag{19}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E\left(d_{\mathrm{t}}\right) &\equiv E\left(\neg A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}\right) \\ &\equiv \left(\neg A_{\mathrm{t}} \times \neg B_{\mathrm{t}}\right) \times p(\neg A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}) \\ &\equiv \left(\left(+0+1\right) \times \left(+0+1\right)\right) \times p\left(\neg A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}\right) \\ &\equiv p(\neg A_{\mathrm{t}} \wedge \neg B_{\mathrm{t}}) \\ &\equiv p\left(d_{\mathrm{t}}\right) \end{aligned} \tag{20}$$

In general, it is

$$p\left(a_{\mathrm{t}}\right) + p\left(b_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right) + p\left(d_{\mathrm{t}}\right) \equiv +1 \tag{21}$$

Table 1 provide us with an overview of the definitions above.

In our understanding, it is

$$p\left(B_{\mathrm{t}}\right) + p\left(\Lambda_{\mathrm{t}}\right) \equiv p\left(a_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right) + p\left(\Lambda_{\mathrm{t}}\right) \equiv p\left(a_{\mathrm{t}}\right) + p\left(b_{\mathrm{t}}\right) \equiv p\left(A_{\mathrm{t}}\right) \tag{22}$$

Table 1. The two by two table of Bernoulli random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

or

$$p(c_t) + p(\Lambda_t) \equiv p(b_t) \tag{23}$$

Under conditions of Einstein's general theory of relativity, Λ denotes the Einstein cosmological (Einstein, 1917) 'constant '.

2.2.2. Binomial random variables

The binomial (see Pearson, 1895, p. 351) distribution (see Cramér, 1937) with parameters n and p has been developed by the Swiss mathematician Jakob Bernoulli (1655-1705) in a proof published in his 1713 book Ars Conjectandi (see Bernoulli, 1713) Part 1. In probability theory and statistics, the probability of getting exactly k successes in n independent Bernoulli trials is given by the probability mass function as

$$p(X_t = k) \equiv \binom{n}{k} \cdot p^k \cdot q^{n-k} \tag{24}$$

is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ the binomial coefficient while the cumulative distribution function is given as

$$p(X_t \leq k) \equiv 1 - p(X_t > k) \equiv \sum_{t=0}^{k} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{25}$$

or as

$$p(X_t > k) \equiv 1 - p(X_t \leq k) \equiv 1 - \sum_{t=0}^{k} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{26}$$

Furthermore, it is

$$p(X_t < k) \equiv 1 - p(X_t \geq k) \equiv \sum_{t=0}^{k-1} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{27}$$

or

$$p(X_t \geq k) \equiv 1 - p(X_t < k) \equiv 1 - \sum_{t=0}^{k-1} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{28}$$

The binomial distribution is the mathematical foundation of a binomial test. The random variable X_t is counting for different things. The discrete geometric (see Feller, 1950, p. 61) distribution describes under certain circumstances the number of Bernoulli trials needed to get one success. The probability that the first occurrence of success requires k independent trials, each with success probability p, is given by the equation

$$p(X_{\mathrm{t}} = k) \equiv p \cdot q^{k-1} \tag{29}$$

The negative (see Fisher, 1941, Haldane, 1941) binomial probability is a discrete probability distribution which defines the number of successes (k) in a sequence of independent and identically distributed Bernoulli trials (n) before a specified (non-random) number of failures (denoted r) occurs. The probability mass function of the negative binomial distribution is

$$p(X_{\mathrm{t}} = r) \equiv \binom{k+r-1}{k-1} p^k \cdot q^r \tag{30}$$

where k is the number of successes, r is the number of failures, and p is the probability of success.

Definition 2.2 (**Expectation value and variance of a binomial random variable**).

The variance(see Pearson, 1904a, p. 66) of the binomial distribution with parameters n, the number of independent experiments each asking a yes–no question and p, the probability of a single event, is defined in contrast to Pearson (see Barukčić, 2022c) as

$$\sigma(X_{\mathrm{t}})^2 \equiv X_{\mathrm{t}} \times X_{\mathrm{t}} \times p(X_{\mathrm{t}}) \times (1 - p(X_{\mathrm{t}})) \tag{31}$$

Definition 2.3 (**Two by two table of Binomial random variables**).

Let X_{t} denote a random variable which itself is determined by the expectation values a_{t}, b_{t}, c_{t}, d_{t}, A_{t}, $\underline{A}_{\mathrm{t}}$, B_{t}, and $\underline{B}_{\mathrm{t}}$. Under conditions where *the probability of an event, an outcome, a success et cetera is* ***constant*** *from Bernoulli trial to Bernoulli trial t*, it is

$$\begin{aligned} A &= X_{\mathrm{t}} \times p(A_{\mathrm{t}}) \\ &\equiv E(A_{\mathrm{t}}) \\ &\equiv X_{\mathrm{t}} \times (p(a_{\mathrm{t}}) + p(b_{\mathrm{t}})) \\ &\equiv a_{\mathrm{t}} + b_{\mathrm{t}} \end{aligned} \tag{32}$$

and

$$\begin{aligned} B &= X_{\mathrm{t}} \times p(B_{\mathrm{t}}) \\ &\equiv E(B_{\mathrm{t}}) \\ &\equiv X_{\mathrm{t}} \times (p(a_{\mathrm{t}}) + p(c_{\mathrm{t}})) \\ &\equiv a_{\mathrm{t}} + c_{\mathrm{t}} \end{aligned} \tag{33}$$

where X_{t} might denote the random variable the population or even the sample size.

Against the background of the Einstein's field equations (see also Einstein, 1916), we define the following.

$$\begin{aligned} B_t = X_t \times p(B_t) &\equiv A_t - \Lambda_t \\ &\equiv (X_t \times p(A_t)) - (X_t \times p(\Lambda_t)) \end{aligned} \tag{34}$$

Furthermore, it is

$$\begin{aligned} a_t &\equiv X_t \times p(a_t) \\ &\equiv E(a_t) \\ &\equiv A_t - b_t \\ &\equiv B_t - c_t \end{aligned} \tag{35}$$

There are circumstances where the value of a_t is not known. Under conditions where a joint distribution between A_t and B_t does exist, the maximum value of a_t is given approximately as

$$a_t \equiv Minimum(A_t, B_t) \tag{36}$$

Furthermore, it is

$$\begin{aligned} b_t &\equiv X_t \times p(b_t) \\ &\equiv E(b_t) \\ &\equiv A_t - a_t \\ &\equiv \underline{B}_t - d_t \end{aligned} \tag{37}$$

and

$$\begin{aligned} c_t &\equiv X_t \times p(c_t) \\ &\equiv E(c_t) \\ &\equiv B_t - a_t \\ &\equiv \underline{A}_t - d_t \end{aligned} \tag{38}$$

and

$$\begin{aligned} d_t &\equiv X_t \times p(d_t) \\ &\equiv E(d_t) \\ &\equiv \underline{A}_t - c_t \\ &\equiv \underline{B}_t - b_t \end{aligned} \tag{39}$$

and

$$a_t + b_t + c_t + d_t \equiv A_t + \underline{A}_t \equiv B_t + \underline{B}_t \equiv X_t \tag{40}$$

Under certain circumstances, a random variable X_t can be decomposed and presented in a matrix format that displays the individual components of the same random variable. In statistics and probability theory, such a table (contingency table) is providing a simple picture of the interrelation between the individual components which are determining a random variable X_t. Table 2 provide us again an overview of a two by two contingency (see also Pearson, 1904b, p. 33) table of Binomial random variables.

Table 2. The two by two table of Binomial random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	a_t	b_t	A_t
A_t	FALSE	c_t	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	X_t

"Such a table is termed a contingency table, and the ultimate scientific statement of description of the relation between two things can always be thrown back upon such a contingency table ⋯ Once the reader realizes the nature of such a table, he will have grasped the essence of the conception of association between cause and effect, and the nature of its ideal limit in causation. "

(see also Pearson, 1911, p. 159)

The situation does not change fundamentally, if we regard the square of X_t or $X_t^{\ 2}$ or even the n-dimensional case $X_t^{\ n}$. We obtain

Table 3. Expectation values

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$(X_t^{\ 2} \times p(a_t))$	$(X_t^{\ 2} \times p(b_t))$	$(X_t^{\ 2} \times p(A_t)) = E(A_t^{\ 2})$
A_t	FALSE	$(X_t^{\ 2} \times p(c_t))$	$(X_t^{\ 2} \times p(d_t))$	$(X_t^{\ 2} \times p(\underline{A}_t)) = E(\underline{A}_t^{\ 2})$
		$(X_t^{\ 2} \times p(B_t)) = E(B_t^{\ 2})$	$(X_t^{\ 2} \times p(\underline{B}_t)) = E(\underline{B}_t^{\ 2})$	$X_t^{\ 2}$

2.2.3. Independence

Definition 2.4 (**Independence**).

The philosophical, mathematical(Kolmogoroff, Andreĭ Nikolaevich, 1933) and physical(Einstein, 1948) et cetera concept of independence is of fundamental(Kolmogoroff, Andreĭ Nikolaevich, 1933) importance in (natural) sciences as such. Therefore, it is appropriate to investigate the concept of independence as completely as possible. In fact, de Moivre sums it up in his book The Doctrine of Chances (see also Moivre, 1718). "Two Events are **independent**, when they have no connexion one with the other, and that the happening of one neither forwards nor obstructs the happening of the other. Two events are **dependent**, when they are so connected together as that the Probability of either's happening is alter'd by the happening of the other. "(see also Moivre, 1756, p. 6) We should consider Kolmogorov's position on independence before the mind's eye too. "The concept of mutual independence of two or more experiments holds, in a certain sense, a central position in the theory of probability."(see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 8) Furthermore, it is insightful to recall even Einstein's theoretical approach to the concept of independence. "*Ohne die Annahme einer ⋯ Unabhängigkeit der ⋯ Dinge voneinander ⋯ wäre physikalisches Denken ⋯ nicht möglich.*"(Einstein, 1948). In general, an event A_t at the Bernoulli trial t need not, but can be independent of the existence or of the occurrence, of another event B_t *at the same* Bernoulli trial t. De Moivre brings it to the point. "From what has been said, it follows, that if a Fraction expresses the Probability of an Event, and another Fraction the Probability of another Event, and those two Events are independent ; the Probability that both those Events will Happen, will be the Product of those two Fractions."(see also Moivre, 1718, p. 4). Mathematically, in terms of probability theory, independence (Kolmogoroff, Andreĭ Nikolaevich, 1933) of events at the same (period of) time (i.e. Bernoulli trial) t is defined as

$$
\begin{aligned}
p(A_t \wedge B_t) &\equiv p(A_t) \times p(B_t) \equiv p(a_t) \\
&\equiv \frac{\sum_{t=1}^{N} (A_t \wedge B_t)}{N} \equiv \frac{N \times (p(a_t))}{N} \equiv 1 - p(A_t \mid B_t) \equiv 1 - p(A_t \uparrow B_t)
\end{aligned}
\tag{41}
$$

while $p(A_t \cap B_t)$ is the joint probability of the events A_t and B_t at a same Bernoulli trial t, $p(A_t)$ is the probability of an event A_t at a same Bernoulli trial t, and $p(B_t)$ is the probability of an event B_t at a same Bernoulli trial t. With respect to a two-by-two table , **under conditions of independence**, it is

$$
p(b_t) \equiv p(A_t) \times p(\underline{B}_t) \tag{42}
$$

or

$$
p(c_t) \equiv p(\underline{A}_t) \times p(B_t) \tag{43}
$$

and

$$
p(d_t) \equiv p(\underline{A}_t) \times p(\underline{B}_t) \tag{44}
$$

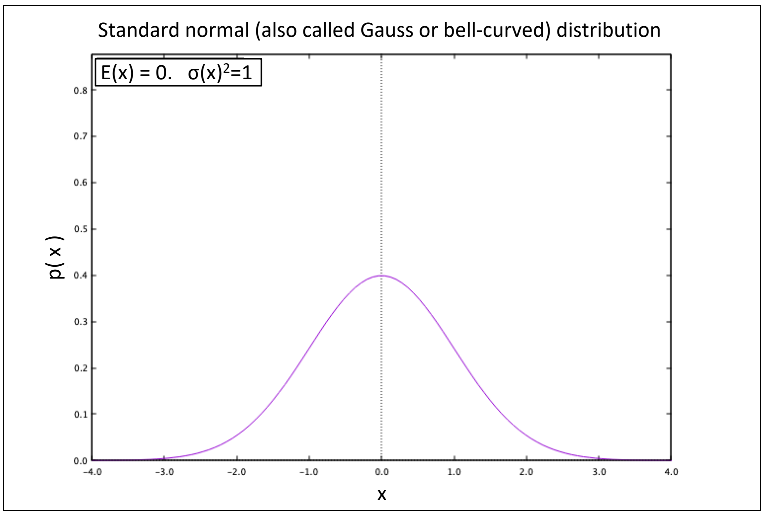
Example. In a narrower sense, the conditio sine qua non relationship concerns itself at the end only with the case whether the presence of an event A_t (condition) enables or guarantees the presence of another event B_t (conditioned). Thus far, as a result of the thoughts before, another question worth asking concerns the relationship between the independence of an event A_t (a condition) and another event B_t (conditioned) and the necessary condition relationship. To be confronted with the danger of bias and equally with the burden of inappropriate conclusions drawn, another fundamental question at this stage is whether is it possible that an event A_t (a condition) is a necessary condition of event B_t (conditioned) even under circumstances where the event A_t (a condition) (a necessary condition) is independent of an event B_t (conditioned)? Meanwhile, this question is more or less already answered to the negative (Barukčić, 2018). An event A_t which is a necessary condition of another event B_t is equally an event without which another event (B_t) could not be, could not occur, and implies as such already a kind of dependence. However, it is not mandatory that such a kind of dependence is a causal one. It is remarkable that **data which provide evidence of a significant conditio sine qua non relationship between two events like A_t and B_t and equally support the hypothesis that A_t and B_t are independent of each other are more or less self-contradictory and of very restricted or of none value for further analysis**. In fact, if the opposite view would be taken as plausible, contradictions are more or less inescapable.

2.2.4. Dependence

Definition 2.5 (**Dependence**).

Whilst it may be true that the occurrence of an event A_t does not affect the occurrence of an other event B_t the contrary is of no minor importance. Under these other conditions, events, trials and random variables et cetera are dependent on each other too. The dependence of events (Barukčić, 1989, p. 57-61) is defined as

$$p\left(\underbrace{A_t \wedge B_t \wedge C_t \wedge \ldots}_{n\ random\ variables}\right) \equiv \sqrt[n]{\underbrace{p(A_t) \times p(B_t) \times p(C_t) \times \ldots}_{n\ random\ variables}} \tag{45}$$



Normal distribution with
E(x) = 0
and
σ(x)^2 = 1

Figure 1. Normal distribution

2.3. Conditions

2.3.1. Distribution and anti-distribution

2.3.1.1. Normal distribution

The origins of the normal distribution, also known as the Gaussian distribution, the second law of Laplace, the law of error et cetera, has been studied at least since the 18th century and can be traced back even to a French mathematician Abraham de Moivre. Johann Carl Friedrich Gauß's (1777-1855) presented 1809 the normal distribution (see Gauß, Carl Friedrich, 1809, p. 244) while illustrating the method of least squares. In the following, Karl Pearson (1857-1936) popularised a new name for Gauß distribution. Pearson wrote: "A frequency-curve, which for practical purposes, can be represented by the error curve, will for the remainder of this paper be termed a normal curve." (see Pearson, 1894, p. 72).

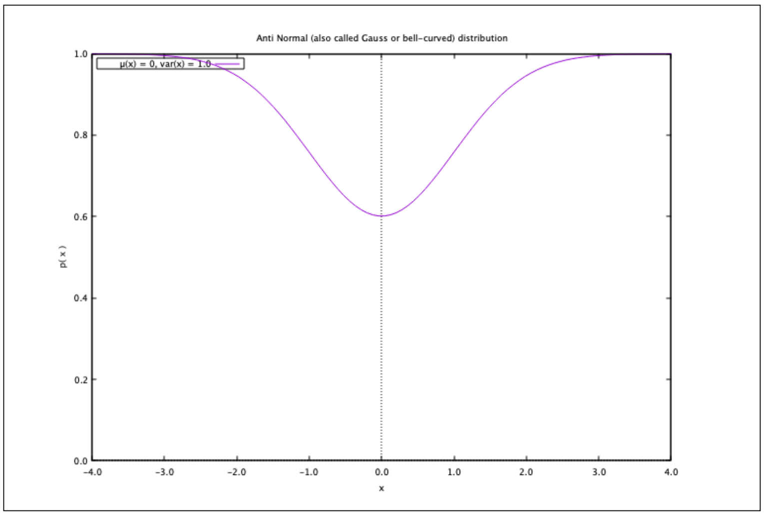
$$p({}_{\mathrm{R}}X_{\mathrm{t}}) = \left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\dfrac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \tag{46}$$

The standard normal distribution is illustrated by figure 1.

Sir Ronald Aylmer Fisher (1890-1962) [11] , a very influential statistician of the first half of the 20th century, presented the case of a normal (see Fisher, Ronald Aylmer, 1912, p. 157) distribution with non-zero mean (see Fisher, Ronald Aylmer, 1920, p. 758) as a typical case. The probability density function (pdf) of an anti-normal distribution is given as

$$p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) = 1 - \left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\dfrac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \tag{47}$$

[11] R. A. Fisher Digital Archive, The University of Adelaide. 5005 AUSTRALIA. copyright@adelaide.edu.au



Anti-normal distribution with
E(x) = 0
and
σ(x)^2 = 1

Figure 2. Anti-normal distribution

as illustrated by figure 2. In general, it is

$$p({}_{\mathrm{R}}X_{\mathrm{t}}) + p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) = 1 \tag{48}$$

The variance of a Gaussian distributed random variable is given as

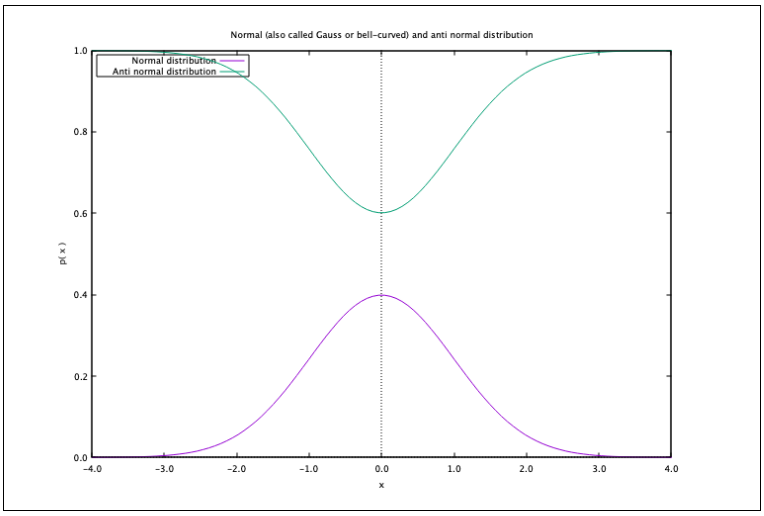
$$\begin{aligned}
\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2 &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \times p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \\
&\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right)\right)
\end{aligned} \tag{49}$$

Under conditions where E($_{\mathrm{R}}$X$_{\mathrm{t}}$) = 0 and $\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2$ = 1, equation 49 becomes

$$\begin{aligned}
\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2 &\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \times p({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \\
&\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right)\right) \\
&\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi \times 1}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - 0)^2}{2 \times 1}}\right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi \times 1}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}} - 0)^2}{2 \times 1}}\right)\right) \\
&\equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(\left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}})^2}{2}}\right) \times \left(1 - \left(\left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{({}_{\mathrm{R}}X_{\mathrm{t}})^2}{2}}\right)\right)
\end{aligned} \tag{50}$$

Standard normal distribution

In general, every normal distribution is a special form of the standard normal distribution. The standard normal distribution is a normal distribution with mean 0 and variance 1. Modern publications



Normal and anti-normal distribution with $E(\ x\ ) = 0$ and $\sigma(\ x\ )^2 = 1$

Figure 3. Normal and anti-normal distribution

often write the density function for the standard normal distribution, 'bell-shaped curve', as

$$p(z) = \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{z^2}{2}} \tag{51}$$

The density function for the anti-standard normal distribution is given as

$$p(\underline{z}) = 1 - p(z) = 1 - \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{z^2}{2}} \tag{52}$$

It is

$$p(z) + p(\underline{z}) = 1 \tag{53}$$

and is illustrated by figure 3.

Normal distributions are of use especially under conditions where the distribution of a random variable is not known. In general, let $_{R}A_{t}$ denote a standard normal distributed condition. Then

$$p({}_{\mathrm{R}}A_{\mathrm{t}}) = \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{{}_{\mathrm{R}}A_{\mathrm{t}}{}^2}{2}} \tag{54}$$

Let $_{R}B_{t}$ denote a standard normal distributed outcome or conditioned. Then

$$p({}_{\mathrm{R}}B_{\mathrm{t}}) = \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{{}_{\mathrm{R}}B_{\mathrm{t}}{}^2}{2}} \tag{55}$$

A bi-variate standard normal distribution is made up of two independent random variables. Two random variables $_{R}A_{t}$ and $_{R}B_{t}$ are independent if and only if the joint probability distribution function satisfies

$$\begin{aligned} p\left({}_{R}A_{t} \wedge {}_{R}B_{t}\right) &\equiv p\left({}_{R}A_{t}\right) \times p\left({}_{R}B_{t}\right) \\ &\equiv \left(\frac{1}{\sqrt{2\pi}}\right) \times e^{-\frac{{}_{R}A_{t}{}^{2}}{2}} \times \left(\frac{1}{\sqrt{2\pi}}\right) \times e^{-\frac{{}_{R}B_{t}{}^{2}}{2}} \\ &\equiv \left(\frac{1}{2 \times \pi}\right) \times e^{-\left(\frac{\left({}_{R}A_{t}\right)^{2}}{2}+\frac{\left({}_{R}B_{t}\right)^{2}}{2}\right)} \\ &\equiv \left(\frac{1}{2 \times \pi}\right) \times e^{-\left(\frac{\left({}_{R}A_{t}\right)^{2}+\left({}_{R}B_{t}\right)^{2}}{2}\right)} \end{aligned} \tag{56}$$

Example.

Let $_{R}B_{t} = 5$. It is

$$\left(\frac{1}{\sqrt{2 \cdot \pi}}\right) \cdot e^{-\left(\left(\frac{9}{(2)}\right)\right)} = 0.004431848412 \tag{57}$$

$$\left(\frac{1}{\sqrt{2 \cdot \pi}}\right) \cdot e^{-\left(\left(\frac{25}{(2)}\right)\right)} = 1.48671951 \cdot 10^{-6} \tag{58}$$

$$\left(\frac{1}{2 \cdot \pi}\right) \cdot e^{-\left(\left(\frac{9}{(2)}\right)+\left(\frac{25}{(2)}\right)\right)} = 6.58891552 \cdot 10^{-9} \tag{59}$$

The formulas above are of used in order to approximate Benford's law, also known as the Newcomb–Benford law or the law of anomalous numbers named after physicist Frank Benford [12] (1883–1948) and Simon Newcomb [13], [14] (1835–1909). More generally, based on the formulas before, analysing certain protocols that prevent unauthorised third parties or the public from reading coded messages becomes more simpler. It was Truman Lee Kelley (1884–1961) who introduced statistical methods into psychological studies [15] and who defined the z-score (see Kelley, 1924, p. 115). In mathematical statistics, a random variable $_{R}X_{t}$ is standardised by subtracting its expected value $E(_{R}X_{t})$ and dividing the difference by its standard deviation $\sigma(_{R}X_{t})$. The z-score or standard score, denoted as $z(_{R}X_{t})$, is defined as

$$z\left({}_{R}X_{t}\right) = \frac{\left({}_{R}X_{t} - E\left({}_{R}X_{t}\right)\right)}{\sigma\left({}_{R}X_{t}\right)} \tag{60}$$

[12] Benford, Frank. "The Law of Anomalous Numbers." Proceedings of the American Philosophical Society 78, no. 4 (1938): 551–72. http://www.jstor.org/stable/984802.

[13] Newcomb, Simon. "Note on the Frequency of Use of the Different Digits in Natural Numbers." American Journal of Mathematics 4, no. 1 (1881): 39–40. https://doi.org/10.2307/2369148.

[14] Formann AK. The Newcomb-Benford law in its relation to some common distributions. PLoS One. 2010 May 7;5(5):e10541. doi: 10.1371/journal.pone.0010541. PMID: 20479878; PMCID: PMC2866333.

[15] McClure WE. Speed and Accuracy of the Feebleminded on Performance Tests. Psychol Clin. 1931 Feb;19(9):265-274. PMID: 28909304; PMCID: PMC5138284.

Simply put, a z-score (also called a standard score) describes how many standard deviations a given quantum mechanical observable or a random variable lies above or below a specific value. Equation 60 changes to

$$z({}_{\mathrm{R}}X_{\mathrm{t}})^2 = \frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})^2}{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}{E({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{{}_{\mathrm{R}}X_{\mathrm{t}} \times (1 - p({}_{\mathrm{R}}X_{\mathrm{t}}))}{{}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{(1 - p({}_{\mathrm{R}}X_{\mathrm{t}}))}{p({}_{\mathrm{R}}X_{\mathrm{t}})} \tag{61}$$

Equation 61 simplifies as

$$E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times E({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{62}$$

We can imagine drawing figure 3 in n dimensions. Under these circumstances we would obtain something similar to an **Einstein–Rosen bridge** or Einstein–Rosen wormhole [16] formulated in terms of the framework of probability theory. Attention should be drawn to circumstances especially of quantum mechanics, where E(${}_{\mathrm{R}}\underline{\mathrm{X}}_{\mathrm{t}}$) indicates something like the expectation value of a 'local hidden variable '. Equation 62 changes slightly. It is

$${}_{\mathrm{R}}X_{\mathrm{t}} \times (1 - p({}_{\mathrm{R}}X_{\mathrm{t}})) = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times {}_{\mathrm{R}}X_{\mathrm{t}} \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{63}$$

and

$$(1 - p({}_{\mathrm{R}}X_{\mathrm{t}})) = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{64}$$

Equation 64 is rearranged as

$$1 = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times p({}_{\mathrm{R}}X_{\mathrm{t}}) + p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{65}$$

or

$$1 = (z({}_{\mathrm{R}}X_{\mathrm{t}})^2 + 1) \times p({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{66}$$

At the end, it follows that

$$p({}_{\mathrm{R}}X_{\mathrm{t}}) = \frac{1}{z({}_{\mathrm{R}}X_{\mathrm{t}})^2 + 1} \tag{67}$$

From equation 61 follows that

$$z({}_{\mathrm{R}}X_{\mathrm{t}})^2 = \frac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})^2}{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}{E({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \times E({}_{\mathrm{R}}X_{\mathrm{t}})}{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}X_{\mathrm{t}})} = \frac{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}{E({}_{\mathrm{R}}X_{\mathrm{t}})^2} \tag{68}$$

Thus far, it is equally

$$\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2 = z({}_{\mathrm{R}}X_{\mathrm{t}})^2 \times E({}_{\mathrm{R}}X_{\mathrm{t}})^2 \tag{69}$$

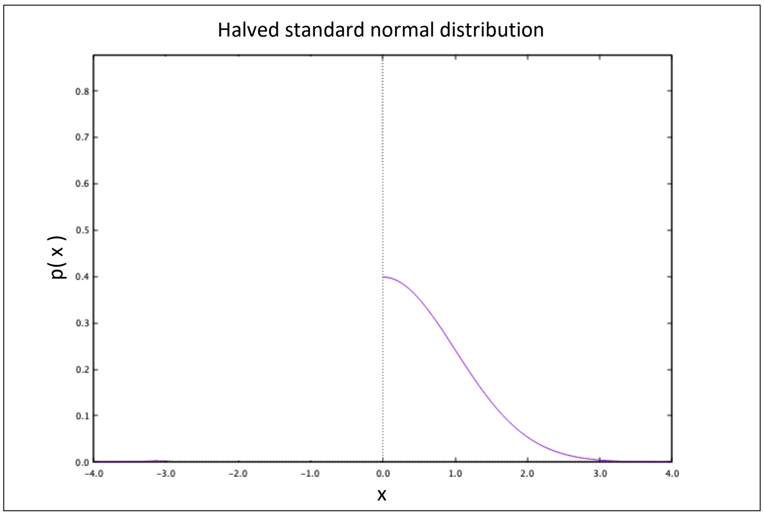
or

$$\sigma({}_{\mathrm{R}}X_{\mathrm{t}}) = z({}_{\mathrm{R}}X_{\mathrm{t}}) \times E({}_{\mathrm{R}}X_{\mathrm{t}}) \tag{70}$$

Per definition, it is

$$E({}_{\mathrm{R}}X_{\mathrm{t}}) = \frac{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})}{z({}_{\mathrm{R}}X_{\mathrm{t}})} \tag{71}$$

[16] Cramer JG, Forward RL, Morris MS, Visser M, Benford G, Landis GA. Natural wormholes as gravitational lenses. Phys Rev D Part Fields. 1995 Mar 15;51(6):3117-3120. doi: 10.1103/physrevd.51.3117. PMID: 10018782.



Halved normal distribution with
E(x) = 0
and
$\sigma(x)^2 = 1$

Figure 4. Halved normal distribution

The probability density of a halved normal distribution for positive x is given as

$$p({}_{\mathrm{R}}X_{\mathrm{t}}) = \left(\frac{2}{\sqrt{2\pi \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}}\right) e^{-\dfrac{({}_{\mathrm{R}}X_{\mathrm{t}} - E({}_{\mathrm{R}}X_{\mathrm{t}}))^2}{2 \times \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^2}} \tag{72}$$

and illustrated by figure 4.

Even if a condition and a cause are deeply related, there are circumstances where a sharp distinction between a cause and a condition is necessary. However, exactly this has been denied by John Stuart Mill's (1806-1873) regularity view of causality (see Mill, 1843b). What might seem to be a theoretical difficulty for many authors is none for Mill. Mill simply reduced a cause to a condition and claimed that "... the real cause of the phenomenon is the assemblage of all its conditions." (see Mill, 1843a, p. 403)

2.3.2. Exclusion relationship

Definition 2.6 (**Exclusion relationship [*EXCL*]**).

Mathematically, the exclusion(see also Barukčić, 2021a) relationship [17] (EXCL), denoted by p(A_t |

[17] Barukčić, Ilija. (2021). Mutually exclusive events. Causation, 16(11), 5–57. https://doi.org/10.5281/zenodo.5746415

B_t) in terms of statistics and probability theory, is defined(see also Barukčić, 1989, p. 68-70) as

$$
\begin{aligned}
p(A_t \mid B_t) &\equiv p(A_t \uparrow B_t) \\
&\equiv p(b_t) + p(c_t) + p(d_t) \\
&\equiv \frac{N \times (p(b_t) + p(c_t) + p(d_t))}{N} \\
&\equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \vee \underline{B}_t)}{N} \equiv \frac{b+c+d}{N} \\
&\equiv \frac{b + \underline{A}}{N} \\
&\equiv \frac{c + \underline{B}}{N} \\
&\equiv +1
\end{aligned}
\tag{73}
$$

Based on the 1913 Henry Maurice Sheffer (1882-1964) relationship, the Sheffer stroke(Nicod, 1917, Sheffer, 1913) usually denoted by ↑, it is $p(A_t \wedge B_t) \equiv 1 - p(A_t \mid B_t)$ (see table 4).

Table 4. A_t excludes B_t and vice versa.

		Conditioned (COVID-19) B_t		
		TRUE	FALSE	
Condition (Vaccine)	TRUE	**+0**	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Example 2.1. *Pfizer Inc. and BioNTech SE announced on Monday, November 09, 2020 - 06:45am results from a Phase 3 COVID-19 vaccine trial with 43.538 participants which provides evidence that their vaccine (BNT162b2) is preventing COVID-19 in participants without evidence of prior SARS-CoV-2 infection. In toto, 170 confirmed cases of COVID-19 were evaluated, with 8 in the vaccine group versus 162 in the placebo group. The exclusion relationship can be calculated as follows.*

$$
\begin{aligned}
p(Vaccine : BNT162b2 \mid COVID - 19(infection)) &\equiv p(b_t) + p(c_t) + p(d_t) \\
&\equiv 1 - p(a_t) \\
&\equiv 1 - \left(\frac{8}{43538}\right) \\
&\equiv +0,99981625
\end{aligned}
\tag{74}
$$

with a P Value = 0,000184.

Following Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables A_t, B_t et cetera at the point t, we obtain

$$\begin{aligned} p(A_t \mid B_t) &\equiv p(\underline{A}_t \cup \underline{B}_t) \\ &\equiv 1 - p(A_t \cap B_t) \\ &\equiv 1 - \int_{-\infty}^{A_t} \int_{-\infty}^{B_t} f(A_t, B_t)\ dA_t\, dB_t \\ &\equiv +1 \end{aligned} \tag{75}$$

while $p(A_t \mid B_t)$ would denote the cumulative distribution function of random variables and $f(A_t, B_t)$ is the joint density function.

2.3.3. Observational study and exclusion relationship

Under conditions of an observational study, the exclusion relationship follows approximately(see Barukčić, 2021a) as

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \geq 1 - \frac{p(a_t)}{p(B_t)} \tag{76}$$

2.3.4. Experimental study and exclusion relationship

Under conditions of an experimental study, the exclusion relationship follows approximately(see Barukčić, 2021a) as

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \geq 1 - \frac{p(a_t)}{p(A_t)} \tag{77}$$

2.3.5. The goodness of fit test of an exclusion relationship

Definition 2.7 (The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship).

Under some well known circumstances, testing hypothesis about an exclusion relationship $p(A_t \mid B_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$
\begin{aligned}
\tilde{\chi}^{2}{}_{\text{Calculated}}\left(\left(A_{\mathrm{t}} \mid B_{\mathrm{t}}\right) \mid A\right) &\equiv \frac{(b-(a+b))^{2}}{A}+ \\
&\frac{((c+d)-\underline{A})^{2}}{\underline{A}} \\
&\equiv \frac{a^{2}}{A}+0 \\
&\equiv \frac{a^{2}}{A}
\end{aligned}
\tag{78}
$$

or equally as

$$
\begin{aligned}
\tilde{\chi}^{2}{}_{\text{Calculated}}\left(\left(A_{\mathrm{t}} \mid B_{\mathrm{t}}\right) \mid B\right) &\equiv \frac{(c-(a+c))^{2}}{B}+ \\
&\frac{((b+d)-\underline{B})^{2}}{\underline{B}} \\
&\equiv \frac{a^{2}}{B}+0 \\
&\equiv \frac{a^{2}}{B}
\end{aligned}
\tag{79}
$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of an exclusion relationship/distribution $p(A_t \mid B_t)$, in which case the null hypothesis has to be accepted. Yate's (Yates, 1934) continuity correction was not used under these circumstances.

2.3.6. The left-tailed p Value of an exclusion relationship

Definition 2.8 (The left-tailed p Value of an exclusion relationship).

It is known that as a sample size, N, increases, a sampling distribution of a special test statistic approaches the normal distribution (central limit theorem). Under these circumstances, the left-tailed (lt) p Value (Barukčić, 2019b) of an exclusion relationship can be calculated as follows.

$$
\begin{aligned}
pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \mid B_{\mathrm{t}}\right) &\equiv 1-e^{-\left(1-p\left(A_{\mathrm{t}} \mid B_{\mathrm{t}}\right)\right)} \\
&\equiv 1-e^{-(a/N)}
\end{aligned}
\tag{80}
$$

A low p-value may provide some evidence of statistical significance.

2.3.7. Neither nor conditions

Definition 2.9 (Neither A_t nor B_t conditions [*NOR*]).

Mathematically, a neither A_t nor B_t condition (or rejection according to the French philosopher and logician Jean George Pierre Nicod (1893-1924), i.e. Jean Nicod's statement (Nicod, 1924)) relationship (NOR), denoted by $p(A_t \downarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p(A_t \downarrow B_t) &\equiv p(d_t) \\ &\equiv \frac{N - \sum_{t=1}^{N} (A_t \vee B_t)}{N} \equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \wedge \underline{B}_t)}{N} \equiv \frac{N \times (p(d_t))}{N} \\ &\equiv \frac{d}{N} \\ &\equiv +1 \end{aligned} \tag{81}$$

2.3.8. The Chi square goodness of fit test of a neither nor condition relationship

Definition 2.10 (The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship).

A neither A_t nor B_t condition relationship $p(A_t \downarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution). The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship with degree of freedom (d. f.) of d. f. = 1 may be calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}((A_t \downarrow B_t) \mid A) &\equiv \frac{(d-(c+d))^2}{\underline{A}} + \\ &\quad \frac{((a+b)-A)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}} + 0 \end{aligned} \tag{82}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}((A_t \downarrow B_t) \mid B) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\ &\quad \frac{((a+c)-B)^2}{B} \\ &\equiv \frac{b^2}{\underline{B}} + 0 \end{aligned} \tag{83}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.9. The left-tailed p Value of a neither nor B condition relationship

Definition 2.11 (The left-tailed p Value of a neither A_t nor B_t condition relationship).

The left-tailed (lt) p Value (Barukčić, 2019b) of a neither $\mathrm{A_t}$ nor $\mathrm{B_t}$ condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) &\equiv 1-e^{-\left(1-p\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right)\right)} \\ &\equiv 1-e^{-p\left(A_{\mathrm{t}} \vee B_{\mathrm{t}}\right)} \\ &\equiv 1-e^{-((a+b+c)/N)} \end{aligned} \tag{84}$$

where $\vee$ may denote disjunction or logical inclusive or. In this context, a low p-value indicates again a statistical significance. In general, it is $p\left(A_{\mathrm{t}} \vee B_{\mathrm{t}}\right) \equiv 1-p\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right)$ (see table 5).

Table 5. Neither $\mathrm{A_t}$ nor $\mathrm{B_t}$ relationship.

		Conditioned $\mathrm{B_t}$		
		YES	NO	
Condition $\mathrm{A_t}$	YES	0	0	0
	NO	0	1	1
		0	1	1

2.3.10. Necessary condition

Definition 2.12 (**Necessary condition [*Conditio sine qua non*]**).

Despite the most extended efforts, the current state of research on conditions and conditioned is still incomplete and very contradictory. However, even thousands of years ago and independently of any human mind and consciousness, water has been and is still a necessary (see Barukčić, 2022b) condition for (human) life. **Without** water, there has been and there is **no** (human) life[18]. It comes therefore as no surprise that one of the first documented attempts to present a rigorous theory of conditions and causation (see also Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica III 2 997a 10 and 13/14) came from the Greek philosopher and scientist Aristotle (384-322 BCE). Thus far, it is amazing that Aristotle himself made already a strict distinction between conditions and causes. Taking Aristotle very seriously, it is necessary to consider that

> "... everything which has a ... ... potency in question ... ... has the potency ... of acting ... not in all circumstances but on certain conditions ... " (see also Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica IX 5 1048a 14-19)

Before going into details, Aristotle went on to define the necessary condition as follows.

> "... necessary ... means ...
>
> without ... a condition, a thing cannot live ... "
>
> (see also Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica V 2 1015a 20-22)

In point of fact, Aristotle developed a theory of conditions and causality commonly referred to as the doctrine of four causes. Many aspects and general features of Aristotle's logical concept of causality are meanwhile extensively and critically debated in secondary literature. However, even if the Greek philosophers Heraclitus, Plato, Aristotle et cetera numbers among the greatest philosophers of all time, the philosophy has evolved. Scientific knowledge and objective reality are deeply interrelated and cannot be reduced only to Greek philosophers like Aristotle. Among many other issues, the specification of necessary conditions has traditionally been part of the philosopher's investigations of different phenomena. However, behind the need of a detailed evidence, it is justified to consider that philosophy or philosophers as such certainly do not possess **a monopoly on the truth** and other areas such as medicine as well as other sciences and technology may transmit truths as well and may be of help to move beyond one's self enclosed unit. Seemingly, **the law's concept of causation** justifies to say few words on this subject, to put some light on some questions. Are there any criteria in law for deciding whether one action or an event A_t has caused another (generally harmful) event B_t? What are these criteria? May causation in legal contexts differ from causation outside the law, for example, in science

[18] Barukčić, Ilija. (2022). Conditio sine qua non (Version 1). Zenodo. https://doi.org/10.5281/zenodo.5854744

or in our everyday life and to what extent? Under which circumstances is it justified to tolerate such differences as may be found to exist? To understand just what is the law's concept of causation, it is useful to re-consider how the highest court of states is dealing with causation. In the case *Hayes v. Michigan Central R. Co., 111 U.S. 228*, the U.S. Supreme Court defined 1884 conditio sine qua non as follows: "... **causa sine qua non – a cause which, if it had not existed, the injury would not have taken place**". (Justice Matthews, Mr., 1884) The German Bundesgerichtshof für Strafsachen stressed once again the importance of conditio sine qua non relationship in his decision by defining the following: "**Ursache eines strafrechtlich bedeutsamen Erfolges jede Bedingung, die nicht hinweggedacht werden kann, ohne daß der Erfolg entfiele**"(Bundesgerichtshof für Strafsachen, 1951) Another lawyer elaborated on the basic issue of **identity and difference between cause and condition**. Von Bar was writing: "Die erste Voraussetzung, welche erforderlich ist, damit eine Erscheinung als die Ursache einer anderen bezeichnet werden könne, ist, daß jene eine der Bedingungen dieser sein. Würde die zweite Erscheinung auch dann eingetreten sein, wenn die erste nicht vorhanden war, so ist sie in keinem Falle Bedingung und noch weniger Ursache. Wo immer ein Kausalzusammenhang behauptet wird, da muß er wenigstens diese Probe aushalten . . . **Jede Ursache ist nothwendig auch eine Bedingung eines Ereignisses; aber nicht jede Bedingung ist Ursache zu nennen**."(Bar, Carl Ludwig von, 1871) Von Bar's position translated into English: *The first requirement, which is required, thus that something could be called as the cause of another, is that the one has to be one of the conditions of the other. If the second something had occurred even if the first one did not exist, so it is by no means a condition and still less a cause. Wherever a causal relationship is claimed, the same must at least withstand this test. . . Every cause is necessarily also a condition of an event too; but not every condition is cause too.* Thus far, let us consider among other the following in order to specify necessary conditions from another, probabilistic point of view. An event (i.e. A_t) which is a necessary condition of another event or outcome (i.e. B_t) must be given, must be present for a conditioned, for an event or for an outcome B_t to occur. A necessary condition (i.e. A_t) is a requirement which need to be fulfilled **at every single Bernoulli trial t**, in order for a conditioned or an outcome (i.e. B_t) to occur, but it alone does not determine the occurrence of such an event. In other words, if a necessary condition (i.e. A_t) is given, an outcome (i.e. B_t) need not to occur. In contrast to a necessary condition, a 'sufficient'condition is the one condition which 'guarantees'that an outcome will take place or will occur for sure. Under which conditions we may infer about the unobserved and whether observations made are able at all to justify predictions about potential observations which have not yet been made or even general claims which my go even beyond the observed (*the 'problem of induction')* is not the issue of the discussion at this point. Besides of the principal necessity of meeting such a challenge, a necessary condition of an event can but need not be at the same Bernoulli trial t a sufficient condition for an event to occur. However, theoretically, it is possible that an event or an outcome is determined by many necessary conditions. Let us focus to some extent on what this means, or in other words how much importance can we attribute to such a special case. *Example*. A human being cannot live without oxygen. A human being cannot live without water. A human being cannot live without a brain. A human being cannot live without kidneys. A human being cannot live without ... et cetera. Thus far, even if oxygen is given, if a brain is given ... et cetera, without water a human being will not survive on the long run. This example is of use to reach the following conclusion. Although it might seem somewhat paradoxical at first sight, **even under circumstances where a condition or an outcome depends on several different necessary conditions it is particularly important that every single of**

these necessary conditions for itself must be given otherwise the conditioned (i.e. the outcome) will not occur. Mathematically, the necessary condition (SINE) relationship, denoted by $p(A_t \leftarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 15-28) as

$$
\begin{split}
p(A_t \leftarrow B_t) \equiv p(A_t \vee \underline{B}_t) &\equiv \frac{\sum_{t=1}^{N} (A_t \vee \underline{B}_t)}{N} \equiv \frac{(A_t \vee \underline{B}_t) \times p(A_t \vee \underline{B}_t)}{(A_t \vee \underline{B}_t)} \\
&\equiv p(a_t) + p(b_t) + p(d_t) \\
&\equiv \frac{N \times (p(a_t) + p(b_t) + p(d_t))}{N} \equiv \frac{E(A_t \leftarrow B_t)}{N} \\
&\equiv \frac{a+b+d}{N} \equiv \frac{E(A_t \vee \underline{B}_t)}{N} \\
&\equiv \frac{A+d}{N} \equiv \frac{E(A_t \leftarrow B_t)}{N} \\
&\equiv \frac{a+\underline{B}}{N} \equiv \frac{E(A_t \vee \underline{B}_t)}{N} \\
&\equiv +1
\end{split}
\tag{85}
$$

where $E(A_t \leftarrow B_t) \equiv E(A_t \vee \underline{B}_t)$ indicates the expectation value of the necessary condition. In general, it is $p(A_t \prec B_t) \equiv 1 - p(A_t \leftarrow B_t)$ (see Table 6).

Table 6. Necessary condition.

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
A_t	FALSE	**+0**	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

A necessary condition A_t is characterised itself by the property that another event B_t will not occur if A_t is not given, if A_t did not occur (Barukčić, 1989, 1997, 2005, 2016, 2017a,b, 2020a,b,c,d, Barukčić and Ufuoma, 2020). Taking into account Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables A_t, B_t et cetera at the (period of) time t, we obtain

$$
\begin{split}
p(A_t \leftarrow B_t) &\equiv +1 \\
&\equiv +1 - p(c_t) \\
&\equiv +1 - p(\underline{A}_t \cap B_t) \\
&\equiv \left(\int_{-\infty}^{A_t} \int_{-\infty}^{B_t} f(A_t, B_t)\, dA_t\, dB_t \right) + \left(1 - \int_{-\infty}^{B_t} f(B_t)\, dB_t \right)
\end{split}
\tag{86}
$$

while $p(A_t \leftarrow B_t)$ would denote the cumulative distribution function of random variables of a necessary condition. Another adequate formulation of a necessary condition is possible too. If certain conditions are met, then necessary conditions and sufficient conditions are one way or another converses of each other, too. It is

$$p\left(A_t \leftarrow B_t\right) \equiv \underbrace{\left(A_t \vee \underline{B_t}\right)}_{\text{(Necessary condition)}} \equiv \underbrace{\left(\underline{B_t} \vee A_t\right)}_{\text{(Sufficient condition)}} \equiv p\left(B_t \rightarrow A_t\right) \tag{87}$$

These relationships are illustrated by the following tables.

Table 7. Without A_t no B_t

		B_t		
		TRUE	FALSE	
A_t	TRUE	a_t	b_t	A_t
	FALSE	$c_t = 0$	d_t	$\underline{A_t}$
		B_t	$\underline{B_t}$	$+1$

Table 8. If B_t then A_t

		A_t		
		TRUE	FALSE	
B_t	TRUE	a_t	$c_t = 0$	B_t
	FALSE	b_t	d_t	$\underline{B_t}$
		A_t	$\underline{A_t}$	$+1$

There are circumstances under which

$$p\left(A_t \leftarrow B_t\right) \equiv \underbrace{\left(A_t \vee \underline{B_t}\right)}_{\text{(Nessessary condition)}} \equiv \underbrace{\left(\underline{A_t} \vee B_t\right)}_{\text{(Sufficient condition)}} \equiv p\left(A_t \rightarrow B_t\right) \tag{88}$$

However, equation 87 does not imply the relationship of equation 88 under any circumstances.

Example I.

A wax candle is characterised by various properties, but is also subject to certain conditions. **Without** sufficient amounts of gaseous oxygen **no** burning wax candle, gaseous oxygen is a necessary condition of a burning candle. However, the converse relationship **if** burning wax candle, **then** sufficient amounts of gaseous oxygen are given is is at the same (period of) time t / Bernoulli trial t true. The following tables are illustrating these relationships.

Table 9. Without gaseous oxygen no burning candle

		Burning candle		
		TRUE	FALSE	
Gaseous	TRUE	a_t	b_t	A_t
oxygen	FALSE	$c_t = 0$	d_t	$\underline{A_t}$
		B_t	$\underline{B_t}$	$+1$

Table 10. If burning candle then gaseous oxygen

		Gaseous oxygen		
		TRUE	FALSE	
Burning	TRUE	a_t	$c_t = 0$	B_t
candle	FALSE	b_t	d_t	$\underline{B_t}$
		A_t	$\underline{A_t}$	$+1$

Example II.

Once again, a human being cannot live without water. A human being cannot live without gaseous oxygen, et cetera. Water itself is a necessary condition for human life. However, gaseous oxygen is a necessary condition for human life too. Thus far, even if water is given and even if water is a necessary condition for human life, without gaseous oxygen there will be no human life. In general, if a conditioned or an outcome B_t depends on the necessary condition A_t and equally on numerous other necessary conditions, an event B_t will not occur if A_t itself is not given independently of the occurrence of other necessary conditions.

Example III.

Another different aspect of a necessary condition relationship is appropriate to be focused upon here. As a direct consequence of a necessary condition **without** sufficient amounts of gaseous oxygen **no** burning wax candle is a special case of an exclusion relationship. The absence of sufficient amounts of gaseous oxygen A_t excludes (see Barukčić, 2021a) a burning wax candle B_t. Thus far, if we want to stop the burning of a wax candle, we would have to significantly reduce the amounts of gaseous oxygen A_t. Under these conditions, a wax candle will stop burning. The following tables (table 11 and table 12) may illustrate this aspect of a necessary condition in more detail.

Table 11. Without gaseous oxygen no burning candle

		Burning candle		
		TRUE	FALSE	
Gaseous	TRUE	a_t	b_t	A_t
oxygen	FALSE	$c_t = 0$	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	+1

Table 12. Absent gaseous oxygen excludes burning wax candle

		Burning candle		
		TRUE	FALSE	
Gaseous	FALSE	$c_t = 0$	d_t	B_t
oxygen	TRUE	a_t	b_t	$\underline{B}_t$
		A_t	$\underline{A}_t$	+1

The necessary condition relationship follows approximately (see Barukčić, 2022b) as

$$p\left(A_t \leftarrow B_t\right) \geq 1 - \frac{p\left(c_t\right)}{p\left(B_t\right)} \tag{89}$$

and as

$$p\left(A_t \leftarrow B_t\right) \geq 1 - \frac{p\left(c_t\right)}{p\left(\underline{A}_t\right)} \tag{90}$$

2.3.11. The Chi-square goodness of fit test of a necessary condition relationship

Definition 2.13 (The $\tilde{\chi}^2$ goodness of fit test of a necessary condition relationship).

Under some well known circumstances, hypothesis about the conditio sine qua non relationship $p(A_t \leftarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or χ^2-distribution), first described by the German statistician Friedrich Robert Helmert (Helmert, 1876) and later rediscovered by Karl Pearson (Pearson, 1900) in the context of a goodness of fit test. The $\tilde{\chi}^2$ goodness of fit test of a conditio sine qua non relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\mathrm{t}} \leftarrow B_{\mathrm{t}} \mid B\right) \equiv & \frac{(a-(a+c))^2}{B}+ \\
& \frac{((b+d)-\underline{B})^2}{\underline{B}} \\
\equiv & \frac{c^2}{B}+0 \\
\equiv & \frac{c^2}{B}
\end{aligned} \tag{91}
$$

or equally as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\mathrm{t}} \leftarrow B_{\mathrm{t}} \mid \underline{A}\right) \equiv & \frac{(d-(c+d))^2}{\underline{A}}+ \\
& \frac{((a+b)-A)^2}{A} \\
\equiv & \frac{c^2}{\underline{A}}+0 \\
\equiv & \frac{c^2}{\underline{A}}
\end{aligned} \tag{92}
$$

and can be compared with a theoretical chi-square value at a certain level of significance α. It has not yet been finally clarified whether the use of Yate's (Yates, 1934) continuity correction is necessary at all.

2.3.12. The left-tailed p Value of the conditio sine qua non relationship

Definition 2.14 (The left-tailed p Value of the conditio sine qua non relationship).

The left-tailed (lt) p Value (Barukčić, 2019b) of the conditio sine qua non relationship can be calculated as follows.

$$
\begin{aligned}
pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \leftarrow B_{\mathrm{t}}\right) & \equiv 1-e^{-\left(1-p\left(A_{\mathrm{t}} \leftarrow B_{\mathrm{t}}\right)\right)} \\
& \equiv 1-e^{-(c/N)}
\end{aligned} \tag{93}
$$

2.3.13. Sufficient condition

Definition 2.15 (**Sufficient condition [*Conditio per quam*]**).

Mathematically, the sufficient (Barukčić, 2021b, p. 68-70) condition (see Barukčić, 2022a) (IMP) relationship, denoted by $\mathrm{p}(\mathrm{A_t} \rightarrow \mathrm{B_t})$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 68-70) as

$$
\begin{aligned}
p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \equiv p(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}) &\equiv \frac{\sum\limits_{t=1}^{N}(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})}{N} \equiv \frac{(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}) \times p(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})}{(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})} \\
&\equiv p(a_{\mathrm{t}}) + p(c_{\mathrm{t}}) + p(d_{\mathrm{t}}) \\
&\frac{N \times (p(a_{\mathrm{t}}) + p(c_{\mathrm{t}}) + p(d_{\mathrm{t}}))}{N} \\
&\equiv \frac{a+c+d}{N} \equiv \frac{E(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})}{N} \\
&\equiv \frac{B+d}{N} \equiv \frac{E(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}})}{N} \\
&\equiv \frac{a+\underline{A}}{N} \\
&\equiv +1
\end{aligned}
\tag{94}
$$

In general, it is $p(A_{\mathrm{t}} \succ\!\!- B_{\mathrm{t}}) \equiv 1 - p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}})$ (see Table 13).

2.3.13.1. Mackie's INUS Condition John Leslie Mackie (1917-1981) critically examined the theories of causation of various (see Ducasse, 1926) philosophers such as Hume (Book I, Part III, of the Treatise) (see Mackie, 1974, pp. 3-28), Kant (as well as Kantian approaches offered by Strawson and Bennett), Mill and other. Mackie rightly claims that Hume's regularity theory of causation offer only an incomplete picture of the nature of causation. Mackie writes: "It seems appropriate to begin by examining and criticizing it, so that we can take over from it whatever seems to be defensible but develop an improved account by correcting its errors and deficiencies." (see Mackie, 1974, p. 3). Nonetheless, in his trial to develop an improved account of Hume's theory of causation, Mackie's own account of the nature of causation follows Hume's principles of causation very closely (see Mackie, 1974, pp. 3-28). Mackie himself proposed already in 1965 that "the so-called cause is ... an *insufficient* but *necessary* part of a condition which is itself *unnecessary* but *sufficient* for the result ... let us call such a condition ... an INUS condition." (see Mackie, 1965, p. 245). However Mackie's account needs modification, and can be modified and when it is modified we can explain much more satisfactorily what Mackie ordinarily take to be a cause. Mackie is of the opinion that "... cause is ... part of a condition ... " (see Mackie, 1965, p. 245) and that "... a condition ... is ... *unnecessary* but *sufficient* for the result [i. e. effect, author]. " (see Mackie, 1965, p. 245). To put it very simply one could say that Mackie reduces a cause to a sufficient condition, "... cause is ... a condition which is itself ... *sufficient* ... " (see Mackie, 1965, p. 245). Indeed, there are circumstances, where several different events [19] might be necessary or sufficient et cetera at the same time in order to determine **a compound/complex sufficient condition relationship**. Thus far, it seems appropriate to take over from Mackie's INUS condition whatever seems to be acceptable but to develop an improved account by correcting its deficiencies and errors in order to do justice to the complexity of affairs. Equation 95 illustrates one real-world example of a compound/complex sufficient condition relationship in more detail.

$$\begin{aligned} p\left(\left(\left({}_1X_{\mathrm{t}} \wedge {}_2X_{\mathrm{t}} \wedge {}_3X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}\right) \rightarrow B_{\mathrm{t}}\right) &\equiv p\left(\underline{\left(\left({}_1X_{\mathrm{t}} \wedge {}_2X_{\mathrm{t}} \wedge {}_3X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}\right)} \vee B_{\mathrm{t}}\right) \\ &\equiv \frac{\sum_{t=1}^{N}\left(\underline{\left(\left({}_1X_{\mathrm{t}} \wedge {}_2X_{\mathrm{t}} \wedge {}_3X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}\right)} \vee B_{\mathrm{t}}\right)}{N} \\ &\equiv +1 \end{aligned} \tag{95}$$

Again, taking into account Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables $\mathrm{A_t}$, $\mathrm{B_t}$ et cetera at the (period of) time t, we obtain

$$\begin{aligned} p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right) &\equiv +1 \\ &\equiv +1 - p\left(b_{\mathrm{t}}\right) \\ &\equiv +1 - p\left(A_{\mathrm{t}} \cap \underline{B}_{\mathrm{t}}\right) \\ &\equiv \left(\int_{-\infty}^{A_{\mathrm{t}}} \int_{-\infty}^{B_{\mathrm{t}}} f\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right)\, dA_{\mathrm{t}}\, dB_{\mathrm{t}}\right) + \left(1 - \int_{-\infty}^{A_{\mathrm{t}}} f\left(A_{\mathrm{t}}\right)\, dA_{\mathrm{t}}\right) \end{aligned} \tag{96}$$

while $p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right)$ would denote the cumulative distribution function of random variables of a sufficient condition. Another adequate formulation of a sufficient condition is possible too.

Table 13. Sufficient condition.

		Conditioned $\mathrm{B_t}$		
		TRUE	FALSE	
Condition	TRUE	$\mathrm{p(a_t)}$	**+0**	$\mathrm{p(A_t)}$
$\mathrm{A_t}$	FALSE	$\mathrm{p(c_t)}$	$\mathrm{p(d_t)}$	$\mathrm{p(\underline{A}_t)}$
		$\mathrm{p(B_t)}$	$\mathrm{p(\underline{B}_t)}$	+1

Remark 2.1. *A sufficient condition A_t is characterized by the property that another event B_t will occur if A_t is given, if A_t itself occured (Barukčić, 1989, 1997, 2005, 2016, 2017a,b, 2020a,b,c,d, Barukčić and Ufuoma, 2020).* ***Example****. The ground, the streets, the trees, human beings and many other objects too will become wet during heavy rain. Especially,* ***if*** *it is raining (event A_t),* ***then*** *human beings will become wet (event B_t). However, even if this is a common human wisdom, a human being equipped with an appropriate umbrella (denoted by R_t) need not become wet even during heavy rain. An appropriate umbrella (R_t) is similar to an event with the potential to counteract the occurrence of another event*

[19] Barukčić, Ilija. (2022). Conditio per quam. Causation, 17(3), 5–86. https://doi.org/10.5281/zenodo.6369831

*(B_t) and can be understood something as an **anti-dot** of another event. In other words, an appropriate umbrella is an antidote of the effect of rain on human body, an appropriate umbrella has the potential to protect humans from the effect of rain on their body. It is a good rule of thumb that the following relationship*

$$p(A_t \rightarrow B_t) + p(R_t \wedge B_t) \equiv +1 \tag{97}$$

indicates that R_t is an antidote of A_t. However, taking a shower, swimming in a lake et cetera may make human hair wet too. More than anything else, however, these events does not affect the final outcome, the effect of raining on human body.

The approximate (see Barukčić, 2022a) value of the material implication is given as

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(A_t)} \tag{98}$$

and alternatively as

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(\underline{B}_t)} \tag{99}$$

2.3.14. The Chi square goodness of fit test of a sufficient condition relationship

Definition 2.16 (The $\tilde{\chi}^2$ goodness of fit test of a sufficient condition relationship).

Under some well known circumstances, testing hypothesis about the conditio per quam relationship p($A_t \rightarrow B_t$) is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a conditio per quam relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(A_t \rightarrow B_t \mid A) &\equiv \frac{(a-(a+b))^2}{A} + \\ &\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + 0 \\ &\equiv \frac{b^2}{A} \end{aligned} \tag{100}$$

or equally as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \rightarrow B_{\text{t}} \mid \underline{B}\right) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\
&\quad \frac{((a+c)-B)^2}{B} \\
&\equiv \frac{b^2}{\underline{B}} + 0 \\
&\equiv \frac{b^2}{\underline{B}}
\end{aligned}
\tag{101}
$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of the conditio per quam relationship/distribution $p(A_t \rightarrow B_t)$, in which case the null hypothesis is accepted. Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.15. The left-tailed p Value of the conditio per quam relationship

Definition 2.17 (The left-tailed p Value of the conditio per quam relationship).

The left-tailed (lt) p Value (Barukčić, 2019b) of the conditio per quam relationship can be calculated as follows.

$$
\begin{aligned}
pValue_{\text{lt}}\left(A_{\text{t}} \rightarrow B_{\text{t}}\right) &\equiv 1 - e^{-(1-p(A_{\text{t}} \rightarrow B_{\text{t}}))} \\
&\equiv 1 - e^{-(b/N)}
\end{aligned}
\tag{102}
$$

Again, a low p-value indicates a statistical significance.

2.3.16. Necessary and sufficient conditions

Definition 2.18 (Necessary and sufficient conditions [*EQV*]).

The necessary and sufficient condition (EQV) relationship, denoted by $p(A_t \leftrightarrow B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$
\begin{aligned}
p\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}\right) &\equiv \frac{\sum_{t=1}^{N}\left(\left(A_{\mathrm{t}} \vee \underline{B}_{\mathrm{t}}\right) \wedge\left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right)\right)}{N} \\
&\equiv p\left(a_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right) \\
&\equiv \frac{N \times\left(p\left(a_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right)\right)}{N} \\
&\equiv \frac{a+d}{N} \\
&\equiv +1
\end{aligned}
\tag{103}
$$

2.3.17. The Chi square goodness of fit test of a necessary and sufficient condition relationship

Definition 2.19 (The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship).

Even the necessary and sufficient condition relationship $p(A_t \leftrightarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}} \mid A\right) \equiv & \frac{(a-(a+b))^2}{A}+ \\
& \frac{d-((c+d))^2}{\underline{A}} \\
\equiv & \frac{b^2}{A}+\frac{c^2}{\underline{A}}
\end{aligned}
\tag{104}
$$

or equally as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}} \mid B\right) \equiv & \frac{(a-(a+c))^2}{B}+ \\
& \frac{d-((b+d))^2}{\underline{B}} \\
\equiv & \frac{c^2}{B}+\frac{b^2}{\underline{B}}
\end{aligned}
\tag{105}
$$

The calculated $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship can be compared with a theoretical chi-square value at a certain level of significance α. Under conditions where the observed values are equal to the expected/theoretical values of a necessary and sufficient condition relationship/distribution $p(A_t \leftrightarrow B_t)$, the $\tilde{\chi}^2$-distribution equals zero. It is to be cleared whether Yate's (Yates, 1934) continuity correction should be used at all.

2.3.18. The left-tailed p Value of a necessary and sufficient condition relationship

Definition 2.20 (The left-tailed p Value of a necessary and sufficient condition relationship).

The left-tailed (lt) p Value (Barukčić, 2019b) of a necessary and sufficient condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}) &\equiv 1 - e^{-(1-p(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}))} \\ &\equiv 1 - e^{-((b+c)/N)} \end{aligned} \tag{106}$$

In this context, a low p-value indicates again a statistical significance. Table 14 may provide an overview of the theoretical distribution of a necessary and sufficient condition.

Table 14. Necessary and sufficient condition.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	1	0	1
	NO	0	1	1
		1	1	2

2.3.19. Either or conditions

Definition 2.21 (Either A_t or B_t conditions [*NEQV*]).

Mathematically, an either A_t or B_t condition relationship (NEQV), denoted by $p(A_t \succ\!\prec B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p(A_{\mathrm{t}} \succ\!\prec B_{\mathrm{t}}) &\equiv \frac{\sum_{t=1}^{N}((A_{\mathrm{t}} \wedge \underline{B}_{\mathrm{t}}) \vee (\underline{A}_{\mathrm{t}} \wedge B_{\mathrm{t}}))}{N} \\ &\equiv p(b_{\mathrm{t}}) + p(c_{\mathrm{t}}) \\ &\equiv \frac{N \times (p(b_{\mathrm{t}}) + p(c_{\mathrm{t}}))}{N} \\ &\equiv \frac{b+c}{N} \\ &\equiv +1 \end{aligned} \tag{107}$$

It is $p(A_t \succ\!\prec B_t) \equiv 1 - p(A_t <\!\!\!-\!\!\!> B_t)$ (see Table 15).

Table 15. Either A_t or B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	1	1
	NO	1	0	1
		1	1	2

2.3.20. The Chi-square goodness of fit test of an either or condition relationship

Definition 2.22 (The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship).

An either or condition relationship p(A_t $\succ\!\!\!\prec$ B_t) can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}((A_t \succ\!\!\!\prec B_t) \mid A) &\equiv \frac{(b-(a+b))^2}{A} + \\ &\quad \frac{c-((c+d))^2}{\underline{A}} \\ &\equiv \frac{a^2}{A} + \frac{d^2}{\underline{A}}\end{aligned} \tag{108}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}((A_t \succ\!\!\!\prec B_t) \mid B) &\equiv \frac{(c-(a+c))^2}{B} + \\ &\quad \frac{b-((b+d))^2}{\underline{B}} \\ &\equiv \frac{a^2}{B} + \frac{d^2}{\underline{B}}\end{aligned} \tag{109}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.21. The left-tailed p Value of an either or condition relationship

Definition 2.23 (The left-tailed p Value of an either or condition relationship).

The left-tailed (lt) p Value (Barukčić, 2019b) of an either or condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}(A_{\mathrm{t}} \succ\!\!\prec B_{\mathrm{t}}) &\equiv 1 - e^{-(1-p(A_{\mathrm{t}} >-< B_{\mathrm{t}}))} \\ &\equiv 1 - e^{-((a+d)/N)} \end{aligned} \tag{110}$$

In this context, a low p-value indicates again a statistical significance.

2.4. Axioms

Whether science needs new and obviously generally valid statements (axioms) which are able to assure the truth of theorems proved from them may remain an unanswered question. In order to be accepted, a new axiom candidate (see Easwaran, 2008) should be at least as simple as possible and logically consistent to enable advances in our knowledge of nature. The importance of axioms is particularly emphasized by Albert Einstein. "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**." (see Einstein, 1919, p. 17). In general, lex identitatis, lex contradictionis and lex negationis have the potential to denote the most simple, the most general and the most far-reaching axioms of science, the foundation of our today's and of our future scientific inquiry.

2.4.1. Principium identitatis (Axiom I)

Principium identitatis or **lex identitatis** or axiom I, is closely related to central problems of metaphysics, epistemology and of science as such. It turns out that it is more than rightful to assume that

$$+1 \equiv +1 \tag{111}$$

is true, otherwise there is every good reason to suppose that nothing can be discovered at all.

Identity as the epitome of a self-identical or of self-reference is at the same time different from difference, identity is free from difference, identity is not difference, identity is at the same time the other of itself, identity is non-identity. Identity as simple equality with itself is determined by a non-being, by a non-being of its own other, by a non-being of difference, identity is different from difference. Identity is in its very own nature different and is in its own self the opposite of itself (symmetry). It is equally

$$-1 \equiv -1 \tag{112}$$

In general, +1 and -1 are distinguished, however these distinct are related to one and the same 1. Identity as a vanishing of otherness, therefore, is this distinguishedness in one relation. It is

$$0 \equiv +1 - 1 \equiv 0 \times 1 \equiv 0 \tag{113}$$

Identity, as the unity of something and its own other is in its own self a separation from difference, and as a moment of separation might pass over into an equivalence relation which itself is reflexive, symmetric and transitive. Nonetheless, backed by thousands of years of often bitter human experience, the scientific development has taught us all that human knowledge is relative too. Even if experiments and other suitable proofs are of help to encourage us more and more in our belief of the correctness of a theory, it is difficult to prove the correctness of a theorem or of a theory et cetera once and for all. The challenge for all the science is the need to comply with Einstein's position: **"Niemals aber kann die Wahrheit einer Theorie erwiesen werden. Denn niemals weiß man, daß auch in Zukunft eine Erfahrung bekannt werden wird, die Ihren Folgerungen widerspricht..."** (Einstein, 1919).

Albert Einstein's position translated into English: 'But the truth of a theory can never be proven. For one never knows if future experience will contradict its conclusion; and furthermore, there are always other conceptual systems imaginable which might coordinate the very same facts.'Our human experience tells us that everything in life is more or less transitory, and that nothing lasts. As a result of our knowledge and experience, several scientific theories have a glorious past to look back on, but all the glory of such scientific theories might remain in the past if scientist don't continue to innovate. In a word, theories can be refuted by time.

"No amount of experimentation can ever prove me right;
a single experiment can prove me wrong."

(Albert Einstein according to: Robertson, 1998, p. 114)

In the light of the foregoing, it is clear that appropriate axioms and conclusions derived from the same are a main logical foundation of any 'theory'.

"**Grundgesetz (Axiome)** und **Folgerungen** zusammen bilden das was man **eine 'Theorie'** nennt. "

(Einstein, 1919)

However, another point is worth being considered again. One single experiment can be enough to refute a whole theory. Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, an axiom as a free creation of the human mind which precedes all science should be like all other axioms, as simple as possible and as self-evident as possible. Historically, the earliest documented use of **the law of identity** can be found in Plato's dialogue Theaetetus (185a) as "... each of the two is different from the other and the same as itself "[20] . However, Aristotle (384–322 B.C.E.), Plato's pupil and equally one of the greatest philosophers of all time, elaborated on the law of identity too. In Metaphysica, Aristotle wrote:

"... all things ... have some unity and identity. "

(see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaphysica, Chapter IV, 999a, 25-30, p. 66)

[20] Plato's dialogue Theaetetus (185a), p. 104.

In Prior Analytics, [21], [22] Aristotle, a tutor of Alexander, the thirteen-year-old son of Philip, the king of Macedon, is writing: "When A applies to the whole of B and of C, and is other predicated of nothing else, and B also applies to all C, A and B must be convertible. For since A is stated only of B and C, and B is predicated both of itself and of C, it is evident that B will also be stated of all subjects of which A is stated, except A itself. "[23], [24] For the sake of completeness, it should be noted at the outset that Aristotle himself preferred **the law of contradiction** and **the law of excluded middle** as examples of fundamental axioms. Nonetheless, it is worth noting that **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally the most simple axiom of science but has been treated by Aristotle in an inadequate manner without having any clear and determined meaning for Aristotle himself. Nonetheless, something which is really just itself is equally different from everything else. In point of fact, is such an equivalence (Degen, 1741) which everything has to itself inherent or must the same be constructed by human mind and consciousness. Can and how can something be **identical with itself** (Förster and Melamed, 2012, Hegel, Georg Wilhelm Friedrich, 1812a, Koch, 1999, Newstadt, 2015) and in the same respect different from itself. An increasingly popular view on identity is the one advocated by Gottfried Wilhelm Leibniz (1646-1716):

"Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra
A est A,
B est B. "
(Leibniz, 1765, p. 327)

or **A = A**, **B = B** or **+1 = +1**. In other words, a thing is what it is (Leibniz, 1765, p. 327). Leibniz' **principium identitatis indiscernibilium** (p.i.i.), the principle of the indistinguishable, occupies a central position in Leibniz' logic and metaphysics and was formulated by Leibniz himself in different ways in different passages (1663, 1686, 1704, 1715/16). All in all, Leibniz writes:

"C'est
le principe des indiscernables,
en vertu duquel
il ne saurait exister dans la nature deux êtres identiques.
...
Il n'y a point deux individus indiscernables. "
(see Leibniz, Gottfried Wilhelm, 1886, p. 45)

Exactly in complete compliance with Leibniz, Johann Gottlieb Fichte (1762 - 1814) elaborates on this subject as follows:

[21] Aristotle, Prior Analytics, Book II, Part 22, 68a
[22] Kenneth T. Barnes. Aristotle on Identity and Its Problems. Phronesis. Vol. 22, No. 1 (1977), pp. 48-62 (15 pages)
[23] Aristotle, Prior Analytics, Book II, Part 22, 68a, p. 511.
[24] Ivo Thomas. On a passage of Aristotle. Notre Dame J. Formal Logic 15(2): 347-348 (April 1974). DOI: 10.1305/ndjfl/1093891315

> “**Each thing is what it is ;**
> **it has those realities which are posited when it is posited,**
> **(A = A.)** ”
> (Fichte, 1889)

Georg Wilhelm Friedrich Hegel (1770 – 1831) himself objected the Law of Identity by claiming that “A = A is ... an empty tautology. ”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 413) provided an example of his own mechanical understanding of the Law of Identity. “the empty tautology: nothing is nothing; ... from nothing only nothing becomes ... nothing remains nothing. ”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 84). Nonetheless, Hegel preferred to reformulate an own version of Leibniz principium identitatis indiscernibilium in his own way by writing that “All things are different, or: there are no two things like each other. ”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422). Much of the debate about identity is still a matter of controversy. This issue has attracted the attention of many authors and has been discussed by Hegel too. In this context, it is worth to consider Hegel’s radical position on identity.

> “The other expression of the law of identity: A cannot at the same time be A and not-A, has a negative form; it is called
> the law of contradiction. ”
> (Hegel, Georg Wilhelm Friedrich, 1991, p. 416)

We may, usefully (see Barukčić, 2019a), state Russell’s position with respect to the identity law as mentioned in his book ‘The problems of philosophy ’ (see Russell, 1912). In particular, according to Russell,

> “...principles have been singled out by tradition under the name of ‘Laws of Thought.’ They are as follows:
> (1) **The law of identity**: ‘Whatever is,is.
> (2)**The law of contradiction**: ‘Nothing can both be and not be.’
> (3) The law of excluded middle: ‘Everything must either be or not be.’
> These three laws are samples of self-evident logical principles, but are not really more fundamental or more self-evident than various other similar principles: for instance, the one we considered just now, which states that what follows from a true premise is true. The name ‘laws of thought’ is also misleading, for what is important is not the fact that we think in accordance with these laws, but the fact that **things behave in accordance with them**; ”
>
> (see Russell, 1912, p. 113)

Russell’s critique, that we tend too much to focus only on the formal aspects of the ‘Laws of Thoughts’ with the consequence that “... we thing in accordance with these laws” (see Russell, 1912, p. 113) is justified. Judged solely in terms of this aspect, it is of course necessary to think in accordance with the 'Laws of Thoughts'. But this is not the only aspect of the 'Laws of Thoughts'. The other and may be much more important aspect of these 'Laws of Thoughts'is the fact that quantum mechanical objects or that "... things behave in accordance with them" (see Russell, 1912, p. 113).

2.4.2. Principium contradictionis (Axiom II)

Principium contradictionis or **lex contradictionis** [25], [26], [27] or axiom II, the other of lex identitatis, the negative of lex identitatis, the opposite of lex identitatis, a complementary of lex identitatis, can be expressed mathematically as

$$+0 \equiv 0 \times 1 \equiv +1 \tag{114}$$

In addition to the above, from the point of view of mathematics, axiom II (equation 114) is equally the most simple mathematical expression and formulation of a contradiction. However, there is too much practical and theoretical evidence that a lot of 'secured'mathematical knowledge and rules differ too generously from real world processes, and the question may be asked whether mathematical truths can be treated as absolute truths at all. Many of the basic principle of today's mathematics allow every single author defining the real world events and processes et cetera in a way as everyone likes it for himself. Consequentially, a resulting dogmatic epistemological subjectivism and at the end agnosticism too, after all, is one of the reasons why we should rightly heed the following words of wisdom of Albert Einstein.

"I don't believe in mathematics."

(Albert Einstein cited according to Brian, 1996, p. 76)

In the long term, however, the above attitude of mathematics is not sustainable. History has taught us time and time again that objective reality has the potential to correct wrong human thinking slowly but surely, and many more than this. Objective reality has demonstrably corrected wrong human thinking again and again in the past.

[25] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

[26] Barukčić I. Aristotle's law of contradiction and Einstein's special theory of relativity. Journal of Drug Delivery and Therapeutics (JDDT). 15Mar.2019;9(2):125-43. https://jddtonline.info/index.php/jddt/article/view/2389

[27] Barukčić, Ilija. (2020, December 28). The contradiction is exsiting objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

Despite all the adversities, it is necessary and crucial to consider that a self-identical as the opposite of itself is no longer only self-identity but a difference of itself from itself within itself. In other words, “All things are different, or: there are no two things like each other ... is, in fact, opposed to the law of identity ...”(see Hegel, Georg Wilhelm Friedrich, 1991, p. 422) Each on its own and without any respect to the other is distinctive within itself and from itself and not only from another. As the opposite of its own something, is no longer only self-identity, but also a negation of itself out of itself and therefore a difference of itself from itself within itself. In other words, in opposition, a self-identical is able to return into simple unity with itself, with the consequence that even as a self-identical the same self-identical is inherently self-contradictory. A question of fundamental theoretical importance is, however, why should something be itself and at the same time the other of itself, the opposite of itself, not itself? Is something like this even possible at all and if so, why and how? These and similar questions have occupied many thinkers, including Hegel.

“Something is therefore
alive only in so far as it contains contradiction within it,
and moreover is this power to
hold and endure the contradiction within it. ”

(see Hegel, Georg Wilhelm Friedrich, 1991, p. 440)

However, as directed against identity, contradiction itself is also at the same time a source of self-changes of a self-identical out of itself.

“... contradiction
is the root of all movement and vitality;
it is only in so far as something has a contradiction within it
that it moves, has an urge and activity. ”

(see Hegel, Georg Wilhelm Friedrich, 1991, p. 439)

The further advance of science will throw any contribution to scientific progress of each of us back into scientific insignificance, as long as principium contradictionis is not given enough and the right attention. **The contradiction [28] is existing objectively and real and is the heartbeat of every self-identical.** We have reason to be delighted by the fact that very different aspects of principium contradictionis have been examined since centuries from different angles by various authors. According to Aristotle, principium contradictionis applies to everything that is, it is the first and the firmest of all principles of philosophy.

[28] Barukčić, Ilija. (2020, December 28). The contradiction is existing objectively and real (Version 1). Zenodo. https://doi.org/10.5281/zenodo.4396106

"... the same ... cannot at the same time belong and not belong to the same
... in the same respect ... This, then, is
the most certain of all principles "

(see Aristotle, of Stageira (384-322 B.C.E), 1908, Metaph., IV, 3, 1005b, 16–22)

Principium contradictionis or axiom II has many facets. As long as we follow Leibniz in this regard, we should consider that "**Le principe de contradiction est en general ...** "(Leibniz, 1765, p. 327). Scientist inevitably have false beliefs and make mistakes. In order to prevent scientific results from falling into logical inconsistency or logical absurdity, it is necessary to posses among other the methodological possibility to start a reasoning with a (logical) contradiction too. However and in contrast to the way of reasoning with inconsistent premises as proposed by para-consistent (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) and other logic, in the absence of technical and other errors of reasoning, the contradiction itself need to be preserved. In other words, **from a contradiction does not anything follows but the contradiction itself** while the theoretical question is indeed justified "What is so Bad about Contradictions? " (Priest, 1998). Historically, **the principle of (deductive) explosion** (Carnielli and Marcos, 2001, Priest, 1998, Priest et al., 1989), coined by 12th-century French philosopher William of Soissons, demand us to accept that anything, including its own negation, can be proven or can be inferred from a contradiction. In short, according to **ex falso sequitur quodlibet**, a (logical) contradiction implies anything. Respecting the principle of explosion, the existence of a contradiction (or the existence of logical inconsistency) in a scientific theorem, rule et cetera is disastrous. However, the historical development of science shows that scientist inevitably revise the theories, false positions and claims are identified once and again, and we all make different kind of mistakes. In order to avert disproportionately great damage to science and to prevent reducing science into pure subjective belief, a negation of the principle of explosion is required. Nonetheless, a justified negation of the **ex contradictione quodlibet principle** (Carnielli and Marcos, 2001) does not imply the correctness of para consistent logic (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989, Quesada, 1977) as such as advocated especially by the Peruvian philosopher Francisco Miró Quesada (Quesada, 1977) and other (Carnielli and Marcos, 2001, da Costa, 1974, 1958, Priest, 1998, Priest et al., 1989). In general, scientific theories appear to progress from lower and simpler to higher and more complex levels. However, high level theories cannot be taken for granted because high level theories are grounded on a lot of assumptions, definitions and other procedures and may rest upon too much erroneous stuff even if still not identified. Therefore, it should be considered to check at lower at simpler levels like with like.

2.4.2.1. Zero power zero

Theorem 2.1. *In general, it is*

$$+0^{+2} \equiv +0 \tag{115}$$

is false.

Proof by direct proof. The premise

$$+0 \equiv +1 \tag{116}$$

is false. In the following, any rearrangement of the premise which is free of (technical) errors, need to end up at a contradiction. In other words, the contradiction will be preserved. We obtain

$$+0 \times +0 \equiv +1 \times +0 \tag{117}$$

Equation 117 becomes

$$+0^{+2} \equiv +0 \tag{118}$$

□

2.4.2.2. Zero divided by zero

Theorem 2.2. *In general,*

$$\frac{1}{0} \equiv \frac{0}{0} \tag{119}$$

is false.

Proof by direct proof. If the premise

$$+1 \equiv +0 \tag{120}$$

is false, then the relationship

$$\frac{1}{0} \equiv \frac{0}{0} \tag{121}$$

is also false.

□

2.4.3. Principium negationis (Axiom III)

Lex negationis or axiom III, is often mismatched with simple opposition. However, from the point of view of philosophy and other sciences, identity, contradiction, negation and similar notions are equally mathematical descriptions of the most simple laws of objective reality. What sort of natural process is negation at the end? Mathematically, we define principium negationis or lex negationis or axiom III as

$$\text{Negation}(0) \times 0 \equiv \neg(0) \times 0 \equiv +1 \tag{122}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, Fritz H., 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990b). In this context, there is some evidence that

$$\text{Negation}(1) \times 1 \equiv \neg(1) \times 1 = 0 \tag{123}$$

Logically, it follows that

$$\text{Negation}(1) \equiv 0 \tag{124}$$

In the following we assume that axiom I is universal. Under this assumption, the following theorem follows inevitably.

Theorem 2.3 (Zero divided by zero)**.** *According to classical logic, it is*

$$\frac{0}{0} \equiv 1 \tag{125}$$

Proof by direct proof. The premise

$$1 \equiv 1 \tag{126}$$

is true. It follows that

$$\begin{aligned} 0 &\equiv 0 \\ &\equiv 0 \times 1 \end{aligned} \tag{127}$$

In the following, we rearrange the premise (see equation 122, p. 52). We obtain

$$0 \times (\text{Negation}(0) \times 0) \equiv 0 \tag{128}$$

Equation 128 changes slightly (see equation 123, p. 52). It is

$$(\text{Negation}(1) \times 1) \times (\text{Negation}(0) \times 0) \equiv 0 \tag{129}$$

Equation 129 demands that

$$(\text{Negation}(1)) \times (\text{Negation}(0)) \times 0 \equiv 0 \tag{130}$$

Equation 130 is logically possible (see equation 113, p. 44) only if

$$(\text{Negation}(1)) \times (\text{Negation}(0)) \equiv 1 \tag{131}$$

(see theorem 2.1, equation 115) whatever the meaning of Negation(1) or of Negation(0) might be, equation 131 demands that

$$\text{Negation}(0) \equiv \frac{1}{\text{Negation}(1)} \tag{132}$$

and that

$$\text{Negation}(1) \equiv \frac{1}{\text{Negation}(0)} \tag{133}$$

Equation 132 simplifies as (see equation 124, p. 52)

$$\begin{aligned} \text{Negation}(0) &\equiv \frac{+1}{\text{Negation}(1)} \\ &\equiv \frac{+1}{+0} \end{aligned} \tag{134}$$

It follows that

$$\neg(0) \times 0 \equiv \frac{1}{0} \times 0 \equiv \frac{0}{0} \equiv 1 \tag{135}$$

To bring it to the point. Classical logic, assumed as generally valid, demands that

$$\frac{0}{0} \equiv 1 \tag{136}$$

□

Concepts like identity, difference, negation, opposition et cetera engaged the attention of scholars at least over the last twenty-three centuries (see also Horn, 1989, Speranza and Horn, 2010). As long as we first and foremost follow Josiah Royce, negatio or negation "is one of the simplest and most fundamental relations known to the human mind. For the study of logic, no more important and fruitful relation is known." (see also Royce, 1917, p. 265) But, do we really know what, for sure, what negation is? Based on what we know about negation, Aristotle (see also Wedin, 1990a) has been one of the first to present a theory of negation, which can be found in discontinuous chunks in his works the Metaphysics, the Categories, De Interpretatione, and the Prior Analytics (see also Horn, 1989, p. 1). Negation (see also Newstadt, 2015) as a fundamental philosophical concept found its own very special melting point especially in Hegel's dialectic and is more than just a formal logical process or operation which converts true to false or false to true. Negation as such is a natural process too and equally '**an engine of changes of objective reality** " (see also Barukčić, 2019a). However, it remains an open question to establish a generally accepted link between this fundamental philosophical concept and an adequate counterpart in physics, mathematics and mathematical statistics et cetera. Especially the relationship between creation and conservation or creatio ex nihilio (see also Donnelly, 1970, Ehrhardt, 1950, Ford, 1983), determination and negation (see also Ayer, 1952, Hedwig, 1980, Heinemann, Fritz H., 1943, Kunen, 1987) has been discussed in science since ancient (see also Horn, 1989, Speranza and Horn, 2010) times too. Why and how does an event occur or why and how is an event created (creation), why and how does an event maintain its own existence over time (conservation)? The development of the notion of negation leads from Aristotle to Meister Eckhart (see also Eckhart, 1986) von Hochheim (1260-1328), commonly known as Meister Eckhart (see also Tsopurashvili, 2012) or Eckehart, to Spinoza (1632 – 1677), to Immanuel Kant (1724-1804) and finally to Georg Wilhelm Friedrich Hegel (1770-1831) and other authors too. One point is worth being noted, even if it does not come as a surprise, it was especially Benedict de Spinoza (1632 – 1677) as one of the philosophical founding fathers of the Age of Enlightenment who addressed the relationship between determination and negation in his lost letter of June 2, 1674 to his friend Jarig Jelles (see also Förster and Melamed, 2012) by the discovery of his fundamental insight that " **determinatio negatio est**" (see also Spinoza, 1674, p. 634). Hegel went even so far as to extended the slogan raised by Spinoza into to "Omnis determinatio est negatio" (see also Hegel, Georg Wilhelm Friedrich, 1812b, 2010, p. 87). Finally, it did not take too long, and the notion of negation entered the world of mathematics and mathematical logic at least with Boole's (see also Boole, 1854) publication in the year 1854. "Let us, for simplicity of conception, give to the symbol x the particular interpretation of men, then 1 - x will represent the class of 'not-men'." (see also Boole, 1854, p. 49). Finally, the philosophical notion negation found its own way into physics by the contributions of authors like Woldemar Voigt (see Voigt, 1887), George Francis FitzGerald (see FitzGerald, 1889), Hendrik Antoon Lorentz (see Lorentz, 1892, 1899), Joseph Larmor (see Larmor, 1897), Jules Henri Poincaré (see Poincaré, 1905) and Albert Einstein (see Einstein, 1905) by contributions to the physical notion "Lorentz factor".

3. Results

3.1. Anti Gödel - Refutation of Gödel's first incompleteness theorem

Gödel's first incompleteness theorem (see Satz VI Gödel, 1931, p. 187-191) demand us to accept at the end that any ... formal system is either contradictory or incomplete. The essence of Gödel's first incompleteness theorem is illustrated by table 16.

Table 16. Gödel's first incompleteness theorem

A formal system		Incomplete B_t YES	NO	
Contradictory	YES	$p(a_t) = 0$	$p(b_t)$	$p(A_t)$
A_t	NO	$p(c_t)$	$p(d_t) = 0$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	$+1$

Gödel's first incompleteness theorem forces us to accept that a formal system which is contradictory cannot be complete. However, we have justified reason to become increasingly confident that this could not be further away from the truth. There are more than enough formal ideological systems which are contradictory but at the same complete. Furthermore, Gödel's first incompleteness theorem excludes the possibility that there are formal systems which are not contradictory and at the same time complete. Also from this point of view Gödel's first incompleteness theorem is not really convincing. As long as we accept and follow Gödel's train of thought we find our self encircled by a typical logically fallacy. Gödel's first incompleteness theorem feed our delusion that incompleteness and contradiction of a formal system are determined by each other. Nonetheless, the existence of an the incompleteness of a formal system is not not the cause of the contradictions determining such a formal system and vice versa.

Proof by an counter-example.

A formal system based on an axiom +1 = +1 is logically consistent and not contradictory. A logical system which describes the addition of the number +1 to the number +1 is logically complete too or +1 +1 = +1 +1 or +2 = +2. Such a system is both, logically not contradictory and complete but not due to Gödel's first incompleteness theorem. It's more than obvious that Gödel himslef is in confusion with notions like something is not in and for itself provable with the notion something is not yet proven. Gödel's stunning intellectual achievements is based on an either or logical fallacy or black or white fallacy and is ultimately because of this without any recognisable epistemological or other value.

Quod erat demonstrandum.

In other words, a set of axioms which is posited as a possible foundation of a theory need not inevitably be incomplete.

3.2. Anti Gödel - Refutation of Gödel's second incompleteness theorem

At this point, we must ask ourselves: Do we have to submit ourselves unconditionally and forever to the dictation that whatever is proved depends at the end on the starting assumptions but not on some fundamental truth as determined by objective reality itself?

After all, we should pay attention that there is a question of fundamental importance that should be mentioned here. Objective reality is that what it is and exists the way the same exists, independently of any axioms and incompleteness theorems. Gödel's second incompleteness theorem (see Satz XI Gödel, 1931, p. 196) states at the end that a logically consistent formal system cannot prove its own consistency. The essence of Gödel's second incompleteness theorem is illustrated by table 17.

In other words, being a logically consistent system and being able to prove its own logical consistency are excluding each other. The other side of Gödel's second incompleteness theorem is viewed by table 18.

According to Gödel's second incompleteness theorem it is generally valid that

Table 17. Gödel's second incompleteness theorem

A formal system		Prove own consistency B_t YES	NO	
Consistent	YES	$p(a_t) = 0$	$p(b_t)$	$p(A_t)$
A_t	NO	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Table 18. Gödel's second incompleteness theorem II

A formal system		Prove own consistency B_t YES	NO	
Consistent	NO	$p(c_t)$	$p(d_t)$	$p(A_t)$
A_t	YES	$p(a_t) = 0$	$p(b_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

without

logical inconsistency of a formal system

no

prove of its own logical consistency by itself.

Gödel's second incompleteness theorem demand us to accept that a formal system need to be logically not consistent in order to be able to prove the same logical system by itself that the same logical system is logically consistent. Clearly, if we accept Gödel's second incompleteness theorem as true, we must accept logical contradictions too. To bring it to the point, all of these consequences of Gödel's second incompleteness theorem are without any sense. Gödel's second incompleteness theorem is without any epistemological value and an expression of a chaotic and non-systematic scientific investigation.

3.3. Without gaseous oxygen no burning wax candle

What makes a candle build out of wax burn, the wax, the wick or something else (see Byzantium, 2nd century BCE)? While there are several ways to present an answer to this question, sometimes an appropriate experiment is of use to get the facts right. In other words, the outcomes of an experiment which is repeated several times, denoted by N, under identical or similar conditions is of great value in order to re-evaluate the fundamentals of our thinking and of our scientific enquiry. [29]

[29] Michael Sendivogius (1566 – 1636), Novum Lumen Chymicum : E Naturae Fonte Et manuali experientia depromptum, & in duodecim Tractatus divisum, ac iam primum in Germania editum. Cui accessit Dialogus, Mercurii, Alchymistae & Naturae, perquam utilis. Coloniae: Apud Antonium Boetzerus ; 1610. COMMENT: Sendivogius discovered that air contains a life-giving substance—later

Proof by experiment By doing a simple (thought) experiment, we re-investigated the relationship between gaseous oxygen and burning wax candle. All in all, 75 different candles were exposed to sufficient amounts of gaseous oxygen for a certain period of time. As a result, a_t = 25 out of $E(A_t)$ = 75 candles were able to ignite and to produce a candle flame, while b_t = 50 out of $E(A_t)$ = 75 candles exposed to sufficient amounts of gaseous oxygen did not burn. Furthermore, the experiment assured that $E(\underline{A}_t)$ = 25 candles were exposed to non-oxygen. In the absence of gaseous oxygen, non of these $E(\underline{A}_t)$ = 25 candles caught fire ((d_t) = 25). Table 19 illustrates the data obtained by the experiment.

Table 19. Without gaseous oxygen, no burning wax candle

		Wax candle (burning) B_t		
		YES	NO	
Gaseous oxygen	YES	$a_t = 25$	$b_t = 50$	$E(A_t) = 75$
A_t	NO	$c_t = 0$	$d_t = 25$	$E(\underline{A}_t) = 25$
		$E(B_t) = 25$	$E(\underline{B}_t) = 75$	$N = 100$

The necessary condition (SINE) relationship, denoted by $p(A_t \leftarrow B_t)$, between sufficient amounts of gaseous oxygen, denoted by A_t, and a burning wax candle, denoted by B_t), follows as (Barukčić, 1989, p. 15-28)

$$
\begin{aligned}
p\left(A_t \leftarrow B_t\right) \equiv p\left(A_t \vee \underline{B}_t\right) &\equiv \frac{\sum_{t=1}^{N}\left(A_t \vee \underline{B}_t\right)}{N} \equiv \frac{\left(A_t \vee \underline{B}_t\right) \times p\left(A_t \vee \underline{B}_t\right)}{\left(A_t \vee \underline{B}_t\right)} \\
&\equiv p\left(a_t\right) + p\left(b_t\right) + p\left(d_t\right) \\
&\equiv \frac{N \times \left(p\left(a_t\right) + p\left(b_t\right) + p\left(d_t\right)\right)}{N} \equiv \frac{E\left(A_t \leftarrow B_t\right)}{N} \\
&\equiv \frac{E\left(A_t \vee \underline{B}_t\right)}{N} \\
&\equiv \frac{A+d}{N} \equiv \frac{E\left(A_t \leftarrow B_t\right)}{N} \\
&\equiv \frac{a+\underline{B}}{N} \equiv \frac{E\left(A_t \vee \underline{B}_t\right)}{N} \\
&\equiv \frac{a+b+d}{N} \\
&\equiv \frac{25+50+25}{100} \\
&\equiv +1
\end{aligned}
\tag{137}
$$

The conclusion is inescapable, **without** sufficient amounts of gaseous oxygen **no** burning wax candle.

called oxygen.

Quod erat demonstrandum.

The previous experiment is simple enough and can easily be repeated anytime anywhere on our planet with the aim of convincing one self of the relationship given.

Induction and logical inference

Nonetheless, despite the enormous epistemological importance of the results of the previous scientific experiment conducted, a critic might still remark that the observations made before does not justify neither predictions about observations we have not yet made nor claims which are going even beyond the observed. A possible reason for such an attitude might be Hume's position. It needs to be made clear that the so called "Hume's problem of induction"(see also David Hume, A Treatise of Human Nature, 1739; Book 1, part iii, section 6) is not the main focus of this work. However, the possibility of a reasoning from the premises to the conclusion is supported by experiments (see table 19) too. At the end, our confidence into the relationship between gaseous oxygen and burning wax candle identified just before increases with every single run of an experiment. We are absolutely opposed to any critic who forbids us to follow up such a chain of thoughts under the pretext of fallacious grounds who equally calls into question the justification of one of our most fundamental ways in which we, the humans, form knowledge. To state it once again in all clarity, as long as we are not confronted by at least one single experiment conducted somewhere on our planet which provides reliable evidence that a wax candle is burning in the absence of gaseous oxygen, critical arguments in this respect cannot be taken seriously. Yet, even if many regarded Hume's problem of induction as one of the most profound theoretical challenges in science, Hume's problem of induction is a relative one and not an absolute one.

3.4. Modus sine

The process between gaseous oxygen and burning wax candle described above forms the basis for further evaluation. In the following, we just replace gaseous oxygen by the notion premise and the outcome: burning wax candle by the notion conclusion. We obtain the following picture (see table 20).

Table 20. Modus sine - Without (premise is true), no (conclusion is true)

		Conclusion B_t		
		TRUE	FALSE	
Premise	TRUE	$a_t = 25$	$b_t = 50$	$E(A_t) = 75$
A_t	FALSE	$c_t = 0$	$d_t = 25$	$E(\underline{A}_t) = 25$
		$E(B_t) = 25$	$E(\underline{B}_t) = 75$	$N = 100$

The basic inference schemata of modus sine is as follows.

Proof by modus sine.

$P1_t$:

without ($P1_t$ is true) no (C_t is true).

$P2_t$:

$P1_t$ is false.

Therefore,

C_t:

C_t is false.

Quod erat demonstrandum.

In contrast to modus ponens, modus sine is less desperate about a true premise and does not pay any respect to a premise which is true. Modus sine expects that a premise is true. Only if a premise is true, a true conclusion can be drawn. Modus sine is focusing on circumstances where a premise is false. Under these conditions, the conclusion itself is false.

3.5. Modus inversus

Our hope is running high that modus inversus, which is starting with a false premise, is of decisive importance and a method of great help to conduct effective scientific proofs (see Toohey, 1948).

Theorem.

However, modus inversus can be derived from a necessary condition too.

Proof based on optical criteria.

As outlined before, the necessary condition relationship between gaseous oxygen and burning wax candle is regarded as secured. Nonetheless, the same relationship could easily be understood quite different. Changing the order of the columns of table 19 without changing the interior relationship between the two events in question, we obtain the following picture.

Table 21. Without gaseous oxygen, no burning wax candle: another point of view

		Wax candle (burning) B_t		
		NO	YES	
Gaseous oxygen	YES	$b_t = 50$	$a_t = 25$	$E(A_t) = 75$
A_t	NO	$d_t = 25$	$c_t = 0$	$E(\underline{A}_t) = 25$
		$E(\underline{B}_t) = 75$	$E(B_t) = 25$	$N = 100$

As a next step, we change the order of the rows of table 21 without changing the interior relationship between the two events in question and do obtain the next picture.

Table 22. Without gaseous oxygen, no burning wax candle: another point of view

		Wax candle (burning) B_t		
		NO	YES	
Gaseous oxygen	NO	$d_t = 25$	$c_t = 0$	$E(\underline{A}_t) = 25$
A_t	YES	$b_t = 50$	$a_t = 25$	$E(A_t) = 75$
		$E(\underline{B}_t) = 75$	$E(B_t) = 25$	$N = 100$

It can be seen immediately that the other side of a necessary condition relationship is the following, new knowledge.

If

sufficient amounts of gaseous oxygen are not given,

then

wax candle is not burning.

In general,

If

premise is false,

then

conclusion is false.

Quod erat demonstrandum.

This discovery has and will have far-reaching consequences, especially for mathematical sciences. We might therefore be inclined to consider the following basic inference schemata of modus inversus.

Proof by modus inversus.

$P1_t$:

if ($P1_t$ is false) then (C_t is false).

$P2_t$:

$P1_t$ is false.

Therefore,

C_t:

C_t is false.

Quod erat demonstrandum.

3.6. Modus inversus - Examples

Theorem.

Peano's axiom of the multiplication by zero is refuted.

Proof by modus inversus.

$P1: \ if\ (+3 = +1)\ is\ false,\ then\ (+3 \times 0 = +1 \times 0)\ is\ false.$

$P2: \ (+3 = +1)\ is\ false.$

Conclusion.

$(+3 \times 0 = +1 \times 0) = 0\ is\ false.$

Quod erat demonstrandum.

Again, we feel compelled to point out to the following wisdom of Albert Einstein.

"A theory is the more impressive the greater the simplicity of its premises is ... "

(see Einstein, 1949, p. 12)

The following theorem should be considered in the light of the above.

Theorem.

Zero divided by zero does not equal zero.

Proof by modus inversus.

$P1: \ if\ (+0 = +1)\ is\ false,\ then\ \frac{+0}{+0} = \frac{+1}{+0}\ is\ false.$

$P2: (+0 = +1)$ *is false*.

Conclusion.

$$\frac{+0}{+0} = \frac{+1}{+0} \quad \textit{is false}.$$

Quod erat demonstrandum.

Theorem.

Minus multiplied by minus does yield minus but not plus.

Proof by modus inversus.

$P1: \textit{if } (-2 = +2) \textit{ is false, then } (-2)^2 = (+2)^2 \textit{ is false}.$

$P2: (-2 = +2)$ *is false*.

Conclusion.

$$(-2)^2 = (+2)^2 \quad \textit{is false}.$$

Quod erat demonstrandum.

Meanwhile, there are a myriad of published examples of modus inversus. Even a blind man is forced to see that all mathematics and mathematical rules should be completely reviewed and rewritten as necessary in order to be able to transfer mathematics from the world of pure belief into the world of pure science.

3.7. Modus inversus - Counterexample

I will make it quite clear that in my opinion it is quite conceivable, if not a necessity, that a possible critic of modus inversus should provide us all with a convincing real word example in order to refute modus inversus once an for all. Amongst other real-word counter-examples with respect to modus inversus, the following one could suffice.

Modus inversus - Counterexample

A 2 meters tall, alive, mentally not confused magician, a folk healer with his heart and his soul a committed advocate and steadfast defender of bizarre forms of quantum mechanics living somewhere out there in the middle of nowhere, claims publicly that the power of human will and pure human imagination is much stronger than any natural law, stronger than any law of classical logic including modus inversus and other natural laws. In order to convince the amazed public and view stubbornly unbelieving and misguided scientists once and for all of supernatural powers, of “spuckhafte Fernwirkung”he proposes to conducts the following experiment publicly.

Experimental setup

A 5x5x5 meters tall, transparent pool is located on a stage for everyone to see and is filled with healthy, normal water right up to the edge. Once and for all, it is for sure that no gaseous oxygen is contained in this water.

Without any risk of confusion in the broader sense, the magician claims, I will bind myself visible for all to a 200 kg heavy iron ball and climb together with the same ball and without any further tools and without the possibility to escape from the transparent pool, into the transparent swimming pool. Only with the power of my will and the non-local connection with the gaseous oxygen outside the swimming pool and without any further tools, I will linger in this water which is free of gaseous oxygen at the bottom of the pool for as long as desired, but at least for 10 hours. I forbid others to interfere in the course of this historical experiment. After 10 hours, I will leave the water and as everyone will be able to testify, I will be alive. This experiment will prove once and for all that there are supernatural powers and that the pure human will of a human being is much stronger than any natural laws including the laws of classical logic et cetera.

A slightly confused and highly concerned scientist is asking the magician to draw his attention at least to the following.

The scientist points out that according to modus inversus, **if** gaseous oxygen is false (or not given for a certain period of time), **then** human being is alive is false too. The magician might expose himself unnecessarily to grave danger. Unfortunately, the magician is not open to any objections and carries out the experiment in all public. After 10 hours have passed, the public waits for the magician to come out of the transparent pool as promised. Nothing happens. Finally, the magician is recovered dead by other.

In general, a critic of modus inversus might repeat the experiment above. However, each critic may decide for himself whether it makes any sense to carry out the experiment above before he embarks on such a dangerous path. As long as an experiment as described before is not conducted successfully at least once, there is no other possibility but to accept that modus inversus is and remains universally valid.

3.8. If burning wax candle, then gaseous oxygen

As in daily life, so also in science too. A lot of scientific evidence is a question of the viewpoint one adopts. Even the nature of the relationship between gaseous oxygen and burning candle is a question that depends on our point of view. Therefore, it can be beneficial to see some specific aspects of the relationship gaseous oxygen and burning candle from different perspectives and through different eyes which refer us back to the beginning of our investigation. In the following we would like to conduct a special study involving in-depth scientific analyses to determine the nature of the relationship between burning candle and gaseous oxygen as seen through ‘the eyes ’of a burning candle.

Proof by experiment.

By conducting a simple (thought) experiment, the relationship between burning wax candle and gaseous oxygen is re-investigated. All in all, 100 different burning candles were investigated. At the same time, it was measured whether sufficient amounts of gaseous oxygen were given or whether sufficient amounts of gaseous oxygen were not given. As a result, $a_t = 25$ candles were burning while at the same (period of) time $E(A_t) = 75$ times sufficient amounts of gaseous oxygen were given. We were not able to observer one single case were a wax candle was burning while at the same (period of) time t there were no sufficient amounts of gaseous oxygen given ($c_t = 0$). Another $E(\underline{B}_t) = 75$ candles, which were not burning, were investigated. Strange enough $b_t = 50$ out of $E(\underline{B}_t) = 75$ candles had more than enough sufficient amounts of gaseous oxygen but did not burn. In other words, even if there are sufficient amounts of gaseous oxygen, a wax candle need not to burn. Furthermore, additional $d_t = 25$ out of $E(\underline{B}_t) = 75$ candles were not burning while at the same time there were not enough sufficient amounts of gaseous oxygen. Table 23 illustrates the data obtained by this experiment once again.

Table 23. If burning wax candle, then gaseous oxygen

		Gaseous oxygen A_t		
		YES	NO	
Wax candle (burning)	YES	$a_t = 25$	$c_t = 0$	$E(B_t) = 25$
B_t	NO	$b_t = 50$	$d_t = 25$	$E(\underline{B}_t) = 75$
		$E(A_t) = 75$	$E(\underline{A}_t) = 25$	$N = 100$

The sufficient (Barukčić, 2021b, p. 68-70) condition (see Barukčić, 2022a) (IMP) relationship, denoted by $p(B_t \to A_t)$ is calculated (Barukčić, 1989, p. 68-70) as

$$
\begin{aligned}
p(B_t \to A_t) \equiv p(\underline{B}_t \vee A_t) &\equiv \frac{\sum_{t=1}^{N} (\underline{B}_t \vee A_t)}{N} \equiv \frac{(\underline{B}_t \vee A_t) \times p(\underline{B}_t \vee A_t)}{(\underline{B}_t \vee A_t)} \\
&\equiv p(a_t) + p(b_t) + p(d_t) \\
&\equiv \frac{N \times (p(a_t) + p(b_t) + p(d_t))}{N} \\
&\equiv \frac{E(\underline{B}_t \vee A_t)}{N} \\
&\equiv \frac{A + d}{N} \equiv \frac{E(B_t \to A_t)}{N} \\
&\equiv \frac{a + \underline{B}}{N} \\
&\equiv \frac{a + b + d}{N} \\
&\equiv \frac{25 + 50 + 25}{100} \\
&\equiv +1
\end{aligned}
\tag{138}
$$

The relationship between burning wax candle and gaseous oxygen has been checked and confirmed by an experiment. Our inescapable logical conclusion which is reached as a result of this experiment is the following.

If burning wax candle, **then** sufficient amounts of gaseous oxygen.

Quod erat demonstrandum.

The specification of necessary conditions and of sufficient conditions is one of the valuable tools in the search for the truth. Nonetheless, the truth of a relationship between certain events should not depend on the point of view. In other words, it is necessary to preserve the truth independently of any point of view. It cannot be stressed enough that a sufficient condition (conditio per quam) is an important part of causation and the causal relationship but not identical with causation (see table 23). The relationship *if* burning wax candle, *then* sufficient amounts of gaseous oxygen are given is true. However, there can surely be no doubts that a burning wax candle is not the cause or even a cause of gaseous oxygen. This simple *counter-example* provides evidence that a cause and a sufficient conditions are not absolutely identical, both are different too. Furthermore, as has been demonstrated beyond all possible doubt, necessary conditions and sufficient conditions are **at the same (period of) time t** converses (see Gomes, Gilberto, 2009) of each other. In other words, A_t (i. e. gaseous oxygen) being a necessary condition of B_t (i. e. burning wax candle) is equivalent to B_t (i. e. burning wax candle) being a sufficient condition of A_t (i. e. gaseous oxygen) and vice versa. In general, if a special relationship between A_t and B_t at the same (period of) time t is given, then the converse relationship is given too. Therefore, as long as we are just concerned with the preservation of truth, the converse relationship as outline before is uncontroversial. It is

$$(A_t \vee \underline{B}_t) \equiv (\underline{B}_t \vee A_t) \tag{139}$$

3.9. Modus ponens - If (premise is true) then (conclusion is true)

An valid inference from the premises to the conclusion is based on natural processes as demonstrated by table 23. Modus ponens is a particular relationship between the premises and the conclusion, which logically guarantee that the conclusion follows necessarily from the premises. In the following, premises are labelled as P, and sub conclusions and conclusions as C. In the course of further discussions, consider the following basic inference schemata of modus ponens.

Proof by modus ponens.

$P1_t$:

if ($P1_t$ is true) then (C_t is true).

$P2_t$:

$P1_t$ is true.

Therefore,

C_t:

C_t is true.

Quod erat demonstrandum.

Modus ponens is sometimes also called modus ponendo ponens.

The following table (see table 24) is illustrating modus ponens.

Table 24. Modus ponens.

		Conclusion A_t		
		TRUE	FALSE	
Premise	TRUE	$a_t = 25$	$c_t = 0$	$E(B_t) = 25$
B_t	FALSE	$b_t = 50$	$d_t = 25$	$E(\underline{B}_t) = 75$
		$E(A_t) = 75$	$E(\underline{A}_t) = 25$	$N = 100$

As can be seen, modus ponens doesn't care about circumstances were the premise is false. Under these circumstances, modus ponens tolerates both, a true conclusion and equally a false conclusion. Modus ponens is blind in one eye and has nothing to say about these particular circumstances. However, this does not mean that circumstances were the premise is false are without any sense. All this, however, does not exclude that other methods of logical inference can bring down this ‘wall of silence’, which is imposed on us by modus ponens.

Example.

Proof by modus ponens.

P1:

if burning wax candle is true, *then* sufficient amounts of gaseous oxygen are given is true.

P2:

Burning wax candle is true.

C:

Sufficient amounts of gaseous oxygen are given is true.

Quod erat demonstrandum.

3.10. Modus tollens

Modus tollens, sometimes also called modus tollendo tollens, is a rule of inference, which is directly related to modus ponens. The following basic inference schemata of modus tollens is given.

Proof by modus tollens.

$P1_t$:

if ($P1_t$ is true) then (C_t is true).

$P2_t$:

C_t is false.

Therefore,

C_t:

$P1_t$ is false.

Quod erat demonstrandum.

Modus tollens is illustrated by a table (see table 25) too.

Table 25. Modus tollens.

		Conclusion A_t		
		TRUE	FALSE	
Premise	TRUE	$a_t = 25$	$c_t = 0$	$E(B_t) = 25$
B_t	FALSE	$b_t = 50$	$d_t = 25$	$E(\underline{B}_t) = 75$
		$E(A_t) = 75$	$E(\underline{A}_t) = 25$	$N = 100$

Example.

Proof by modus tollens.

P1:

if burning wax candle is true, *then* sufficient amounts of gaseous oxygen are given is true.

P2:

Sufficient amounts of gaseous oxygen are given is false.

C:

Burning wax candle is false.

Quod erat demonstrandum.

In contrast to the foregoing, let $\mathrm{A_t}$ denote the premise and let $\mathrm{B_t}$ denote the conclusion. Without limiting the foregoing, modus tollens is not to be confused with modus inversus. There is a fundamental difference between modus tollens and modus inversus. The natural foundation of modus tollens is a secured real-word relationship i. e. of the form

$$(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \tag{140}$$

In contrast to modus tollens, modus inversus itself is based on the relationship

$$(\underline{A}_{\mathrm{t}} \rightarrow \underline{B}_{\mathrm{t}}) \tag{141}$$

4. Discussion

One driving force behind any human cognition and science is the recurring effort to correct misconceptions too. Thus far, slowly but surely, step by step, scientific progress is achieved by time. However, why should and how can erroneous human thinking be identified as such for sure and as a result be corrected? Clearly, we need a reliable technology and methods which can be of use to help us to identify erroneous human reasoning (i. e. logical fallacies). Deduction and induction are some of these methods. In contrast to deduction, the conclusion from a single observation to a more general law or knowledge is called induction. Induction has been known and debated at least since the time of Aristotle (384-322). As science progressed authors like David Hume tried to approach to the issue of induction (see also Book 1, part III, section 6 in Hume, 1739, pp. 86-95) too, from his point of view. At long last, *a non-problem has been declared to a problem* by Hume himself. Nonetheless, induction was and still is one of our fundamental ways in which we humans form knowledge. However, David Hume's attack on induction has sewed some doubts on the adequacy of induction as a scientific method. In any case and no matter how things have to go, the starting point of Hume's chain of reasoning (see also Book 1, part III, section 6, pp Hume, 1739, pp. 86-95) on induction is based on a far-reaching, sceptical, wrongly formulated and at the end false premise of the following kind:

"There is no object, which implies the existence of any other ... '

(see also Hume, 1739, p. 88)

In fact, many, and perhaps too many, authors have been tempted by Hume's inductive fallacy and are usually united behind Hume full of conviction. However, Hume's starting point with respect to induction could not be further away from the truth because Hume's starting point rules out any possibility of objective reality based human reasoning. Hume demonstrably spent a great deal of his very complex chains of thought to lead us all into epistemological darkness and to lock us all up forever in a logical prison. In the last consequence, those who follow Hume's chains of thought are neither enabled nor permitted to discover the truth, but are forced to cover up the truth. Even Karl Raimund Popper (1902-1994) [30] has not really distinguished himself on this issue from others while trying to demonstrate several points: induction itself is an illusion; in reality only deduction should be used; deduction itself is sufficient. Nonetheless, how do we become that what we are, how do we gain new knowledge? In point of fact, the basic question whether objective reality itself has the power and the ability to correct erroneous human thinking keeps pressing for an answer.

Hume's flame and heat example

Hume reduced an objective relationship between a flame and heat to a pure mental process. 'We ... call to mind their constant conjunction in all past instances. ' (see also Hume, 1739, p. 87) However, in what way does an objective natural process like the heat of a flame occurring without any

[30] Karl R. Popper, David W. Miller. A proof of the impossibility of inductive probability. Nature, 302 (1983), 687–688. https://doi.org/10.1038/302687a0

human intervention somewhere on our earth depend on that what we humans think or know about such processes?

Anti Hume - Counterexamples

If it is raining, **then** the street is wet.

Under certain circumstances, raining is such an object which implies another object, the wetness of a street independently of any human mind and consciousness. However, Hume's fallacious understanding of induction, Hume's induction fallacy, doesn't allow us to recognise even such a simple, everyday relationship. Another simple example is the relationship between gaseous oxygen and human life.

Without gaseous oxygen

no human life.

As we are all aware, gaseous oxygen is one important element for living human beings because it sustains life on our planet earth. Under today's conditions, the relationship **without** gaseous oxygen **no** human life is given for sure, for every single human being, for hundreds of human beings, for all human beings, for newborn human beings, for old human beings et cetera. This is one simple example of the natural foundations of induction. In his own analysis of induction, Hume takes only a perfunctory note of this fact.

"... what we learn not from one object, we can never learn from a hundred, which are all of the same kind, and are perfectly resembling in every circumstance ... From the mere repetition ... even to infinity, there never will arise any new ... necessary connexion ; and ... has ... no more effect than if we confin'd ourselves to one only. "

(see also Hume, 1739, p. 88)

Hume raised unnecessary and artificial doubts about inductive inferences and offered equally his inadequate explanation of what induction is driven by. With the best will in the world, it is at the end not possible to follow Hume on this point. Again, the relationship between gaseous oxygen and human life is clear and secured. However, objective reality is of course chaining all the time and what is valid today does not have to be valid tomorrow too. For this reason, among others, we have the tool *P Value* in science, which can give us information about how sure we can be about our knowledge gained by induction. In spite of all conceivable adversities, as long as there is no evidence to the contrary, we may rely with some relative confidence on what is already secured.

5. Conclusion

Modus inversus as one of the methods of the logical inference represents a possibility with which we humans can reach logically unobjectionable knowledge.

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Conflict of interest statement

No conflict of interest to declare.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

Erratum July 31, 2022

In the previous publications,

the definition of the two by two table of Binomial random variables

was partly incorrect.

This has been corrected in this publication.

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I was born October, 1st 1961 in Novo Selo, Bosnia and Herzegovina, former Yogoslavia. I am of Croatian origin. From 1982-1989 C.E., I studied human medicine at the University of Hamburg, Germany. Meanwhile, I am working as a specialist of internal medicine. My basic field of research since my high school days at the Wirtschaftsgymnasium Bruchsal, Baden Württemberg, Germany is the mathematization of the relationship between a cause and an effect valid without any restriction under any circumstances including the conditions of classical logic, probability theory, quantum mechanics, special and general theory of relativity, human medicine et cetera. I endeavour to investigate positions of quantum mechanics, relativity theory, mathematics et cetera, only insofar as these positions put into question or endanger **the general validity of the principle of causality**.

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