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Geometry and probability unified

Research article

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Abstract:
Background:
We may ask a number of questions about the nature of the relationship between geometry and probability. Can both accurately model objective reality at all while the same is changing permanently?
Methods:
Various aspects of the relationship between geometry and probability have been re-investigated while relying on a hypothetico deductive method.
Results:
It turns out that an interplay between two basic theorems of geometry, Euclid's theorem and Pythagorean theorem, is a possible theoretical foundation of probability theory too.
Conclusion:
Geometry as a framework of systematic logical thinking is unified with probability theory.

Keywords: Classical logic; Geometry; Probability theory; Unification

1. Introduction

What is geometry, what is probability? Can we identify anything both have in common? A satisfactory answer of this question is complicated by the fact that there are various alternative formalizations, especially of probability theory. Moreover, as is generally known, some of the interpretations of probability do not obey all of Kolmogorov's axioms. Thus far, is it at the end, as Bertrand Russell (1872-1970) once remarked?

> "Probability is the most important concept in modern science, especially as nobody has the slightest notion what it means. "

(Bertrand Russell, 1929 Lecture cited in Bell, E. T., 1945, p. 587) [1]

Probability theory is a mathematical and conceptual framework which focuses on the investigation and description of (one part or the whole) objective reality from its own methodological point of view. However, geometry itself is a mathematical framework too, which is investigating and describing (the same part or the same whole) objective reality with its own and many times completely different scientific methods. What both mathematical frameworks have in common is the focus on a very precise description of (one part of) objective reality. Therefore, are any contradictory results in this context justified? In order to avoid any contradictions between these two scientific frameworks, it is necessary to clarify that the unity of nature is at the end the basis for the unity of science and of our human knowledge. The same unity of nature forces us every day once and again to avoid or at least to minimise any possible contradictions in our thinking. However, a generally accepted unification of geometry and probability theory into a one and single, powerful mathematical framework is nowhere near in sight. Nonetheless, painstaking investigations carried out on probabilistic geometry [2], [3] and geometric probability [4], [5], [6], [7], [8] are not completely in vain. In point of fact, especially in order to unify quantum theory and relativity theory into a one and unique mathematical framework, such an undertaking is desired to be successful.

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2. Material and methods

Scientific knowledge and objective reality are more than interrelated. Objective reality is the foundation of any scientific knowledge. Our human experience teaches us however that seen by light, grey is never merely simply grey, and looked at from different angles, many paths may lead to climb up a certain mountain. In general, it is appropriate to ensure as much as possible a broader consideration of a research question and to take into account the different facets and viewpoints of an issue investigated in order to reach a goal.

2.1. Methods

Definitions should help us to provide and assure a systematic approach to a mathematical formulation of different relationships. It also goes without the need of further saying that a definition must be logically consistent and correct.

2.1.1. Random variables

Let a **random variable**(Gosset, 1914) X denote something like a function defined on a probability space, which itself maps from the sample space(Neyman and Pearson, 1933) to the real numbers.

2.1.2. The Expectation of a Random Variable

Definition 2.1 (**The First Moment Expectation of a Random Variable**). *Summaries of an entire distribution of a random variable(see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 22) X, such as the expected value, or average value, are useful in order to identify where X is expected to be without describing the entire distribution. For practical and other reasons, we shall limit ourselves here to discrete random variables, while the basic properties of the expectation value of a random variable X will not be investigated. Thus far, let X be a discrete random variable with the probability p(X). The relationship between the first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of X, denoted by E(X), and the probability p(X), is given by the equation:*

$$\begin{aligned} E(X) &\equiv X \times p(X) \\ &\equiv \Psi(X) \times X \times \Psi^{*}(X) \end{aligned} \tag{1}$$

where $\Psi(X)$ is the wave-function (see Born, 1926, Schrödinger, Erwin Rudolf Josef Alexander, 1926) of X, $\Psi^{*}(X)$ is the complex conjugate wave-function of X. The first moment expectation value squared of a random variable X follows as

$$
\begin{aligned}
E(X)^2 &\equiv p(X) \times X \times p(X) \times X \\
&\equiv p(X) \times p(X) \times X \times X \\
&\equiv (p(X) \times X)^2 \\
&\equiv E(X) \times E(X)
\end{aligned} \tag{2}
$$

The ongoing progress with artificial intelligence has the potential to transform human society far beyond any imaginable border of human recognition and can help even to solve problems that otherwise would not be tractable. No wonder, scientist and systems are confronted with large volumes of data (big data) of various natures and from different sources. The use of tensor technology can simplify and accelerate Big data analysis. In other words, let $X_{kl\mu\nu\ldots}$ denote an n-th index co-variant tensor with the probability $p(X_{kl\mu\nu\ldots})$. The first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of $X_{kl\mu\nu\ldots}$, denoted by $E(X_{kl\mu\nu\ldots})$, is a number defined as follows:

$$
E(X_{kl\mu\nu\ldots}) \equiv p(X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots} \equiv p(X_{kl\mu\nu\ldots}) \cap X_{kl\mu\nu\ldots} \tag{3}
$$

while $\times$ or $\cap$ might denote the commutative multiplications of tensors. The first moment expectation value squared of a random variable X follows as

$$
\begin{aligned}
{}^2E(X_{kl\mu\nu\ldots}) &\equiv p(X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots} \times p(X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots} \\
&\equiv p(X_{kl\mu\nu\ldots}) \times p(X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots} \times X_{kl\mu\nu\ldots} \\
&\equiv {}^2(p(X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots}) \\
&\equiv E(X_{kl\mu\nu\ldots}) \times E(X_{kl\mu\nu\ldots})
\end{aligned} \tag{4}
$$

Definition 2.2 (The Second Moment Expectation of a Random Variable). *The second(see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42) moment expectation value (or more or less arithmetic mean) of a (large) number of independent realizations of a random variable X follows as:*

$$
\begin{aligned}
E(X^2) &\equiv p(X) \times X^2 \\
&\equiv (p(X) \times X) \times X \\
&\equiv E(X) \times X \\
&\equiv X \times E(X)
\end{aligned} \tag{5}
$$

From the point of view of tensor algebra it is

$$
\begin{aligned}
E\left({}^2X_{kl\mu\nu\ldots}\right) &\equiv p(X_{kl\mu\nu\ldots}) \times {}^2X_{kl\mu\nu\ldots} \\
&\equiv (p(X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots} \\
&\equiv E(X_{kl\mu\nu\ldots}) \times X_{kl\mu\nu\ldots} \\
&\equiv X_{kl\mu\nu\ldots} \times E(X_{kl\mu\nu\ldots})
\end{aligned} \tag{6}
$$

Definition 2.3 (**The n-th Moment Expectation of a Random Variable**). *The n-th(see Barukčić, 2020a, 2021) moment expectation value of a (large) number of independent realizations of a random variable X follows as:*

$$\begin{aligned} E\left(X^{n}\right) &\equiv p\left(X\right) \times X^{n} \\ &\equiv \left(p\left(X\right) \times X\right) \times X^{n\text{-}1} \\ &\equiv E\left(X\right) \times X^{n\text{-}1} \end{aligned} \tag{7}$$

2.1.3. Probability of a Random Variable

What is the nature of the probability of an event, or what is the relationship between probability and geometry or between the probability of an event and notions like false or true. At a first pass, various authors answer this question, one way or another. For authors like De Morgan, probability is only a degree of confidence, or credences or of belief. "By degree of probability, we really mean, or ought to mean, degree of belief" (see De Morgan, 1847, p. 172). Such a purely subjective (or personalist or Bayesian (see Bayes, 1763)) interpretation of probabilities as degrees of confidence, or credences finds its own scientific opposition, moreover, in Kolmogorov's axiomatization of probability theory. However, perhaps we can do better, then, to think that Kolmogorov's axiomatization of probability theory is the last word spoken on probability theory. Nobody seriously considers that Kolmogorov's conceptual apparatus of probability theory has solved the basic problem of any probability theory, the relationship between classical logic or geometry and probability theory. One very massive disadvantage of Kolmogorov's axiomatization of probability theory is that it is very silent especially on this issue. Any unification of geometry and probability theory into one unique mathematical framework might prove very difficult as long as we rely purely on Kolmogorov's understanding of probability theory. It's not surprising that the probability of an event bear at least directly, and sometimes indirectly, upon central philosophical and scientific concerns. A correct understanding of probability is one of the most important foundational scientific problems. Now let us strengthen our position with respect to the probability of an event. In our understanding, the probability of an event is something objectively and real. The probability of an event is the truth value of something or the degree to which something, i.e. a random variable X, is determined by its own expectation value. The probability p(X) of a random variable X follows as (see equation 1)

$$\begin{aligned}
p(X) &\equiv \frac{X \times p(X)}{X} \equiv \frac{E(X)}{X} \equiv p(X) \\
&\equiv \frac{X \times X \times p(X)}{X \times X} \equiv \frac{X \times E(X)}{X \times X} \equiv \frac{E\left(X^2\right)}{X^2} \\
&\equiv \frac{E(X)}{X} \equiv \frac{E(X) \times E(X)}{X \times E(X)} \equiv \frac{E(X)^2}{E(X^2)} \\
&\equiv \frac{E(X)}{X} \equiv \frac{E(X) \times E(\underline{X})}{X \times E(\underline{X})} \equiv \frac{\sigma(X)^2}{X \times X \times (1 - p(X))} \equiv \frac{\sigma(X)^2}{E(\underline{X}^2)} \\
&\equiv \Psi(X) \times \Psi^*(X)
\end{aligned} \tag{8}$$

where $\Psi(X)$ is the wave-function of X, $\Psi^*(X)$ is the complex conjugate wave-function of X. As soon as the probability p(X) of an event X is determined, the probability of its own other, 1 - p(X), the complementary of X, the opposite of X, anti X, is determined too. We obtain

$$\begin{aligned}
1 - p(X) &\equiv 1 - \frac{X \times p(X)}{X} \equiv 1 - \frac{E(X)}{X} \equiv \frac{X}{X} - \frac{E(X)}{X} \equiv \frac{X - E(X)}{X} \equiv \frac{E(\underline{X})}{X} \equiv p(\underline{X}) \\
&\equiv 1 - \frac{X \times X \times p(X)}{X \times X} \equiv 1 - \frac{X \times E(X)}{X \times X} \equiv 1 - \frac{E\left(X^2\right)}{X^2} \equiv \frac{X^2}{X^2} - \frac{E\left(X^2\right)}{X^2} \equiv \frac{X^2 - E\left(X^2\right)}{X^2} \\
&\equiv 1 - \frac{E(X)}{X} \equiv 1 - \frac{E(X) \times E(X)}{X \times E(X)} \equiv 1 - \frac{E(X)^2}{E(X^2)} \\
&\equiv 1 - \frac{E(X)}{X} \equiv 1 - \frac{E(X) \times E(\underline{X})}{X \times E(\underline{X})} \equiv 1 - \frac{\sigma(X)^2}{X \times X \times (1 - p(X))} \equiv 1 - \frac{\sigma(X)^2}{E(\underline{X}^2)} \\
&\equiv 1 - \Psi(X) \times \Psi^*(X)
\end{aligned} \tag{9}$$

In our understanding, there are conditions where probability theory / statistics is related with geometry (i.e. Pythagorean theorem, Euclid's theorem et cetera) (see also figure 1) by the equation:

$$a^2 \equiv E\left(X^2\right) \tag{10}$$

Further research should be able and might provide convincing evidence whether - and to what extent - equation 10 makes any sense at all. However, none of this relieves us of our duty to seriously consider the possibility of negative probabilities (see theorem 3.38 Barukčić, 2019b, pp. 67-68) like

$$-p(X) \equiv \frac{-E(X)}{-X} \tag{11}$$

It is

$$+1 \equiv p(X) + 1 - p(X) \equiv p(X) + p(\underline{X}) \equiv C \tag{12}$$

as illustrated by figure 1 and equally

$$\begin{aligned}
+1^{+2} &\equiv (1 \times p(X)) + (1 \times (1 - p(X))) \equiv (1 \times (p(X) + p(\underline{X}))) \equiv C^2 \\
&\equiv a^2 + b^2 \\
&\equiv C^2
\end{aligned} \tag{13}$$

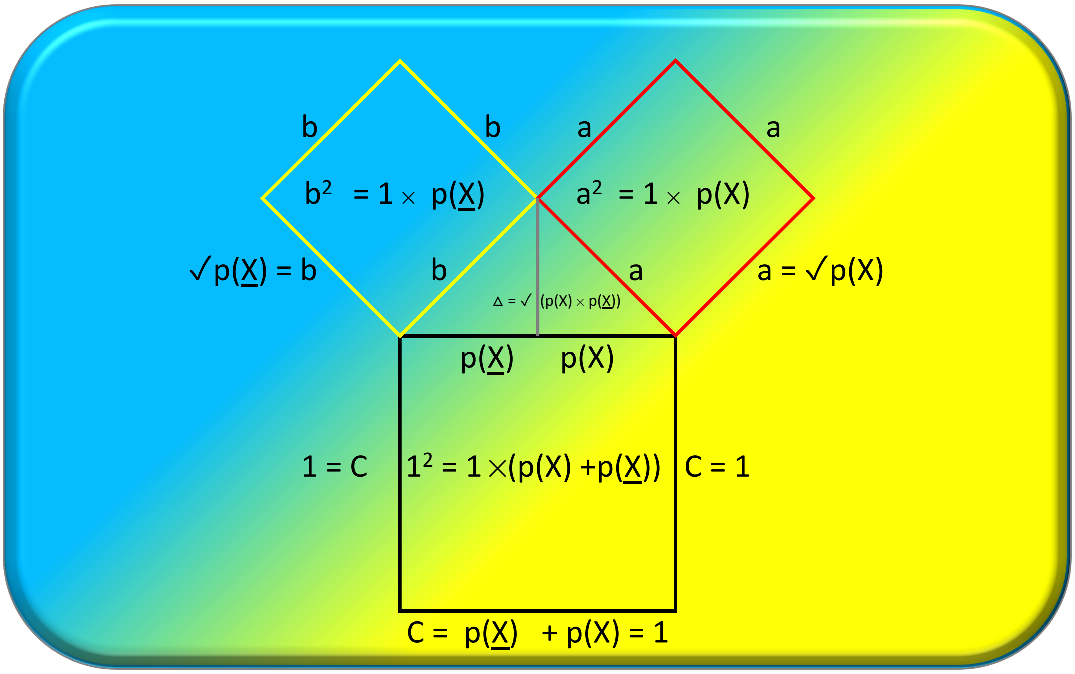

Figure 1. Geometry and probability theory.

The distributions of properties of geometric objects like length, area, volume, etc. is studied by **geometric probability** (see Klain and Rota, 1997, Milman, Vitali D., 2008, Solomon, 1978) too. In other words, probability is involved in geometry. **Example**. Let the length of a line C be C = 10 cm. Let X denote the length of a sub-line of C. Let X = 5 cm. The probability p(C = X) is calculated as p(C = X) = 5 / 10 = 1/2. However, as can be seen by figure 1, probability and geometry are not only deeply interrelated. In contrast to Menger's approach to **probabilistic geometry** (see Menger, 1951, 2003, Milman, Vitali D., 2008, Špaček, 1956), probability theory can be defined by geometry, completely and potentially vice versa too. The known trigonometrical functions are one geometric way to formulate probabilities of events. In consideration of the preceding figure 1 before and the general definition of the function sine, denoted as sin, it is

$$sin\ \alpha \equiv \frac{a}{c} \equiv \frac{a}{1} \equiv \frac{\sqrt[2]{p(X)}}{1} \equiv \sqrt[2]{p(X)} \tag{14}$$

and

$$sin^2\ \alpha \equiv (sin\ \alpha)^2 \equiv sin\ \alpha \times sin\ \alpha \equiv \left(\frac{a}{c}\right)^2 \equiv \left(\frac{a}{1}\right)^2 \equiv a^2 \equiv p(X) \equiv \Psi(X) \times \Psi^*(X) \tag{15}$$

Against the background of figure 1 and the general definition of the function cosecant, denoted as csc, it is

$$csc\ \alpha \equiv \frac{c}{a} \equiv \frac{1}{a} \equiv \frac{1}{\sqrt[2]{p(X)}} \tag{16}$$

and equally.

$$csc^2\ \alpha \equiv (csc\ \alpha)^2 \equiv csc\ \alpha \times csc\ \alpha \equiv \left(\frac{c}{a}\right)^2 \equiv \left(\frac{1}{a}\right)^2 \equiv \frac{1}{a^2} \equiv \frac{1}{p(X)} \tag{17}$$

In general it is

$$sin\ \alpha \times csc\ \alpha \equiv +1 \tag{18}$$

In the light of figure 1 above, and the definition the function cosine, denoted as cos, it is

$$cos\ \alpha \equiv \frac{b}{c} \equiv \frac{b}{1} \equiv \frac{\sqrt[2]{p(\underline{X})}}{1} \equiv \sqrt[2]{p(\underline{X})} \tag{19}$$

and at the same time

$$cos^2\ \alpha \equiv (cos\ \alpha)^2 \equiv cos\ \alpha \times cos\ \alpha \equiv \left(\frac{b}{c}\right)^2 \equiv \left(\frac{b}{1}\right)^2 \equiv b^2 \equiv p(\underline{X}) \equiv 1 - \Psi(X) \times \Psi^*(X) \tag{20}$$

Claudius Ptolemy (c. 85 – c. 165 CE), a very influential Greek astronomers of his time, developed a geocentric theory of our solar system (Almagest, (see Ptolemaeus, Claudius, 1952)) that prevailed for more than 1400 years until overthrown by the heliocentric theory of Copernicus in the De revolutionibus of 1543 (see Copernici, Nicolai, 1543). Ptolemy's 'Almagest'is a scientific text longer in use than Isaac Newton's Principia (see Newton, 1687). The first known astronomical observation made by Ptolemy was on 26 March 127 while the last one was made on 2 February 141. In fact, it must be treated as relatively sure that Ptolemy already knew about the relationship

$$sin^2\ \alpha + cos^2\ \alpha \equiv +1 \tag{21}$$

known as **Ptolemy's theorem** (see Ptolemaeus, Claudius, 1952, Book 1, Chapter 10) which is meanwhile identified in more detail as

$$sin^2\ \alpha + cos^2\ \alpha \equiv p(X) + p(\underline{X}) \equiv +1 \tag{22}$$

or the Pythagorean theorem in the language of trigonometry. We are justified in asking whether the expectation value of an angle α, denoted as E(α), might be given by the equation

$$E(\alpha) \equiv \alpha \times \left(sin^2\ \alpha\right) \tag{23}$$

whether E(α^2) would be given by the equation

$$E\left(\alpha^2\right) \equiv \alpha \times \alpha \times \left(sin^2\ \alpha\right) \tag{24}$$

Under these assumptions, the variance $\sigma(\alpha)^2$ of an angle would follow as

$$\sigma(\alpha)^2 \equiv E\left(\alpha^2\right) - E(\alpha)^2 \equiv \alpha \times \alpha \times \left(sin^2\ \alpha\right) \times \left(1 - \left(sin^2\ \alpha\right)\right) \tag{25}$$

Having regard to figure 1 above and on the basis of the definition of the function secant, denoted by sec, it is

$$sec\ \alpha \equiv \frac{c}{b} \equiv \frac{1}{b} \equiv \frac{1}{\sqrt[2]{p(\underline{X})}} \tag{26}$$

and equally

$$sec^2\ \alpha \equiv (sec\ \alpha)^2 \equiv sec\ \alpha \times sec\ \alpha \equiv \left(\frac{c}{b}\right)^2 \equiv \left(\frac{1}{b}\right)^2 \equiv \frac{1}{p(\underline{X})} \tag{27}$$

$$cos\ \alpha \times sec\ \alpha \equiv +1 \tag{28}$$

On the basis of a presentation by figure 1 and the known definition of the function tangent, denoted as tan, it is

$$tan\ \alpha \equiv \frac{sin\ \alpha}{cos\ \alpha} \equiv \frac{\frac{a}{c}}{\frac{b}{c}} \equiv \frac{a}{b} \equiv \frac{\sqrt[2]{p(X)}}{\sqrt[2]{p(\underline{X})}} \equiv \sqrt[2]{\frac{p(X)}{p(\underline{X})}} \tag{29}$$

and equally

$$tan^2\ \alpha \equiv (tan\ \alpha)^2 \equiv (tan\ \alpha) \times (tan\ \alpha) \equiv \left(\frac{sin\ \alpha}{cos\ \alpha}\right) \times \left(\frac{sin\ \alpha}{cos\ \alpha}\right) \equiv \frac{sin^2\ \alpha}{cos^2\ \alpha} \equiv \frac{\frac{a^2}{c^2}}{\frac{b^2}{c^2}} \equiv \frac{a^2}{b^2} \equiv \frac{p(X)}{p(\underline{X})} \tag{30}$$

In view of figure 1 and the definition of cotangent, denoted as cot, it is

$$cot\ \alpha \equiv \frac{b}{a} \equiv \frac{\sqrt[2]{p(\underline{X})}}{\sqrt[2]{p(X)}} \equiv \sqrt[2]{\frac{p(\underline{X})}{p(X)}} \tag{31}$$

Furthermore, it is

$$cot^2\ \alpha \equiv (cot\ \alpha)^2 \equiv (cot\ \alpha) \times (cot\ \alpha) \equiv cot^2\ \alpha \equiv \frac{b^2}{a^2} \equiv \frac{p(\underline{X})}{p(X)} \tag{32}$$

Based on the findings as explained before and by figure 1 it is

$$tan\ \alpha \times cot\ \alpha \equiv +1 \tag{33}$$

Some relationships before are demonstrated by a unit circle (see figure 2), i.e. a circle of unit radius — that is, a radius of 1.

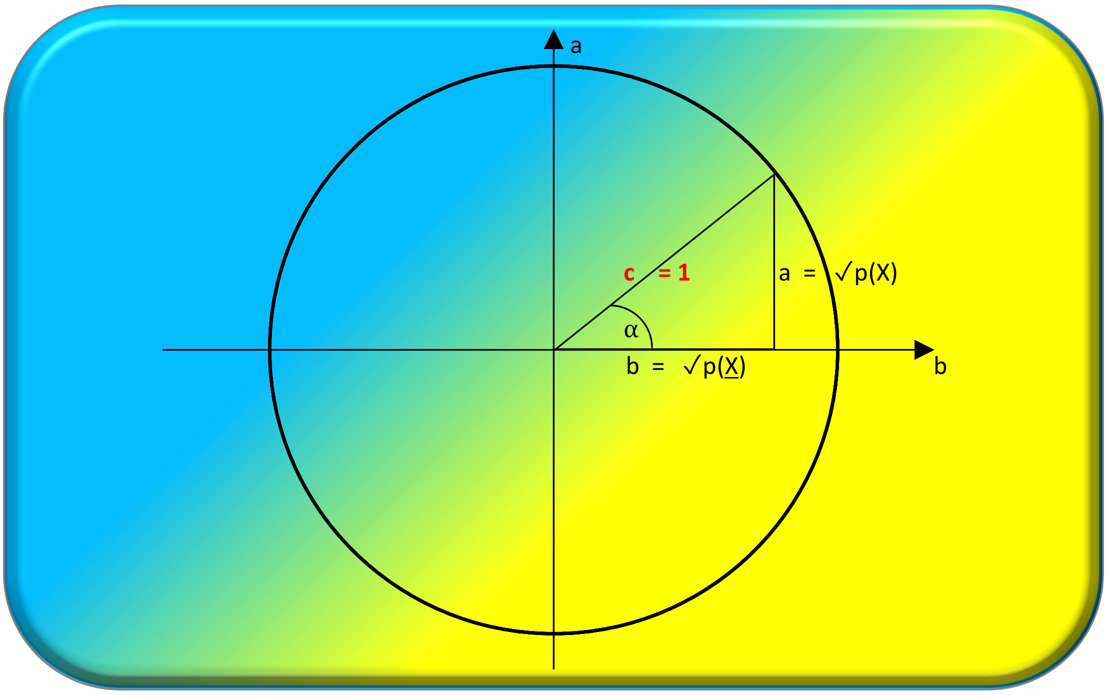

Figure 2. Geometry, probability theory and unit circle.

An undeniable consequence of the previous explanations is that the "local hidden variable" (see Bohm, 1952, De Broglie, Louis, 1927), denoted as $E(\underline{X})$, is determined by the relationship

$$E(\underline{X}) \equiv X \times cos^2\ \alpha \equiv \frac{\sigma(X)^2}{E(X)} \equiv \frac{\sigma(X)^2}{\Psi(X) \times X \times \Psi^*(X)} \tag{34}$$

while the variance from the point of view of geometry is given as

$$\begin{aligned} \sigma(X)^2 &\equiv E\left(X^2\right) - E(X)^2 \\ &\equiv (X \times p(X)) \times X \times (1 - p(X)) \\ &\equiv \left(X \times sin^2\ \alpha\right) \times X \times \left(1 - sin^2\ \alpha\right) \\ &\equiv \left(X \times sin^2\ \alpha\right) \times X \times \left(cos^2\ \alpha\right) \\ &\equiv X^2 \times \left(sin^2\ \alpha\right) \times \left(cos^2\ \alpha\right) \end{aligned} \tag{35}$$

From the point of view of tensor algebra, we obtain

$$\begin{aligned} p\left(X_{\mathrm{kl}\mu\nu\ldots}\right) &\equiv \frac{X_{\mathrm{kl}\mu\nu\ldots} \times p\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{X_{\mathrm{kl}\mu\nu\ldots}} \equiv \frac{E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{X_{\mathrm{kl}\mu\nu\ldots}} \\ &\equiv \frac{X_{\mathrm{kl}\mu\nu\ldots} \times X_{\mathrm{kl}\mu\nu\ldots} \times p\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{X_{\mathrm{kl}\mu\nu\ldots} \times X_{\mathrm{kl}\mu\nu\ldots}} \equiv \frac{E\left({}^{2}X_{\mathrm{kl}\mu\nu\ldots}\right)}{{}^{2}X_{\mathrm{kl}\mu\nu\ldots}} \\ &\equiv \frac{E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times X_{\mathrm{kl}\mu\nu\ldots}} \equiv \frac{{}^{2}E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{E\left({}^{2}X_{\mathrm{kl}\mu\nu\ldots}\right)} \\ &\equiv \Psi\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times \Psi^{*}\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \end{aligned} \tag{36}$$

where $\Psi\left(X_{\mathrm{kl}\mu\nu\ldots}\right)$ is the wave-function tensor of $\mathrm{X}_{\mathrm{kl}\mu\nu\ldots}$, $\Psi^{*}\left(X_{\mathrm{kl}\mu\nu\ldots}\right)$ is the complex conjugate wave-function tensor of $\mathrm{X}_{\mathrm{kl}\mu\nu\ldots}$.

2.1.4. Variance of a Random Variable

Definition 2.4 (**The Variance of a Random Variable**). *Johann Carl Friedrich Gauß (1777-1855) introduced the normal distribution and the error of mean squared in his 1809 monograph(see Gauß, Carl Friedrich, 1809). In the following, Karl Pearson (1857-1936) coined the term "standard deviation"in 1893. Pearson is writing: "Then σ will be termed its standard-deviation (error of mean square)."(see Pearson, 1894, p. 80). Finally, the term variance was introduced by Sir Ronald Aylmer Fisher (1890-1962) in the year 1918.*

> *"The ... deviations of a ... measurement from its mean ... may be ... measured by the standard deviation corresponding to the square root of the mean square error ... It is ... desirable* ***in analysing the causes*** *... to deal with the square of the standard deviation as the measure of variability. We shall term this quantity the Variance... "*
>
> *(see Fisher, Ronald Aylmer, 1919, p. 399)*

The deviation of a random variable X from its population mean or sample mean E(X) has a central role in statistics and is one important measure of dispersion. The variance $\sigma(X)^2$ (see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42), the second central moment of a distribution, is the expectation value of the squared deviation of a random variable X from its own expectation value E(X) and is determined in general as (see equation 5):

$$\begin{aligned}\sigma(X)^2 &\equiv E\left(X^2\right) - E(X)^2 \\ &\equiv (X \times E(X)) - E(X)^2 \\ &\equiv E(X) \times (X - E(X)) \\ &\equiv E(X) \times E(\underline{X})\end{aligned} \tag{37}$$

while $E(\underline{X}) \equiv X - E(X)$. From the point of view of tensor algebra, it is

$$\begin{aligned}{}^{2}\sigma\left(X_{\mathrm{kl}\mu\nu\ldots}\right) &\equiv E\left({}^{2}X_{\mathrm{kl}\mu\nu\ldots}\right) - {}^{2}E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \\ &\equiv \left(X_{\mathrm{kl}\mu\nu\ldots} \times E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)\right) - {}^{2}E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \\ &\equiv E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times \left(X_{\mathrm{kl}\mu\nu\ldots} - E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)\right) \\ &\equiv E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times E\left(\underline{X}_{\mathrm{kl}\mu\nu\ldots}\right)\end{aligned} \tag{38}$$

while $E\left(\underline{X}_{\mathrm{kl}\mu\nu\ldots}\right) \equiv X_{\mathrm{kl}\mu\nu\ldots} - E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)$. As demonstrated by equation 38, variance depends not just on the expectation value of what has actually been observed $E\left(\left(X_{\mathrm{kl}\mu\nu\ldots}\right)\right)$, but also on the expectation value that could have been observed but were not $\left(E\left(\underline{X}_{\mathrm{kl}\mu\nu\ldots}\right)\right)$. There are circumstances in quantum mechanics where this fact is called the local hidden variable. Even if his might strike us as peculiar, variance [9] is primarily a mathematical method which is of use in order to evaluate specific hypotheses in the light of some empirical facts. However, as a mathematical tool or method, variance is also a scientific description of a certain part of objective reality too. In this context, as a general mathematical principle, one fundamental meaning of variance is to provide a logically consistent link between something and its own other, between X and anti X.

"The variance in this sense is a measure of the inner contradictions of a random variable, of changes, of struggle within this random variable itself, or the greater $\sigma(X)^2$ of a random variable, the greater the inner contradictions of this random variable. "

(see Barukčić, 2006a, p. 57)

All things considered, we can safely say that, on the whole, **the variance is a mathematical description of the philosophical notion of the inner contradiction of a random variable X** (see Hegel, 1812, 1813, 1816) . Based on equation 37, it is

$$E\left(X^2\right) \equiv E(X)^2 + \sigma(X)^2 \tag{39}$$

or

$$\frac{E(X)^2}{E(X^2)} + \frac{\sigma(X)^2}{E(X^2)} \equiv p(X) + \frac{\sigma(X)^2}{E(X^2)} \equiv +1 \tag{40}$$

In other words, the variance (see Barukčić, 2006b) of a random variable is a determining part of the probability of a random variable. The wave function Ψ follows in general, as

$$\begin{aligned}
\Psi(X) &\equiv \frac{1}{\Psi^*(X)} - \frac{\sigma(X)^2}{(\Psi^*(X) \times E(X^2))} \\
&\equiv \frac{\left(E(X^2) - \sigma(X)^2\right)}{(\Psi^*(X) \times E(X^2))} \\
&\equiv \frac{1}{(\Psi^*(X) \times E(X^2))} \times \left(E\left(X^2\right) - \sigma(X)^2\right) \\
&\equiv \frac{1}{(\Psi^*(X) \times E(X^2))} \times E(X)^2 \\
&\equiv \frac{1}{\Psi^*(X)} \times \frac{E(X)^2}{E(X^2)} \\
&\equiv \frac{1}{\Psi^*(X) \times X} \times E(X)
\end{aligned} \tag{41}$$

The wave function (see Born, 1926) of a quantum-mechanical system is a central determining part of the Schrödinger wave equation (see Schrödinger, Erwin Rudolf Josef Alexander, 1926, 1929, 1952).

[9] Romeijn, Jan-Willem, "Philosophy of Statistics", The Stanford Encyclopedia of Philosophy (Spring 2022 Edition), Edward N. Zalta (ed.), forthcoming URL = https://plato.stanford.edu/archives/spr2022/entries/statistics/.

Definition 2.5 (**The First Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**). *In general, let $E(\underline{X})$ be defined as*

$$E(\underline{X}) \equiv X - E(X) \equiv X - (X \times p(X)) \equiv X \times (+1 - p(X)) \tag{42}$$

and denote an expectation value of a (discrete) random variable anti X with the probability

$$p(\underline{X}) \equiv 1 - p(X) \tag{43}$$

The first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of anti X, denoted as E($\underline{X}$), is a number defined as follows:

$$E(\underline{X}) \equiv X - (X \times p(X)) \equiv X \times (1 - p(X)) \equiv X \times p(\underline{X}) \tag{44}$$

The first moment expectation value squared of a random variable anti X follows as

$$\begin{aligned} E(\underline{X})^2 &\equiv p(\underline{X}) \times X \times p(\underline{X}) \times X \\ &\equiv p(\underline{X}) \times p(\underline{X}) \times X \times X \\ &\equiv (p(\underline{X}) \times X)^2 \\ &\equiv E(\underline{X}) \times E(\underline{X}) \end{aligned} \tag{45}$$

Definition 2.6 (**The Second Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**). *The second(see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42) moment expectation value (or more or less arithmetic mean) of a (large) number of independent realizations of a random variable anti X follows as:*

$$\begin{aligned} E(\underline{X^2}) &\equiv p(\underline{X}) \times X^2 \\ &\equiv (p(\underline{X}) \times X) \times X \\ &\equiv E(\underline{X}) \times X \\ &\equiv X \times E(\underline{X}) \end{aligned} \tag{46}$$

Definition 2.7 (**The n-th Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**). *The n-th(see Barukčić, 2020a, 2021) moment expectation value of a (large) number of independent realizations of a random variable anti X follows as:*

$$\begin{aligned} E(\underline{X^n}) &\equiv p(\underline{X}) \times X^n \\ &\equiv (p(\underline{X}) \times X) \times X^{n\text{-}1} \\ &\equiv E(\underline{X}) \times X^{n\text{-}1} \end{aligned} \tag{47}$$

Definition 2.8 (**The Co-Variance of a Random Variable**). *Sir Ronald Aylmer Fisher (1890 -1962) introduced the term covariance (see Bailey, 1931) in the year 1930 in his book as follows:*

> *"It is obvious too that where a considerable fraction of the variance is contributed by chance causes, the variance of any group of individuals will be inflated in comparison with the covariances between related groups ... "*
>
> *(see Fisher, Ronald Aylmer, 1930, p. 195)*

In general, the co-variance is defined as given by equation 48.

$$\sigma(X,Y) \equiv E(X,Y) - (E(X) \times E(Y)) \tag{48}$$

From the point of view of tensor algebra, it is

$$\sigma\left(X_{kl\mu\nu\,\ldots}, Y_{kl\mu\nu\,\ldots}\right) \equiv E\left(X_{kl\mu\nu\,\ldots}, Y_{kl\mu\nu\,\ldots}\right) - \left(E\left(X_{kl\mu\nu\,\ldots}\right) \times E\left(Y_{kl\mu\nu\,\ldots}\right)\right) \tag{49}$$

2.1.5. Geometry

2.1.5.1. Euclid's theorem In general, Euclid's (ca. 360-280 BC) so-called right triangle altitude theorem or the geometric mean theorem or Euclid's theorem, published as a corollary to proposition 8 in Book VI of Euclid's Elements (see also Euclid, of Alexandria (300 BCE), 1893) and used in proposition 14 of Book II to square a rectangle is defined (see Barukčić, 2013, 2015, 2016) as

$$\begin{aligned}
{}_{\mathrm{R}}\Delta_{\mathrm{t}}{}^{2} &\equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \\
&\equiv \frac{\left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \times {}_{\mathrm{R}}X_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}}} \\
&\equiv \frac{\left({}_{\mathrm{R}}a_{\mathrm{t}}\right)^{2} \times \left({}_{\mathrm{R}}b_{\mathrm{t}}\right)^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} \\
&\equiv \sigma\left(X_{\mathrm{t}}\right)^{2}
\end{aligned} \tag{50}$$

where $\sigma(X_{\mathrm{t}})^{2}$ is the variance of the random variable X_t. The variance ${}_{\mathrm{R}}\Delta_{\mathrm{t}}{}^{2} \equiv \sigma(X_{\mathrm{t}})^{2}$ of a right-angled triangle is illustrated by Fig. 3 in more detail.

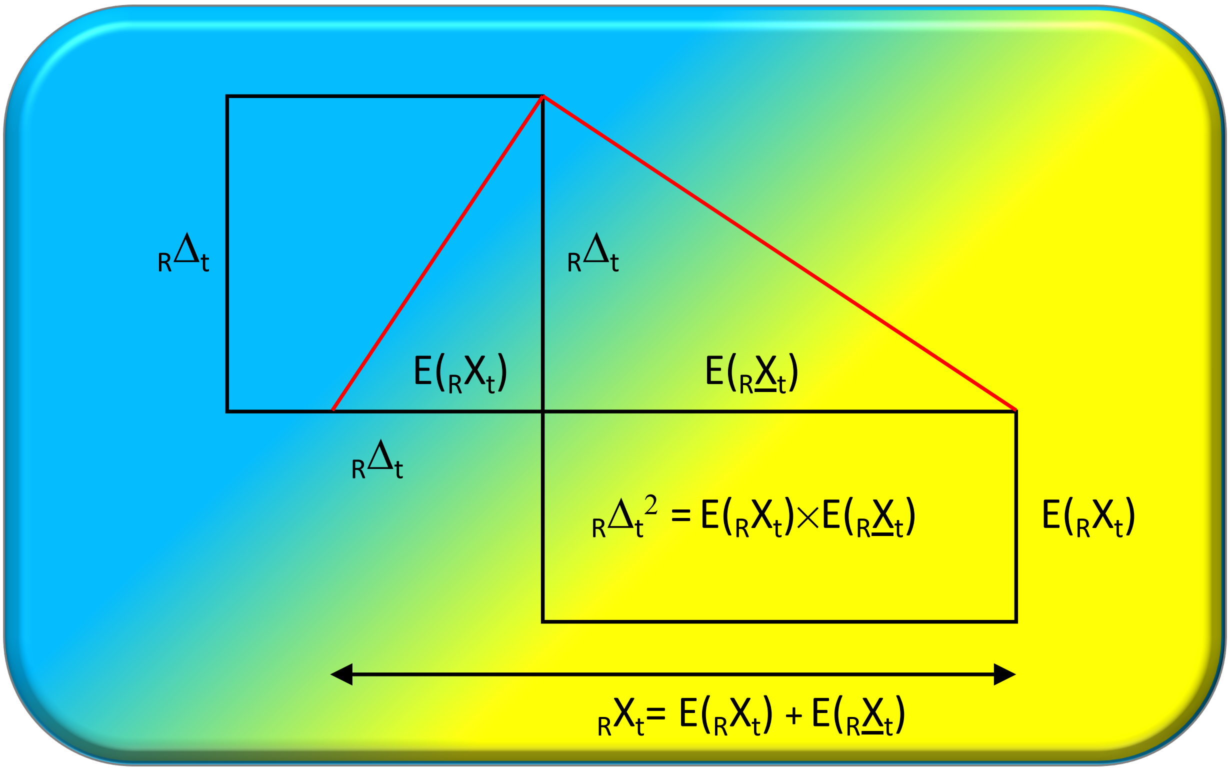

Figure 3. Euclid's theorem.

2.1.5.2. Pythagorean theorem

Definition 2.9 (**The right-angled triangle**). *A right-angled triangle is a triangle in which one angle is a 90-degree angle. Let* $_{R}X_{t}$ *denote the hypotenuse, the side opposite the right angle (side* $_{R}X_{t}$ *inside figure 4). The sides* $_{R}a_{t}$ *and* $_{R}b_{t}$ *are called legs of the triangle. In a right-angled triangle ABC, the side AC, which is abbreviated as* $_{R}b_{t}$*, is the side which is adjacent to the angle* α*, while the side CB, denoted as* $_{R}a_{t}$*, is the side opposite to the angle* α*. Figure 4 might illustrate a right-angled triangle (see Bettinger and Englund, 1960). The relation between the sides and angles of a right-angled triangle are known to be the basis for trigonometry, but are the basis of probability theory too.*

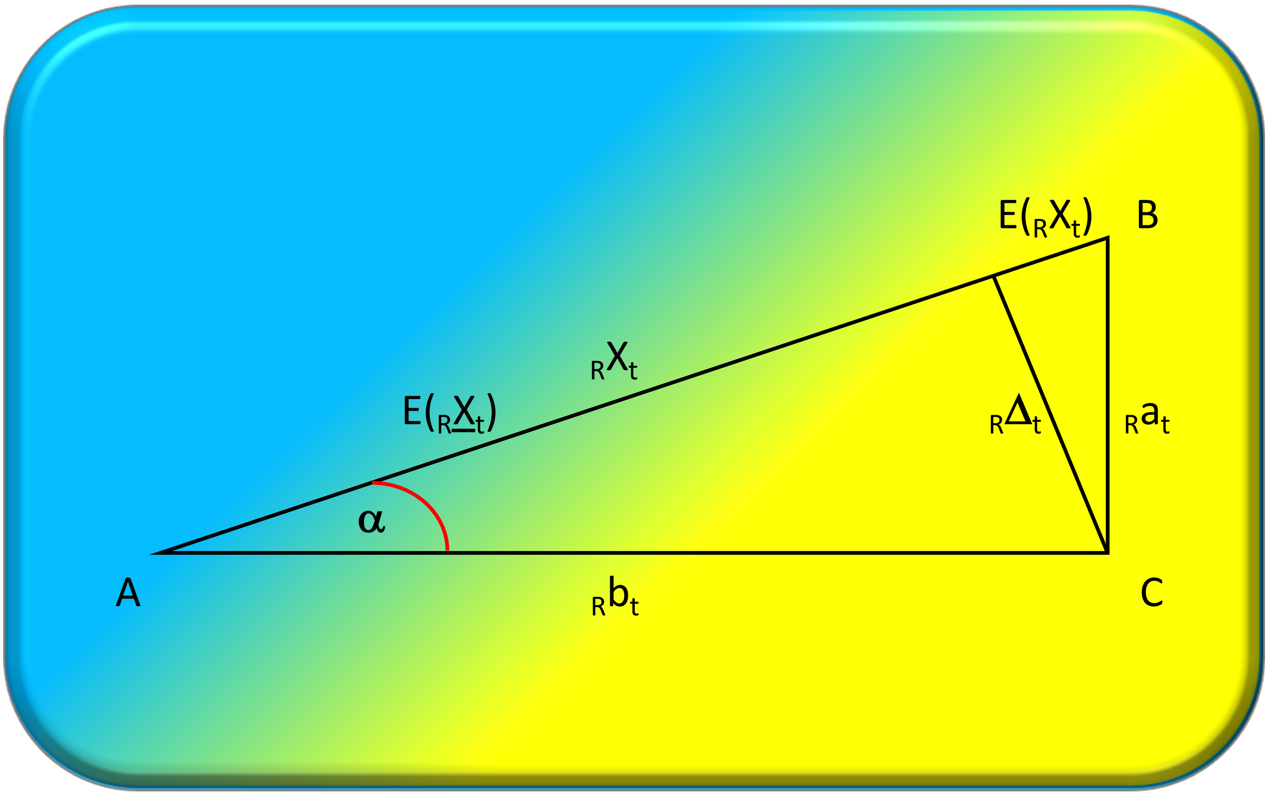

Figure 4. Right-angled triangle. $_{R}X_{t}$ **is hypotenuse,** $_{R}a_{t}$ **and** $_{R}b_{t}$ **are called triangles legs.**

Again, $_{R}X_{t}$ is in the state of superposition, a law which has been re-formulated by the Danish geologist Nicolaus Steno (see Stenonis, Nicolai, 1669) in his 1696 book 'De Solido Intra Naturaliter Contento Dissertationis Prodomus '. Thus far, how big is the chance or probability that three random points like A, B, C in space-time are able to form a certain, stable right-angled triangle? Problems of similar type have been studied in the 18th century under the notion of geometric probability (see Milman, Vitali D., 2008, Solomon, 1978). Geometry and probability are deeply interrelated.

2.1.5.2.1. The Pythagorean theorem in general

Definition 2.10 (**The Pythagorean theorem**).

"The Pythagorean Theorem is arguably **the most famous statement in mathematics**, and **the fourth most beautiful equation**" (see Ratner, 2009). In general, the Pythagorean theorem is defined as

$$ {}_{R}a_{t}^{2} + {}_{R}b_{t}^{2} \equiv {}_{R}C_{t}^{2} \equiv {}_{R}X_{t}^{2} \tag{51} $$

Fig. 5 is illustrating the Pythagorean theorem in all its splendour and beauty in more detail in a grossly simplified form.

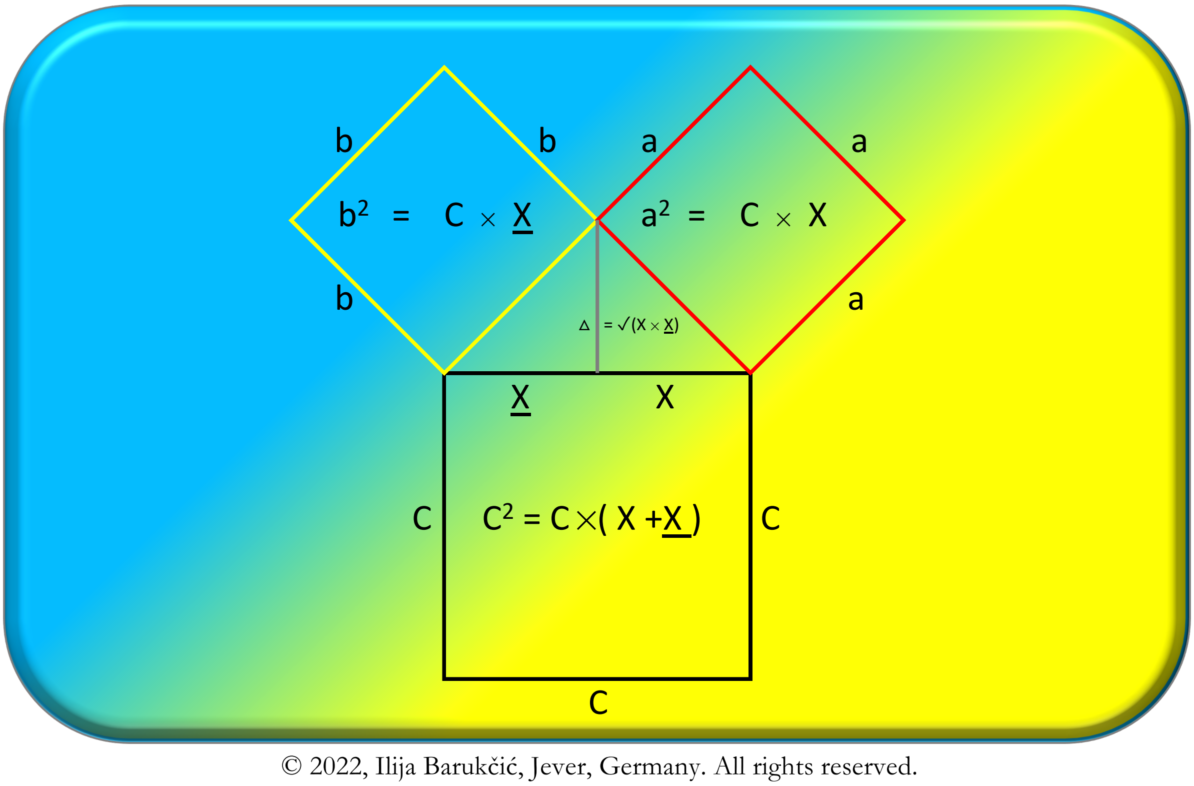

Figure 5. Pythagorean theorem.

The famous Pythagorean theorem of Euclidean geometry is attributed to the Greek thinker Pythagoras of Samos (570–490 BCE). Pythagoras, the first mathematician, wrote [10] nothing. Pythagoras lived long before the time of Plato (427–347 BCE) and Aristotle (384–322 BCE). However, even if attributed to Pythagoras, the theorem has been known to the Babylonians (see Maor, 2007) more than a thousand years before Pythagoras. Thus far, it remains controversial whether Pythagoras himself has been

[10] Huffman, Carl, "Pythagoras", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2018/entries/pythagoras/>.

the first in history to introduce the so-called Pythagorean theorem. This gives rise to serious doubts concerning the extent of the historical glorification of Pythagoras.

2.2. *Axioms*

2.2.1. Axiom I. Lex identitatis

In this context, we define axiom I as the expression

$$+1 = +1 \tag{52}$$

2.2.2. Axiom II. Lex contradictionis

In this context, axiom II or **lex contradictionis**, the negative of lex identitatis, or

$$+0 = +1 \tag{53}$$

and equally the most simple form of a contradiction formulated.

2.2.3. Axiom III. Lex negationis

$$\neg(0) \times 0 = 1 \tag{54}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, Fritz H., 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990). In this context, there is some evidence that $\neg(1) \times 1 = 0$. In other words, it is $(\neg(1) \times 1) \times (\neg(0) \times 0) = 1$

3. Results

3.1. *Probability and the law of the excluded middle*

Theorem 3.1 (Probability and the law of the excluded middle)**.** *Let t denote a single Bernoulli trial or run of an experiment. In general, it is*

$$p({}_{R}X_{t}) \equiv p({}_{0}X_{t}) + p({}_{0}\underline{X}_{t}) \tag{55}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{56}$$

is true. In the following, we rearrange this premise. We obtain

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{57}$$

At the same Bernoulli trial t $p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$ can be determined by various, different probabilities. We obtain the identity

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{0}X_{\mathrm{t}}\right)+p\left({}_{1}X_{\mathrm{t}}\right)+p\left({}_{2}X_{\mathrm{t}}\right)+p\left({}_{3}X_{\mathrm{t}}\right)+\cdots \tag{58}$$

Equation 58 does not exclude circumstances where $p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv +1$. However, in order to bring back Aristotle's law of the excluded middle [11], [12] into memory, there are conditions where "**... there cannot be an intermediate between contradictories ...** " (see also Aristotle, of Stageira (384-322 B.C.E), 1908, (Metaphysica, Chapter VII, 1011b, 23-24)). In other words, **tertium non datur**, a third [13] is not given. Equation 58 is rearranged as

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)-p\left({}_{0}X_{\mathrm{t}}\right) \equiv p\left({}_{1}X_{\mathrm{t}}\right)+p\left({}_{2}X_{\mathrm{t}}\right)+p\left({}_{3}X_{\mathrm{t}}\right)+\cdots \tag{59}$$

Therefore, we define in general

$$p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)-p\left({}_{0}X_{\mathrm{t}}\right) \equiv p\left({}_{1}X_{\mathrm{t}}\right)+p\left({}_{2}X_{\mathrm{t}}\right)+p\left({}_{3}X_{\mathrm{t}}\right)+\cdots \tag{60}$$

Equation 60 simplifies as

$$p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)-p\left({}_{0}X_{\mathrm{t}}\right) \tag{61}$$

In general, the relationship between something and its own other, its own opposite, its own complementary at one and the same Bernoulli trial t, at one and the same run of an experiment et cetera, is given by the equation

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{0}X_{\mathrm{t}}\right)+p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{62}$$

□

3.2. *Negative probabilities*

Theorem 3.2 (Negative probabilities)**.** *It appears to be reasonable to assume the existence of negative probabilities.*

Proof by direct proof. In agreement with equation 62, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{0}X_{\mathrm{t}}\right)+p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{63}$$

Rearranging equation 63, it is

$$+0 \equiv p\left({}_{0}X_{\mathrm{t}}\right)+p\left({}_{0}\underline{X}_{\mathrm{t}}\right)-p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{64}$$

[11] Gottlieb, Paula, "Aristotle on Non-contradiction", The Stanford Encyclopedia of Philosophy (Spring 2019 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2019/entries/aristotle-noncontradiction/.

[12] Bown A. Epicurus on Bivalence and the Excluded Middle. Archiv für Geschichte der Philosophie. 2016;98(3): 239-271. https://doi.org/10.1515/agph-2016-0012

[13] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

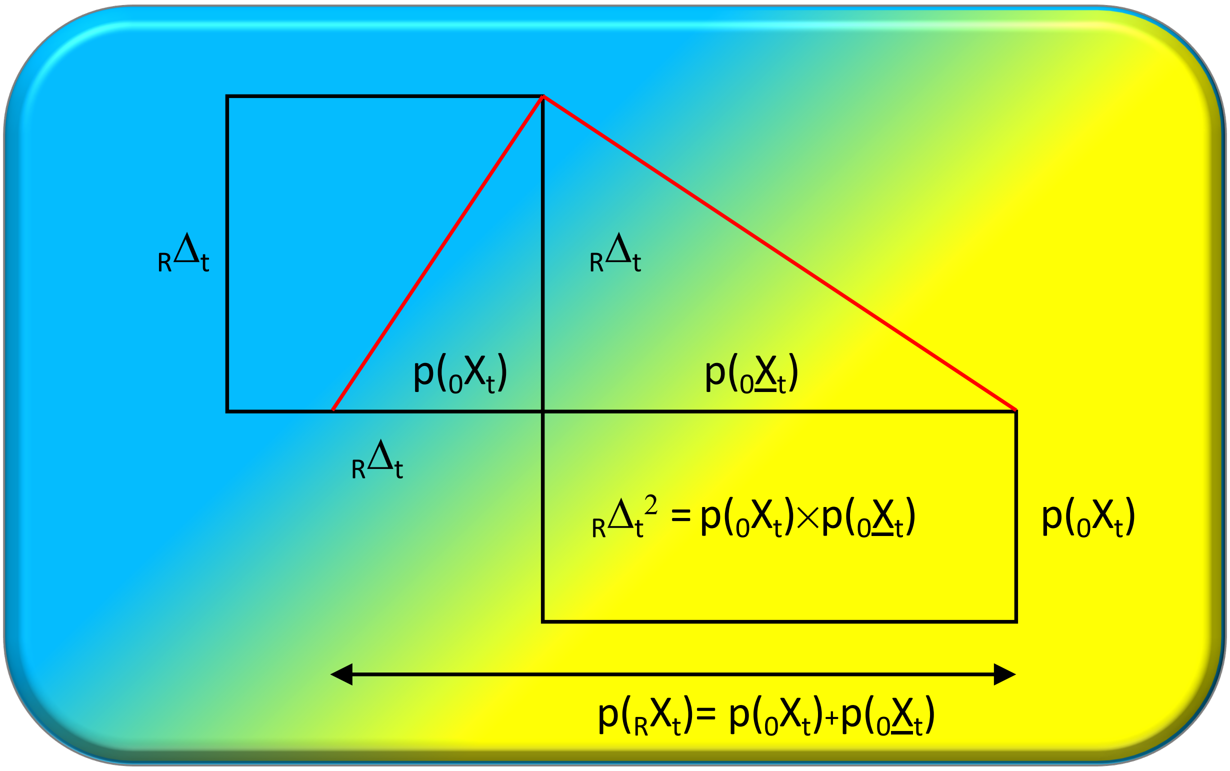

Figure 6. Euclid's theorem and probability theory.

and finally

$$-p\left({}_{0}X_{t}\right)-p\left({}_{0}\underline{X}_{t}\right)\equiv -p\left({}_{R}X_{t}\right) \tag{65}$$

□

3.3. Determinatio est negatio

Anti ${}_{0}X_{t}$ is usually opposed to ${}_{0}X_{t}$. However, the occurrence or measurement of ${}_{0}X_{t}$ is already determinate and is distinguished from another ${}_{0}X_{t}$. Yet, we cannot but express our disappointment that the anti ${}_{0}X_{t}$ which is opposed to the ${}_{0}X_{t}$ is potentially also an anti ${}_{0}X_{t}$ of a particular ${}_{0}X_{t}$, a determinate anti ${}_{0}X_{t}$. Here, however, anti ${}_{0}X_{t}$ is to be taken as it is in its simplicity. Furthermore, we are dealing with ${}_{0}X_{t}$ which in realizing itself in the form of an event at the same time resolves itself, ${}_{0}X_{t}$ has for its result its own negation. In general, ${}_{0}X_{t}$ passes over into the negation (see also Hegel, 1812, 1813, 1816) of its particular content and contains therefore to some extent that from which it results.

Theorem 3.3 (Determinatio est negatio)**.** *In general, the relationship between something and its own other is determined by the natural process of negation, as*

$$p\left({}_{0}X_{t}\right) \equiv p\left({}_{R}X_{t}\right) \times \left(1 - \left(\frac{p\left({}_{0}\underline{X}_{t}\right)}{p\left({}_{R}X_{t}\right)}\right)\right) \tag{66}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{67}$$

is true. In the following, we rearrange this premise. We obtain

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{68}$$

or

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) + 0 \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) - p\left({}_{0}X_{\mathrm{t}}\right) + p\left({}_{0}X_{\mathrm{t}}\right) \tag{69}$$

Again it is $p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) - p\left({}_{0}X_{\mathrm{t}}\right)$ or according to Aristotle, "**... there cannot be an intermediate between contradictories ...** " (see also Aristotle, of Stageira (384-322 B.C.E), 1908, (Metaphysica, Chapter VII, 1011b, 23-24)), **tertium non datur**, a third [14] is not given. Equation 68 becomes

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{0}X_{\mathrm{t}}\right) + p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{70}$$

Equation 70 changes to

$$\frac{p\left({}_{0}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} + \frac{p\left({}_{0}\underline{X}_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv \frac{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv +1 \tag{71}$$

and to

$$\frac{p\left({}_{0}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv 1 - \frac{p\left({}_{0}\underline{X}_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \tag{72}$$

At the end of the proof, we arrived at Spinoza's "**... determinatio negatio est ...** " (see also Spinoza, Benedictus de, 1674, p. 634). Equation 72 demands that

$$p\left({}_{0}X_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(1 - \left(\frac{p\left({}_{0}\underline{X}_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}\right)\right) \tag{73}$$

□

3.4. Expectation value I

Theorem 3.4 (Expectation value I)**.** *Let t denote a single Bernoulli trial or run of an experiment. In general, it is*

$$E\left({}_{R}X_{t}\right) \equiv E\left({}_{0}X_{t}\right) + E\left({}_{0}\underline{X}_{t}\right) \tag{74}$$

[14] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{75}$$

is true. In the following, we rearrange this premise. We obtain

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{76}$$

or

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) + 0 \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) - E\left({}_{0}X_{\mathrm{t}}\right) + E\left({}_{0}X_{\mathrm{t}}\right) \tag{77}$$

We define $E\left({}_{0}\underline{X}_{\mathrm{t}}\right) \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) - E\left({}_{0}X_{\mathrm{t}}\right)$. In other words, "**... there cannot be an intermediate between contradictories ...** " (see also Aristotle, of Stageira (384-322 B.C.E), 1908, (Metaphysica, Chapter VII, 1011b, 23-24)), **tertium non datur**, a third [15] is not given. Equation 76 becomes

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv E\left({}_{0}X_{\mathrm{t}}\right) + E\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{78}$$

□

Figure 7 might illustrate the result of theorem 3.4 in more detail.

Theorem 3.5 (Expectation value II)**.**

Proof by direct proof. In general, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{0}X_{\mathrm{t}}\right) + p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{79}$$

After considering all of the circumstances which are known at this point in time, theoretically an unknown parameter Y can be given, which permanently ensures that

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv Y \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{80}$$

Equation 79 changes only slightly and becomes

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \left(Y \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right) \equiv \left(Y \times p\left({}_{0}X_{\mathrm{t}}\right)\right) + \left(Y \times p\left({}_{0}\underline{X}_{\mathrm{t}}\right)\right) \tag{81}$$

□

Theorem 3.6 (Expectation value III)**.**

Proof by direct proof. In general, it is

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv E\left({}_{0}X_{\mathrm{t}}\right) + E\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{82}$$

[15] Horn, Laurence R., "Contradiction", The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2018/entries/contradiction/.

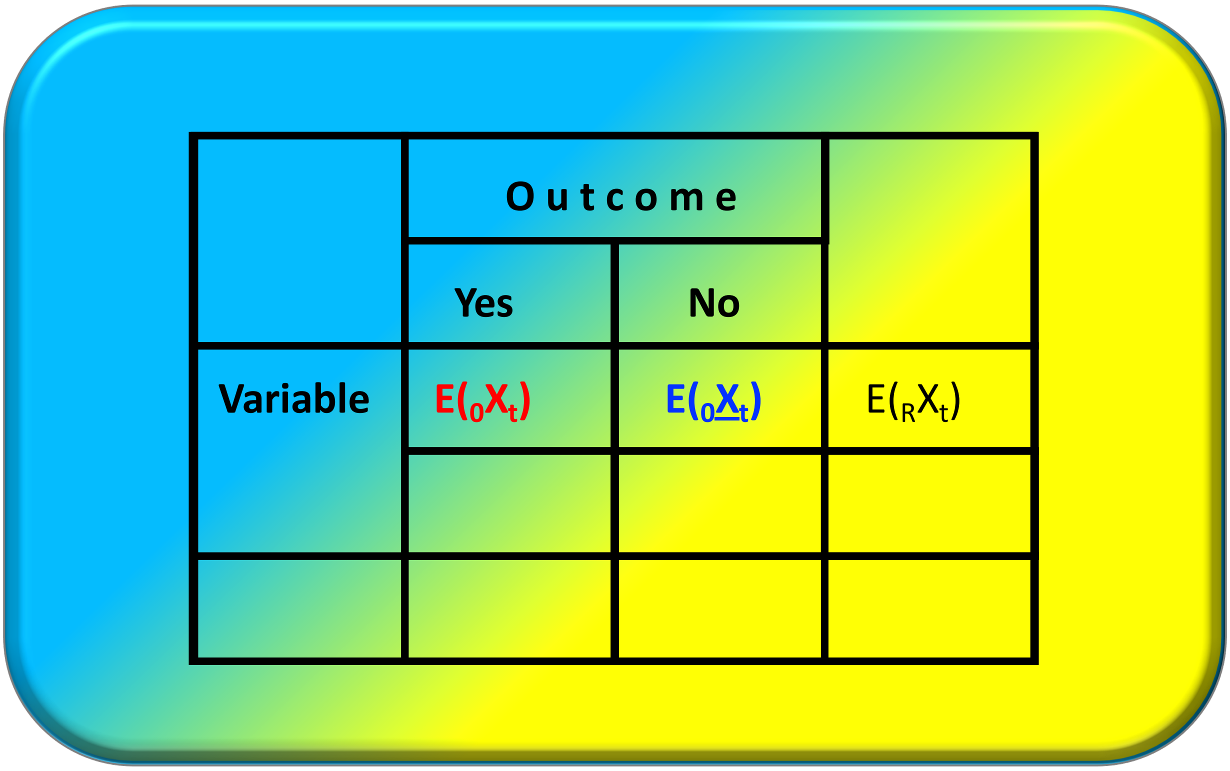

Figure 7. Euclid's theorem and expectation values.

On this occasion, there is an unknown parameter Z, which permanently ensures that

$$E\left({}_{0}X_{t}\right) \equiv Z \times p\left({}_{0}X_{t}\right) \tag{83}$$

and simultaneously also

$$E\left({}_{0}\underline{X}_{t}\right) \equiv Z \times p\left({}_{0}\underline{X}_{t}\right) \tag{84}$$

Equation 82 changes slightly and becomes

$$\left(Z \times p\left({}_{0}X_{t}\right)\right) + \left(Z \times p\left({}_{0}\underline{X}_{t}\right)\right) \equiv E\left({}_{R}X_{t}\right) \tag{85}$$

and according to equation 81

$$\left(Z \times p\left({}_{0}X_{t}\right)\right) + \left(Z \times p\left({}_{0}\underline{X}_{t}\right)\right) \equiv \left(Y \times p\left({}_{R}X_{t}\right)\right) \equiv E\left({}_{R}X_{t}\right) \tag{86}$$

Rearranging equation 86according to equation 61, it is

$$\left(Z \times \left(p\left({}_{R}X_{t}\right) - p\left({}_{0}\underline{X}_{t}\right)\right)\right) + \left(Z \times p\left({}_{0}\underline{X}_{t}\right)\right) \equiv \left(Y \times p\left({}_{R}X_{t}\right)\right) \tag{87}$$

or

$$(Z \times p({}_{R}X_{t})) - (Z \times p({}_{0}\underline{X}_{t})) + (Z \times p({}_{0}\underline{X}_{t})) \equiv (Y \times p({}_{R}X_{t})) \tag{88}$$

Equation 88 simplifies as

$$(Z \times p({}_{R}X_{t})) \equiv (Y \times p({}_{R}X_{t})) \tag{89}$$

However, it is also possible and necessary to simplify equation 89 further. In general, it is necessary to accept the mathematical identity

$$Z \equiv Y \tag{90}$$

□

3.5. *The unknown becomes known*

Theorem 3.7 (The unknown becomes known)**.**

Proof by direct proof. In agreement with equation 80 it is

$$E({}_{R}X_{t}) \equiv Y \times p({}_{R}X_{t}) \tag{91}$$

In line with equation 1 it is $(E({}_{R}X_{t}) \equiv p({}_{R}X_{t}) \times {}_{R}X_{t})$. Equation 91 changes slightly and becomes,

$$Y \times p({}_{R}X_{t}) \equiv p({}_{R}X_{t}) \times {}_{R}X_{t} \tag{92}$$

In the final analysis, equation 92 will ultimately lead to the necessity to accept the mathematical identity

$$Y \equiv {}_{R}X_{t} \tag{93}$$

□

3.6. *Expection value IV*

Theorem 3.8 (Expection value IV)**.**

Proof by direct proof. In agreement with equation 83 it is

$$E({}_{0}X_{t}) \equiv Z \times p({}_{0}X_{t}) \tag{94}$$

In line with equation 90 it is $(Z \equiv Y)$. Equation 94 changes slightly and becomes,

$$E({}_{0}X_{t}) \equiv Y \times p({}_{0}X_{t}) \tag{95}$$

In line with equation 93 it is $(Y \equiv {}_{\mathrm{R}}X_{\mathrm{t}})$. In the final analysis, equation 95 demand us to accept the mathematical identity

$$E\left({}_{0}X_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times p\left({}_{0}X_{\mathrm{t}}\right) \tag{96}$$

□

3.7. Expection value V

Theorem 3.9 (Expection value V)**.**

Proof by direct proof. In agreement with equation 84 it is

$$E\left({}_{0}\underline{X}_{\mathrm{t}}\right) \equiv Z \times p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{97}$$

In line with equation 90 it is $(Z \equiv Y)$. Equation 97 changes slightly and becomes,

$$E\left({}_{0}\underline{X}_{\mathrm{t}}\right) \equiv Y \times p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{98}$$

In line with equation 93 it is $(Y \equiv {}_{\mathrm{R}}X_{\mathrm{t}})$. In the final analysis, equation 98 demand us to accept the mathematical identity

$$E\left({}_{0}\underline{X}_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times p\left({}_{0}\underline{X}_{\mathrm{t}}\right) \tag{99}$$

□

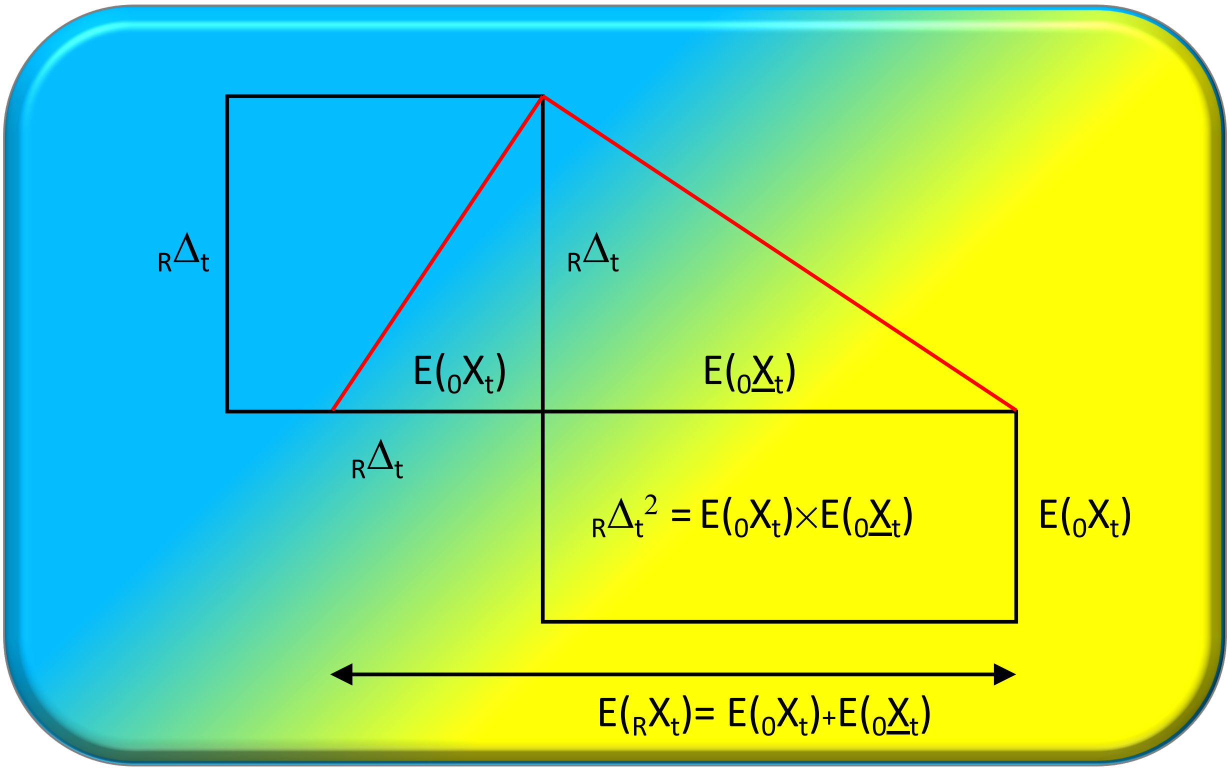

Figure 8. Euclid's theorem and expectation values.

Theorem 3.10 (Expection value VI)**.**

Proof by direct proof. In agreement with equation 96 it is

$$E\left({}_{0}X_{t}\right) \equiv {}_{R}X_{t} \times p\left({}_{0}X_{t}\right) \tag{100}$$

This result can be generalized. We set i = 0. Equation 100 changes slightly. It is

$$E\left({}_{i}X_{t}\right) \equiv {}_{R}X_{t} \times p\left({}_{i}X_{t}\right) \tag{101}$$

Dividing equation 101 by $\left({}_{R}X_{t}\right)$, it is

$$\frac{E\left({}_{i}X_{t}\right)}{{}_{R}X_{t}} \equiv p\left({}_{i}X_{t}\right) \tag{102}$$

Dividing equation 102 by $p\left({}_{R}X_{t}\right)$, it is

$$\frac{E\left({}_{i}X_{t}\right)}{{}_{R}X_{t} \times p\left({}_{R}X_{t}\right)} \equiv \frac{p\left({}_{i}X_{t}\right)}{p\left({}_{R}X_{t}\right)} \tag{103}$$

In agreement with equation 96, equation 103 becomes

$$\frac{E\left({}_{\mathrm{i}}X_{\mathrm{t}}\right)}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv \frac{p\left({}_{\mathrm{i}}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \tag{104}$$

Equation 104 changes slightly and becomes

$$E\left({}_{\mathrm{i}}X_{\mathrm{t}}\right) \equiv \frac{p\left({}_{\mathrm{i}}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{105}$$

or in general

$$E\left({}_{\mathrm{i}}X_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{i}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{106}$$

□

3.8. Expectation value and variance

Theorem 3.11 (Expectation value and variance)**.** *In general, it is*

$$\sigma\left({}_{R}X_{t}\right)^{2} \equiv E\left({}_{R}X_{t}\right) \times E\left({}_{R}\underline{X}_{t}\right) \tag{107}$$

Proof by direct proof. Our premise (i.e. axiom) which is equally the theoretical starting point of this theorem, is that

$$+1 \equiv +1 \tag{108}$$

is true. This premise is true. In the following, we rearrange this premise (i.e. axiom). We obtain

$${}_{\mathrm{R}}X_{\mathrm{t}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \tag{109}$$

or

$${}_{\mathrm{R}}X_{\mathrm{t}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times 1 \tag{110}$$

and

$${}_{\mathrm{R}}X_{\mathrm{t}} + 0 \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times (1+0) \tag{111}$$

or equally

$${}_{\mathrm{R}}X_{\mathrm{t}} + 0 \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) + p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right) \tag{112}$$

$$ {}_{\mathrm{R}}X_{\mathrm{t}} + 0 \equiv \left({}_{\mathrm{R}}X_{\mathrm{t}} \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)\right) + \left({}_{\mathrm{R}}X_{\mathrm{t}} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right) \tag{113} $$

It is $E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)$ and $E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$. Equation 113 becomes

$$ {}_{\mathrm{R}}X_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) + E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{114} $$

Multiplying equation 114 by $E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$ it is

$$ {}_{\mathrm{R}}X_{\mathrm{t}} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right) + \left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right) \tag{115} $$

It is ${}_{\mathrm{R}}X_{\mathrm{t}} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)$. Equation 115 becomes

$$ E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} + E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{116} $$

or

$$ E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) - E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{117} $$

It is $\sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) - E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2}$. Equation 117 becomes

$$ \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{118} $$

□

3.9. Superposition of probabilities I

Theorem 3.12 (Superposition of probabilities I)**.** *Let $p\left({}_{R}X_{t}\right)$ denote the probability of quantum mechanical observable, a random variable ${}_{R}X_{t}$ et cetera as seen, measured et cetera from the point of view of a stationary observer R. In general, it is*

$$ E\left({}_{R}X_{t}\right) \equiv \left(E\left({}_{0}X_{t}\right)\right) + \left(E\left({}_{1}X_{t}\right)\right) + \left(E\left({}_{2}X_{t}\right)\right) + \left(E\left({}_{3}X_{t}\right)\right) + \cdots \tag{119} $$

Proof by direct proof. The premise

$$ +1 \equiv +1 \tag{120} $$

is true. In the following, we rearrange the equation 120 (premise or axiom). We obtain

$$ p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{121} $$

Under circumstances where the probability $p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$ is composed out of different individual probabilities ("state of superposition"), it is

$$ p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{0}X_{\mathrm{t}}\right) + p\left({}_{1}X_{\mathrm{t}}\right) + p\left({}_{2}X_{\mathrm{t}}\right) + p\left({}_{3}X_{\mathrm{t}}\right) + \cdots \tag{122} $$

From equation 122 follows, too, that

$$\frac{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv \frac{p\left({}_{0}X_{\mathrm{t}}\right)+p\left({}_{1}X_{\mathrm{t}}\right)+p\left({}_{2}X_{\mathrm{t}}\right)+p\left({}_{3}X_{\mathrm{t}}\right)+\cdots}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \equiv +1$$
$$\equiv \frac{p\left({}_{0}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}+\frac{p\left({}_{1}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}+\frac{p\left({}_{2}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}+\frac{p\left({}_{3}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}+\cdots \equiv +1 \tag{123}$$

There are circumstances in which $p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$ of equation 123 is equal to $p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv +1$. Under these circumstances, the relationships of equation 123 simplifies further to $p\left({}_{0}X_{\mathrm{t}}\right)+p\left({}_{1}X_{\mathrm{t}}\right)+p\left({}_{2}X_{\mathrm{t}}\right)+p\left({}_{3}X_{\mathrm{t}}\right)+\cdots \equiv +1$. However, this requirement need not be given in general. Multiplying equation 122 by $E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$ it is

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$$
$$\equiv \frac{p\left({}_{0}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+\frac{p\left({}_{1}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+\frac{p\left({}_{2}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+\frac{p\left({}_{3}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+\cdots \tag{124}$$

In agreement with equation 105 it is

$$E\left({}_{\mathrm{i}}X_{\mathrm{t}}\right) \equiv \frac{p\left({}_{\mathrm{i}}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{125}$$

Example

$$E\left({}_{0}X_{\mathrm{t}}\right) \equiv \frac{p\left({}_{0}X_{\mathrm{t}}\right)}{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{126}$$

Equation 124 becomes (see also DeGroot and Schervish, 2005, p. 219)

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \left(E\left({}_{0}X_{\mathrm{t}}\right)\right)+\left(E\left({}_{1}X_{\mathrm{t}}\right)\right)+\left(E\left({}_{2}X_{\mathrm{t}}\right)\right)+\left(E\left({}_{3}X_{\mathrm{t}}\right)\right)+\cdots \tag{127}$$

□

3.10. Superposition of probabilities II

Theorem 3.13 (Superposition of probabilities II)**.** *In general, it is*

$$E\left({}_{R}X_{t}\right) \equiv E\left({}_{0}X_{t}\right)+E\left({}_{1}X_{t}\right)+E\left({}_{2}X_{t}\right)+\cdots \tag{128}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{129}$$

is true. In the following, we rearrange the premise. We obtain

$${}_{\mathrm{R}}X_{\mathrm{t}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \tag{130}$$

while $_{R}X_{t}$ might indicate a random variable as seen, regarded, measured et cetera from the point of view of a **stationary observer R** (see also Einstein, 1905) at a Bernoulli trial t (see Uspensky, 1937). Under conditions where a random variable $_{R}X_{t}$ is in the state of superposition, it is

$$_{R}X_{t} \equiv {_{0}X_{t}} + {_{1}X_{t}} + {_{2}X_{t}} + \cdots \tag{131}$$

while $_{0}X_{t}$ might denote the value of the random variable $_{R}X_{t}$ as measured, seen, regarded et cetera from the point of view of a **co-moving observer 0** (see also Einstein, 1905) at a Bernoulli trial t (see Uspensky, 1937). Rearranging equation 131, it is

$$_{R}X_{t} \equiv \frac{_{0}X_{t} \times p\left(_{R}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \frac{_{1}X_{t} \times p\left(_{R}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \frac{_{2}X_{t} \times p\left(_{R}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \cdots \tag{132}$$

It is worth mentioning that equation 132 changes in agreement with equation 96 and equation 105 to

$$_{R}X_{t} \equiv \frac{E\left(_{0}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \frac{E\left(_{1}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \frac{E\left(_{2}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \cdots \tag{133}$$

Simplifying equation 133, it is

$$_{R}X_{t} \times p\left(_{R}X_{t}\right) \equiv E\left(_{0}X_{t}\right) + E\left(_{1}X_{t}\right) + E\left(_{2}X_{t}\right) + \cdots \tag{134}$$

In particular, in order to compute how a wave propagates and behaves like in quantum mechanics, the application of the superposition principle is of advantage. Historically, there is some evidence that **the superposition principle** has been a law which has been re-formulated by the Danish geologist Nicolaus Steno (see Stenonis, Nicolai, 1669) in his 1696 book 'De Solido Intra Naturaliter Contento Dissertationis Prodomus '. and later by Daniel Bernoulli (1700 – 1782) in 1753 ("**Later (1753), Daniel Bernoulli formulated the principle of superposition ...**" (see Leon Brillouin, 1946, p. 2)). In general, the expectation value of the random variable $_{R}X_{t}$ is determined as (see also DeGroot and Schervish, 2005, p. 219)

$$\begin{aligned} E\left(_{R}X_{t}\right) &\equiv E\left(_{0}X_{t} + {_{1}X_{t}} + {_{2}X_{t}} + \cdots\right) \\ &\equiv E\left(_{0}X_{t}\right) + E\left(_{1}X_{t}\right) + E\left(_{2}X_{t}\right) + \cdots \end{aligned} \tag{135}$$

□

In general and as found before (see equation 1), it is

$$E\left(_{R}X_{t}\right) \equiv p\left(_{R}X_{t}\right) \times {_{R}X_{t}} \equiv E\left(_{0}X_{t}\right) + E\left(_{1}X_{t}\right) + E\left(_{2}X_{t}\right) + \cdots \tag{136}$$

Equation 136 changes to

$$_{R}X_{t} \equiv \frac{E\left(_{0}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \frac{E\left(_{1}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \frac{E\left(_{2}X_{t}\right)}{p\left(_{R}X_{t}\right)} + \cdots \tag{137}$$

In general, it is equally (see equation 125)

$$_{R}X_{t} \equiv \left({_{R}X_{t}} \times \frac{p\left(_{0}X_{t}\right)}{p\left(_{R}X_{t}\right)}\right) + \left({_{R}X_{t}} \times \frac{p\left(_{1}X_{t}\right)}{p\left(_{R}X_{t}\right)}\right) + \left({_{R}X_{t}} \times \frac{p\left(_{2}X_{t}\right)}{p\left(_{R}X_{t}\right)}\right) + \cdots \tag{138}$$

3.11. Euclid's theorem normalised

Theorem 3.14. *Euclid's theorem can be normalised. In general, it is*

$$p\left({}_{R}X_{t}\right)+p\left({}_{R}\underline{X}_{t}\right)=+1 \tag{139}$$

Proof. **If** the premise

$$+1=+1 \tag{140}$$

is true, **then** the conclusion

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)=+1 \tag{141}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying equation 140 by $+_{R}X_{t}$ it is

$$+{}_{\mathrm{R}}X_{\mathrm{t}}=+{}_{\mathrm{R}}X_{\mathrm{t}} \tag{142}$$

Rearranging equation 142, we obtain

$$+{}_{\mathrm{R}}X_{\mathrm{t}}-E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)=+{}_{\mathrm{R}}X_{\mathrm{t}}+0 \tag{143}$$

while it is necessary that $E({}_{R}X_{t})$ is for sure one determining part of $+_{R}X_{t}$, whatever $E({}_{R}X_{t})$ and $+_{R}X_{t}$ may denote. In general, we consider without an exception all but $E({}_{R}X_{t})$ at a certain period of or point in time t as anti $E({}_{R}X_{t})$. Anti $E({}_{R}X_{t})$ is denoted by $E({}_{R}\underline{X}_{t})$. Arithmetically, we define $E({}_{R}\underline{X}_{t})$as

$$E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\equiv+\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)-E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{144}$$

Equation 143 changes in perfect agreement with 144. In general and, in particular, it is

$$+E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)={}_{\mathrm{R}}X_{\mathrm{t}} \tag{145}$$

Equation 145 is illustrated in more detail by figure 9. By rearranging equation 145, we obtain the general normalised form of Euclid's theorem as

$$+\left(\frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}}\right)+\left(\frac{E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}}\right)=\left(\frac{{}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}X_{\mathrm{t}}}\right)=+1 \tag{146}$$

From the point of view of geometry, the probability of a single event, an entity, a quantity, a number et cetera is the extent to which $E({}_{R}X_{t})$, this single event, entity, quantity, number et cetera, is a determining part of ${}_{R}X_{t}$. In general, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\equiv\frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}} \tag{147}$$

From the point of view of geometry, the probability of a single anti-event, an anti-entity, an anti-quantity, an anti-number et cetera is the extent to which $E({}_{R}\underline{X}_{t})$, this single anti-event, an anti-entity, an anti-quantity, an anti-number et cetera, is a determining part of ${}_{R}X_{t}$. In general, it is

$$p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\equiv\frac{E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}}=1-\frac{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}}=1-\frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{{}_{\mathrm{R}}X_{\mathrm{t}}}=1-p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{148}$$

Taking into account the previous definitions (equation 147 and equation 148) then equation 146 changes to

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)=+1 \tag{149}$$

□

	Outcome		
	Yes	No	
Variable	$E({}_{0}X_{t})$	$E({}_{0}\underline{X}_{t})$	$E({}_{R}X_{t})$
			$E({}_{R}\underline{X}_{t})$
			${}_{R}X_{t}$

Figure 9. Expectation values at one single Bernoulli trial t.

3.12. Pythagorean theorem and probability theory

It should be remembered, moreover, that Euclid's theorem is related to Thales of Miletus (ca. 624/623–ca.548/545 BCE) theorem. We may now apply Euclid's theorem to the relative latecomer in scientific history, the expectation values (see also fig. 3).

Theorem 3.15 (Pythagorean theorem and probability theory)**.** *In general, the **Pythagorean theorem** as the foundation for geometry and probability theory is given in the language of probability theory* *and statistics by the equation*

$$ {}_{R}X_{t}{}^{2} \equiv E\left({}_{R}X_{t}{}^{2}\right) + E\left({}_{R}\underline{X}_{t}{}^{2}\right) \tag{150} $$

as illustrated by fig. 10.

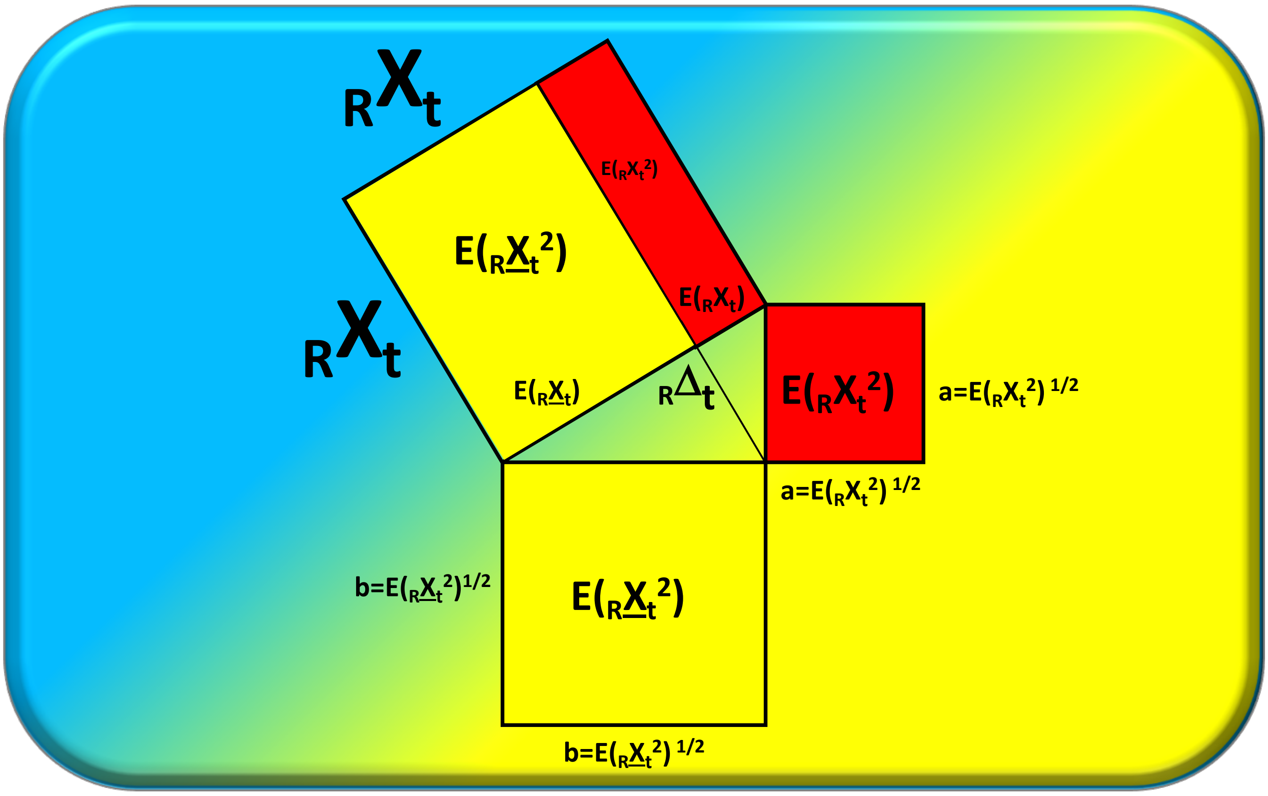

Figure 10. The Pythagorean theorem.

Proof by direct proof. Our premise (i.e. axiom) which is equally the theoretical starting point of this theorem, is that

$$ +1 \equiv +1 \tag{151} $$

is true. Well, obviously this premise is true. In the following, we rearrange this premise (i.e. axiom). We obtain

$$ +1+0 \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) + 1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{152} $$

Rearranging equation 152 , it is

$$ +1 \equiv \frac{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} + \frac{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(+1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)}{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \tag{153} $$

In accordance with equation 8 and equation 9, it is

$$+1 \equiv \frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} + \frac{E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \tag{154}$$

Equation 154 is multiplied by ${}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}$. It is

$${}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \equiv \frac{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} + \frac{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)} \tag{155}$$

or

$${}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) + \left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{156}$$

Equation 156 changes (see equation 5, p. 8) and becomes

$${}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) + \left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{157}$$

Equation 157 changes. The **Pythagorean theorem** follows according to probability theory and statistics as

$${}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) + E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}{}^{2}\right) \tag{158}$$

□

It must also be borne in mind to consider circumstances in which $E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)$ denotes something similar like a 'local hidden variable '.

3.13. Pythagorean theorem and probability theory II

Theorem 3.16 (Pythagorean theorem and probability theory II)**.**

Proof by direct proof. It is

$$+1 \equiv +1 \tag{159}$$

Equation 159 is equal (see equation 139, p. 36) with

$$+1 \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) + p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{160}$$

Multiplying equation 160 by ${}_{\mathrm{R}}C_{\mathrm{t}}{}^{2}$ it is

$${}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \equiv \left({}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right) + \left({}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right) \tag{161}$$

In our opinion, it is

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}C_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \tag{162} $$

Equation 161 changes accordingly. We obtain

$$ {}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \equiv \left({}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}\right) + \left({}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right) \tag{163} $$

In our view, it is

$$ {}_{\mathrm{R}}b_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}C_{\mathrm{t}} \times {}_{\mathrm{R}}\underline{X}_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}{}^{2}\right) \tag{164} $$

Equation 163 becomes

$$ {}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \equiv \left({}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}\right) + \left({}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}\right) \tag{165} $$

□

3.14. Pythagorean theorem and probability theory III

Theorem 3.17 (Pythagorean theorem and probability theory III)**.** *In general, it is*

$$ \frac{{}_{R}X_{t}}{{}_{R}C_{t}} \equiv p\left({}_{R}X_{t}\right) \tag{166} $$

Proof by direct proof. The premise

$$ +1 \equiv +1 \tag{167} $$

is true. Rearranging equation 167, it is

$$ \left({}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}\right) \equiv \left({}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}\right) \tag{168} $$

In general, it is (see equation 162, p. 40) ${}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv {}_{\mathrm{R}}C_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)$ Equation 168 changes slightly and becomes

$$ \left({}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}\right) \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \tag{169} $$

Dividing equation 169, we obtain

$$ \frac{\left({}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}\right)}{{}_{\mathrm{R}}C_{\mathrm{t}}{}^{2}} \equiv \frac{E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)}{{}_{\mathrm{R}}C_{\mathrm{t}}{}^{2}} \tag{170} $$

Simplifying equation 170, it is

$$ \frac{{}_{\mathrm{R}}C_{\mathrm{t}} \times {}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}C_{\mathrm{t}} \times {}_{\mathrm{R}}C_{\mathrm{t}}} \equiv \frac{{}_{\mathrm{R}}C_{\mathrm{t}}{}^{2} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)}{{}_{\mathrm{R}}C_{\mathrm{t}} \times {}_{\mathrm{R}}C_{\mathrm{t}}} \tag{171} $$

The reasoning before is formal and logically consistent from the beginning. In general, it is

$$ \frac{{}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}C_{\mathrm{t}}} \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{172} $$

□

Our sun has risen every day for a long time in the past. However, the question is justified, will the same sun rise tomorrow too, for sure? In the light of such empirical facts, any inference from the known or observed to the unknown or unobserved, has become known as “inductive inferences”. The phrase **per modum inductionis** (see Wallisii, 1656) has been coined by John Wallis in 1656 in his book Arithmetica Infinitorum. Inductive inference is often overshadowed by the possibility of being mistaken and is associated with a certain level of significance (see Arbuthnot, John, 1710, Venn, 1888), often denoted as the p-value (see Pearson, 1900). Historically, it was especially David Hume [16] who put into question in his 1739 Book ‘A Treatise of Human Nature, part iii, section 6’ (see Hume, 1739) any justification in which humans form knowledge which became known as Hume’s ‘problem of induction ’.

Theorem 3.18 (The approximate probability of an event).

In general, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \exp^{-\left(p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)} \tag{173}$$

Proof. **If** the premise

$$+1 = +1 \tag{174}$$

is true, **then** the conclusion

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \exp^{-\left(p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)} \tag{175}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying equation 174 by the probability $p({}_{R}X_{t})$ of an event ${}_{R}X_{t}$ at the (period of) time / Bernoulli trial t, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{176}$$

Equation 176 changes according to equation 149 into

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \left(+1 - p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right) \tag{177}$$

Let us assume that the probability is constant from trial to trial, from experiment to experiment, it is equally

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \left(+1 - \left(\frac{n \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right) \tag{178}$$

While the number of observations increases, we obtain the following picture too.

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \equiv \left(+1 - \left(\frac{n \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right)^{\mathrm{n}} \tag{179}$$

[16] Henderson, Leah, ”The Problem of Induction”, The Stanford Encyclopedia of Philosophy (Spring 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2020/entries/induction-problem/.

Equation 179 can be simplified as

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \equiv \left(+1-\left(\frac{E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right)^{\mathrm{n}} \tag{180}$$

From elementary calculus (see also DeGroot and Schervish, 2005, p. 195) it is known that

$$\lim_{n \to +\infty}\left(+1-\left(\frac{E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}\right)\right)^{\mathrm{n}} \equiv \exp^{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)} \tag{181}$$

According to equation 181, equation 180 is rearranged as

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \equiv \exp^{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)} \tag{182}$$

The probability of a single event follows as

$$\begin{aligned} p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) &\equiv \sqrt[n]{p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}}} \\ &\equiv \sqrt[n]{\exp^{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}} \\ &\equiv \exp^{\frac{-E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)}{n}} \\ &\equiv \exp^{-\frac{\left(n \times p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)}{n}} \end{aligned} \tag{183}$$

Finally, there are circumstances where the probability of a single event (see Barukčić, 2019c, pp. 1843-1844) is given by the equation

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \exp^{-\left(p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)} \tag{184}$$

□

3.16. The quantum mechanical measurement problem

One of the many (philosophical) issues [17] raised by quantum theory is the measurement problem. Nonetheless, even a measuring apparatus which is working properly is not the only possible factor able to disturb 'the true vale of ${}_{R}X_{t}$'. Other, and mostly even unknown events/factors (i.e. super-nova explosion somewhere in deep space, gravitational waves) or other violent chain of events et cetera might possess the possibility to determine the value of a random variable ${}_{R}X_{t}$ too. How many factors determine the value of a random variable ${}_{R}X_{t}$and to what extent at the Bernoulli trial t? Is it possible at all to know anything with absolute certainty [18] (i.e. objectivism vs relativism), and who is the one who knows anything with absolute certainty? Consideration should be given to a new, useful and logically consistent approximation to the quantum mechanical measurement problem (see John von Neumann's projection postulate) [19] on all scales to the extent necessary.

Theorem 3.19 (The quantum mechanical measurement problem)**.** *In general, there are conditions, where the quantum mechanical measurement problem is described completely by the equation*

$$\Psi\left({}_{R}X_{t}\right) \equiv A\left({}_{R}X_{t}\right) \times \exp^{-\left(p\left({}_{R}\underline{X}_{t}\right)\right)} \tag{185}$$

Proof by direct proof. Based on Born's (see Born, 1926) rule (see equation 8, p. 10), we have to consider the relationship

$$p\left({}_{R}X_{t}\right) \equiv \Psi\left({}_{R}X_{t}\right) \times \Psi^{*}\left({}_{R}X_{t}\right) \tag{186}$$

Equation 186 changes to (see equation 184, p. 42)

$$\Psi\left({}_{R}X_{t}\right) \times \Psi^{*}\left({}_{R}X_{t}\right) \equiv \exp^{-\left(p\left({}_{R}\underline{X}_{t}\right)\right)} \tag{187}$$

The wave function of any random variable ${}_{R}X_{t}$ is determined approximately as

$$\Psi\left({}_{R}X_{t}\right) \equiv \frac{1}{\Psi^{*}\left({}_{R}X_{t}\right)} \times \exp^{-\left(p\left({}_{R}\underline{X}_{t}\right)\right)} \tag{188}$$

We define the amplitude as $A\left({}_{R}X_{t}\right) \equiv \frac{1}{\Psi^{*}\left({}_{R}X_{t}\right)}$. Under these assumptions, the general form of a wave function of any random variable ${}_{R}X_{t}$ follows approximately as

$$\Psi\left({}_{R}X_{t}\right) \equiv A\left({}_{R}X_{t}\right) \times \exp^{-\left(p\left({}_{R}\underline{X}_{t}\right)\right)} \tag{189}$$

while $p\left({}_{R}\underline{X}_{t}\right)$ is the probability or the amount to which a local hidden parameter, the measuring apparatus et cetera is a determining part of the wave function of any random variable ${}_{R}X_{t}$.

□

Stock market and other prices are very vulnerable. Many factors are influencing the same random variable, too. Equation 189 should be able to cope even with these circumstances.

[17] Myrvold, Wayne, "Philosophical Issues in Quantum Theory", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2018/entries/qt-issues/.

[18] Kant, Immanuel. Critic der reinen Vernunft. Dritte Auflage. Riga: Johann Friedrich Hartknoch; 1790. p. 884.

[19] Faye, Jan, "Copenhagen Interpretation of Quantum Mechanics", The Stanford Encyclopedia of Philosophy (Winter 2019 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2019/entries/qm-copenhagen/.

3.17. The normalised Pythagorean theorem

In the following, we assume that ${}_{R}X_{t}{}^{2} \equiv {}_{R}C_{t}{}^{2}$.

Theorem 3.20. *The normalised Pythagorean theorem is determined as*

$$\frac{{}_{R}a_{t}{}^{2}}{{}_{R}X_{t}{}^{2}} + \frac{{}_{R}b_{t}{}^{2}}{{}_{R}X_{t}{}^{2}} \equiv +1^{+2} \tag{190}$$

Proof. **If** the premise

$$+1 \equiv +1 \tag{191}$$

is true, **then** the conclusion

$$\frac{{}_{R}a_{t}{}^{2}}{{}_{R}X_{t}{}^{2}} + \frac{{}_{R}b_{t}{}^{2}}{{}_{R}X_{t}{}^{2}} \equiv +1^{+2} \tag{192}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. The Pythagorean theorem is proofed (see equation 51) as ${}_{R}a_{t}{}^{2} + {}_{R}b_{t}{}^{2} \equiv {}_{R}X_{t}{}^{2}$. Equation 191 changes to

$${}_{R}X_{t}{}^{2} \equiv {}_{R}X_{t}{}^{2} \tag{193}$$

and finally to

$${}_{R}a_{t}{}^{2} + {}_{R}b_{t}{}^{2} \equiv {}_{R}X_{t}{}^{2} \tag{194}$$

Dividing equation 194 by ${}_{R}X_{t}{}^{2}$ under conditions where this is possible and allowed, we obtain

$$\frac{{}_{R}a_{t}{}^{2}}{{}_{R}X_{t}{}^{2}} + \frac{{}_{R}b_{t}{}^{2}}{{}_{R}X_{t}{}^{2}} \equiv \frac{{}_{R}X_{t}{}^{2}}{{}_{R}X_{t}{}^{2}} + 1^{+2} \tag{195}$$

□

Theorem 3.21. *In the following, we assume that ${}_{R}X_{t}{}^{2} \equiv {}_{R}C_{t}{}^{2}$. In general, ${}_{R}a_{t}$ is the negation of ${}_{R}b_{t}$ and vice versa. It is*

$${}_{R}a_{t}{}^{2} = \neg\left({}_{R}b_{t}\right) \times {}_{R}X_{t}{}^{2} \tag{196}$$

Proof. **If** the premise

$$+1 = +1 \tag{197}$$

is true, **then** the conclusion

$${}_{R}a_{t} = \sqrt[2]{\left(+1^{+2} - \frac{{}_{R}b_{t}{}^{2}}{{}_{R}X_{t}{}^{2}}\right)} \times {}_{R}X_{t} \tag{198}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. equation 197 is rearranged as

$$+1^{+2} = +1^{+2} \tag{199}$$

The normalised form of the Pythagorean theorem is proofed as (see theorem 3.20, equation 195) as $\frac{{}_{R}a_{t}^{2}}{{}_{R}X_{t}^{2}} + \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}} = +1^{+2}$. Equation 199 changes to

$$\frac{{}_{R}a_{t}^{2}}{{}_{R}X_{t}^{2}} + \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}} = +1^{+2} \tag{200}$$

Rearranging equation 200

$$\frac{{}_{R}a_{t}^{2}}{{}_{R}X_{t}^{2}} + = \left(+1^{+2} - \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}}\right) \tag{201}$$

Simplifying equation 201, it is

$${}_{R}a_{t}^{2} \times +1^{+2} = \left(+1^{+2} - \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}}\right) \times {}_{R}X_{t}^{2} \tag{202}$$

Equation 202 changes to

$${}_{R}a_{t}^{2} = \left(+1^{+2} - \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}}\right) \times {}_{R}X_{t}^{2} \tag{203}$$

and to

$${}_{R}a_{t} = \sqrt[2]{\left(+1^{+2} - \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}}\right)} \times {}_{R}X_{t} \tag{204}$$

We define in general

$$\neg\left({}_{R}b_{t}\right) \equiv \left(+1^{+2} - \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}}\right) \tag{205}$$

Equation 203 changes to

$${}_{R}a_{t}^{2} = \neg\left({}_{R}b_{t}\right) \times {}_{R}X_{t}^{2} \tag{206}$$

□

The negation of ${}_{R}b_{t}$ need to be calculated similarly. We will obtain

$$\neg\left({}_{R}a_{t}\right) \equiv \left(+1^{+2} - \frac{{}_{R}a_{t}^{2}}{{}_{R}X_{t}^{2}}\right) \tag{207}$$

Under conditions of Einstein's special relativity where ${}_{R}a_{t}$ does denote the rest-mass and where ${}_{R}X_{t}$ does denote the relativistic mass, we obtain the identity with reciprocal Lorentz factor or Lorentz term (see also Lorentz, 1899, p. 432) as $\left(\sqrt[2]{\left(+1^{+2} - \frac{{}_{R}b_{t}^{2}}{{}_{R}X_{t}^{2}}\right)}\right) \equiv \left(\sqrt[2]{\left(1 - \frac{v^{2}}{c^{2}}\right)}\right)$ (see also Barukčić, 2019a).

3.18. The n-dimensional Pythagorean theorem

The metric tensor in general relativity theory generalizes the Pythagorean theorem more or less to non-Euclidean geometries. However, the n-dimensional Pythagorean theorem can be derived in another, simple and logically consistent way too. In the following, we assume that ${}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \equiv {}_{\mathrm{R}}C_{\mathrm{t}}{}^{2}$.

Theorem 3.22. *The n-dimensional Pythagorean theorem is determined as*

$$ {}_{R}a_{t}{}^{2n} + {}_{R}b_{t}{}^{2n} \equiv {}_{R}X_{t}{}^{2n} \tag{208}$$

Proof. **If** the premise

$$ +1 = +1 \tag{209}$$

is true, **then** the conclusion

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2\mathrm{n}} + {}_{\mathrm{R}}b_{\mathrm{t}}{}^{2\mathrm{n}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2\mathrm{n}} \tag{210}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Equation 209 change quickly to

$$ {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} = {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \tag{211}$$

Rearranging equation 211 by equation 190 derived as $\left(\frac{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} = {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)$ yields

$$ \frac{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} = {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \tag{212}$$

Several properties of the Pythagorean theorem are already identified. In general, it is proofed that

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2} \equiv E({}_{\mathrm{R}}X_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{213}$$

or that

$$ {}_{\mathrm{R}}a_{\mathrm{t}}{}^{2\mathrm{n}} \equiv E({}_{\mathrm{R}}X_{\mathrm{t}})^{\mathrm{n}} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \equiv (E({}_{\mathrm{R}}X_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}})^{\mathrm{n}} \tag{214}$$

Furthermore, it is

$$ {}_{\mathrm{R}}b_{\mathrm{t}}{}^{2} \equiv E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}} \tag{215}$$

and equally

$$ {}_{\mathrm{R}}b_{\mathrm{t}}{}^{2\mathrm{n}} \equiv E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})^{\mathrm{n}} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \equiv (E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}})^{\mathrm{n}} \tag{216}$$

where n might denote the number of dimensions. Rearranging equation 212 according to the relationship of equation 213 it is

$$ \frac{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + \frac{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} = +1^{2} \tag{217}$$

Rearranging equation 217 according to the relationship of equation 215 it is

$$ \frac{E({}_{\mathrm{R}}X_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} + \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \times {}_{\mathrm{R}}X_{\mathrm{t}}}{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}} = +1^{2} \tag{218}$$

Equation 218 simplifies further, as

$$\frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} + \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} = +1^2 \tag{219}$$

Figure 11 illustrates these relationships in more detail.

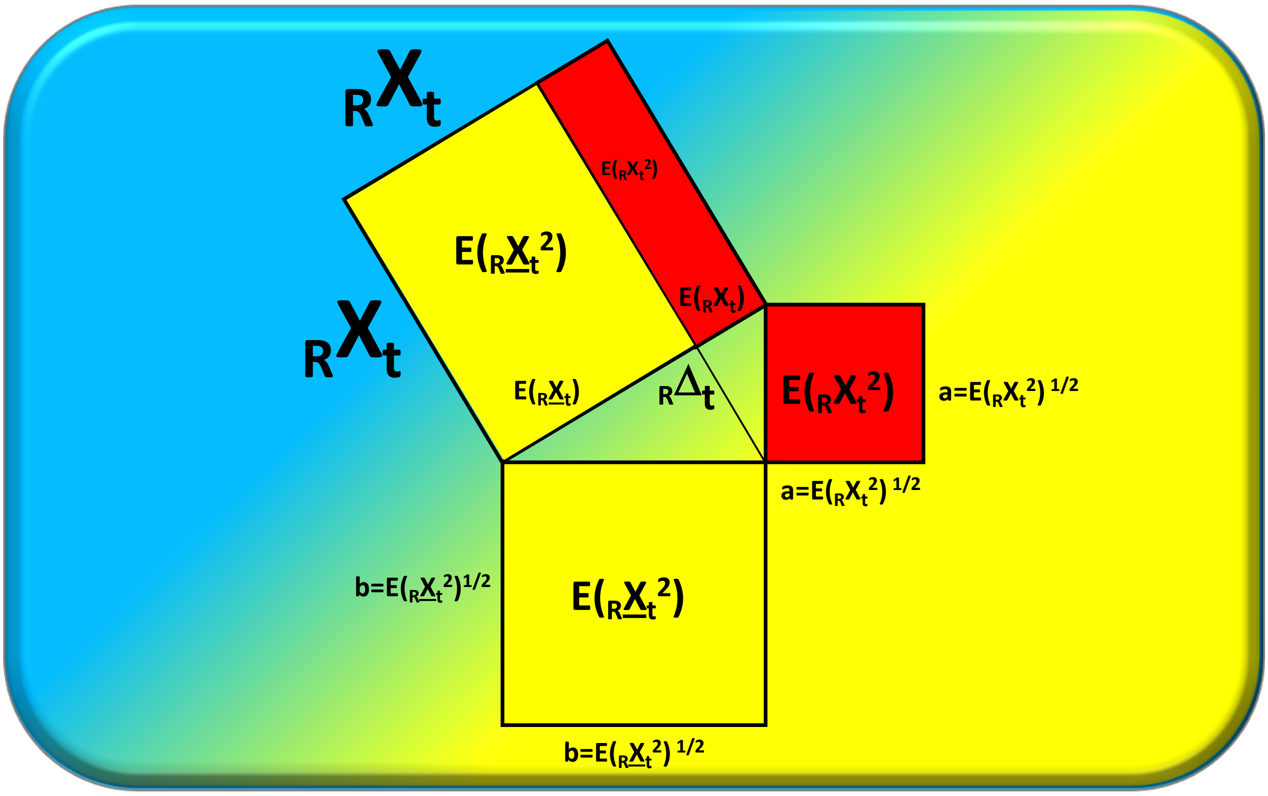

Figure 11. Pythagorean theorem and expectation values.

Simplifying equation 219 it is

$$E({}_{\mathrm{R}}X_{\mathrm{t}}) + E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) = {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times 1^1 \times 1^1 = ({}_{\mathrm{R}}X_{\mathrm{t}} \times 1 \times 1)^1 = {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} = {}_{\mathrm{R}}X_{\mathrm{t}} \tag{220}$$

As known, it is $\left(U^1 \times U^0 \equiv U^{+1} \equiv U\right)$. However, equation 220 simplifies further. The most simple and most general form of the Pythagorean theorem (see Barukčić, 2016) is based on the fundamental relationship,

$$E({}_{\mathrm{R}}X_{\mathrm{t}}) + E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}) \equiv {}_{\mathrm{R}}X_{\mathrm{t}} \tag{221}$$

In particular, the Pythagorean theorem can be extended to higher dimensions (see Yeng et al., 2008) too. In the n-dimensional case (see Barukčić, 2020b), the relationship before becomes

$$\left(E({}_{\mathrm{R}}X_{\mathrm{t}}) + E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})\right)^{\mathrm{n}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \tag{222}$$

Multiplying equation 222 by ${}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}$, the Pythagorean theorem becomes something like

$$\left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)+E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right)\right)^{\mathrm{n}} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \tag{223}$$

or as

$$\underbrace{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{n}} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}}_{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2\mathrm{n}}} + \underbrace{\ldots}_{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2\mathrm{n}}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2\mathrm{n}} \tag{224}$$

In general, the n-dimensional Pythagorean theorem is determined as

$${}_{\mathrm{R}}a_{\mathrm{t}}{}^{2\mathrm{n}} + {}_{\mathrm{R}}b_{\mathrm{t}}{}^{2\mathrm{n}} \equiv {}_{\mathrm{R}}X_{\mathrm{t}}{}^{2\mathrm{n}} \tag{225}$$

□

3.19. Pythagorean theorem and the wave function Ψ

Especially in order to compute how a wave propagates and behaves like in quantum mechanics, the application of the superposition principle is of advantage. There is some evidence that **the superposition principle** has been stated by Daniel Bernoulli (1700 – 1782) in 1753 ("**Later (1753), Daniel Bernoulli formulated the principle of superposition ...**" (see Leon Brillouin, 1946, p. 2)).

Theorem 3.23. *In general, it is*

$$\left(\frac{\Psi({}_{R}X_{t}) \times \Psi(E({}_{R}X_{t}))}{\Psi({}_{R}X_{t}) \times \Psi({}_{R}X_{t})}\right) + \left(\frac{\Psi({}_{R}X_{t}) \times \Psi(E({}_{R}\underline{X}_{t}))}{\Psi({}_{R}X_{t}) \times \Psi({}_{R}X_{t})}\right) \equiv +1^{2} \tag{226}$$

Proof. **If** the premise

$$+1 = +1 \tag{227}$$

is true, **then** the conclusion

$$\left(\frac{\Psi({}_{R}X_{t}) \times \Psi(E({}_{R}X_{t}))}{\Psi({}_{R}X_{t}) \times \Psi({}_{R}X_{t})}\right) + \left(\frac{\Psi({}_{R}X_{t}) \times \Psi(E({}_{R}\underline{X}_{t}))}{\Psi({}_{R}X_{t}) \times \Psi({}_{R}X_{t})}\right) \equiv +1^{2} \tag{228}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying equation 227 by ${}_{R}X_{t}$, it is

$${}_{R}X_{t} = {}_{R}X_{t} \tag{229}$$

Based on theorem 3.22, equation 221, equation 229 changes to

$$E({}_{R}X_{t}) + E({}_{R}\underline{X}_{t}) \equiv {}_{R}X_{t} \tag{230}$$

Theoretically it is necessary to consider the possibility that there are conditions where ${}_{R}X_{t}$ is in a state of superposition of $E({}_{R}X_{t})$ and $E({}_{R}\underline{X}_{t})$. Thus far, under conditions where equation 230 can be described by a (linear) function $\Psi({}_{R}X_{t})$ which satisfies the superposition principle, it is equally

$$\Psi(E({}_{R}X_{t})) + \Psi(E({}_{R}\underline{X}_{t})) \equiv \Psi(E({}_{R}X_{t}) + E({}_{R}\underline{X}_{t})) \equiv \Psi({}_{R}X_{t}) \tag{231}$$

The principle of superposition and the Pythagorean theorem are the two sides of the same coin. It is

$$\Psi(E({}_{R}X_{t})) + \Psi(E({}_{R}\underline{X}_{t})) \equiv \Psi({}_{R}X_{t}) \tag{232}$$

Normalizing the relationship before, equation 232 changes slightly. It is

$$\frac{\Psi(E({}_{R}X_{t}))}{\Psi({}_{R}X_{t})} + \frac{\Psi(E({}_{R}\underline{X}_{t}))}{\Psi({}_{R}X_{t})} \equiv \frac{\Psi({}_{R}X_{t})}{\Psi({}_{R}X_{t})} \equiv +1^{1} \tag{233}$$

Multiplying equation 233 by $\left(\frac{\Psi({}_{R}X_{t})}{\Psi({}_{R}X_{t})}\right)$ it is,

$$\left(\frac{\Psi({}_{R}X_{t}) \times \Psi(E({}_{R}X_{t}))}{\Psi({}_{R}X_{t}) \times \Psi({}_{R}X_{t})}\right) + \left(\frac{\Psi({}_{R}X_{t}) \times \Psi(E({}_{R}\underline{X}_{t}))}{\Psi({}_{R}X_{t}) \times \Psi({}_{R}X_{t})}\right) \equiv +1^{2} \tag{234}$$

□

Theorem 3.24 (The general contradiction law). *In general, it is*

$$E\left({}_{R}\underline{X}_{t}\right) \leq \left(\frac{{}_{R}X_{t}{}^{2} \times \pi^{2} \times \hbar^{2}}{E\left({}_{R}X_{t}\right) \times h^{2}}\right) \tag{235}$$

Proof. **If** the premise

$$+1 = +1 \tag{236}$$

is true, **then** the conclusion

$$E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \leq \left(\frac{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{2} \times \pi^{2} \times \hbar^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times h^{2}}\right) \tag{237}$$

is also true, the absence of any technical errors and other errors of human reasoning presupposed. The starting point of this proof (premise: +1 = +1) is true. Multiplying equation 236 by the variance of ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ denoted as $\sigma({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})^{2}$ (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42), it is

$$\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} \equiv \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} \tag{238}$$

The variance of ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$, denoted as $\sigma({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})^{2}$ (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42), is defined or has been proved as

$$\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} \equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \tag{239}$$

In general, according to theorem 3.14, equation 147, it is

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv \frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} \tag{240}$$

while theorem 3.14, equation 148 demands that

$$p\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \equiv \frac{E({}_{\mathrm{R}}\underline{X}_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} = 1 - \frac{E({}_{\mathrm{R}}X_{\mathrm{t}})}{{}_{\mathrm{R}}X_{\mathrm{t}}} = 1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{241}$$

Therefore, equation 239 changes to

$$\begin{aligned} \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} &\equiv \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \\ &\equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}} - E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)^{2} \\ &\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \times \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(1 - \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)\right)\right) \\ &\equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \end{aligned} \tag{242}$$

equation 242 simplifies as

$$\begin{aligned} \sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2} &\equiv E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}\right) \\ &\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \times \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(1 - \left(p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)\right)\right) \end{aligned} \tag{243}$$

Under conditions, where the probability of a single event is not known, it is

$$(p({}_{R}X_{t}) \times (1-(p({}_{R}X_{t})))) \leq \frac{1}{4} \tag{244}$$

equation 243 changes slightly to

$$\frac{E({}_{R}X_{t}) \times E({}_{R}\underline{X}_{t})}{{}_{R}X_{t}{}^{2}} \leq \left(\frac{1}{2} \times \frac{1}{2}\right) \tag{245}$$

From quantum theory, it is known that

$$\frac{1}{2} \equiv \frac{\pi \times \hbar}{h} \tag{246}$$

equation 245 changes to

$$\frac{E({}_{R}X_{t}) \times E({}_{R}\underline{X}_{t})}{{}_{R}X_{t}{}^{2}} \leq \left(\frac{\pi^{2} \times \hbar^{2}}{h^{2}}\right) \tag{247}$$

The expectation value of anti ${}_{R}X_{t}$, denoted as $E({}_{R}\underline{X}_{t})$, follows approximately as

$$E({}_{R}\underline{X}_{t}) \leq \left(\frac{{}_{R}X_{t}{}^{2} \times \pi^{2} \times \hbar^{2}}{E({}_{R}X_{t}) \times h^{2}}\right) \tag{248}$$

□

Equation 248 does not give any reason for the assumption that there is a kind of uncertainty between ${}_{R}X_{t}$ and ${}_{R}\underline{X}_{t}$ and do not constitute in no way a new uncertainty principle. Under conditions of 4 space-time dimensions of general relativity, it is

$$\frac{1}{g_{\mu\nu} \times g^{\mu\nu}} \equiv \frac{1}{4} \tag{249}$$

Equation 245 changes under these conditions of general relativity to

$$\frac{E({}_{R}X_{t}) \times E({}_{R}\underline{X}_{t})}{{}_{R}X_{t}{}^{2}} \leq \frac{1}{g_{\mu\nu} \times g^{\mu\nu}} \tag{250}$$

or to

$${}_{R}X_{t}{}^{2} \geq E({}_{R}X_{t}) \times g_{\mu\nu} \times E({}_{R}\underline{X}_{t}) \times g^{\mu\nu} \tag{251}$$

Furthermore, under conditions where

$$E({}_{R}X_{t}) + E({}_{R}\underline{X}_{t}) \equiv {}_{R}X_{t} \tag{252}$$

we obtain, the identity (see also Barukčić, 2020a,b, 2021) of

$${}_{R}\Delta_{t}{}^{2} \equiv \sigma({}_{R}X_{t})^{2} \tag{253}$$

Especially, general relativity is related to the Pythagorean theorem. General relativity is a theory of the geometrical properties of space-time to, while the metric tensor $g_{\mu\nu}$ itself is of fundamental importance for general relativity. An important differentiation with respect to the metric tensor $g_{\mu\nu}$ is necessary. The metric tensor $g_{\mu\nu}$ does not describe above all the gravitational field, but the gravitational potential. Einstein himself worded this fact excellently.

"... die ... Komponenten des Gravitationspotentials $g_{\mu\nu}$ ... "

(see also Einstein, 1916, p. 818)

In English: '... the ... components of the gravitational potential $g_{\mu\nu}$ ...'. The metric tensor $g_{\mu\nu}$ is something like the generalization of the Pythagorean theorem. Thus far, it does not appear to be necessary to restrict the validity of the Pythagorean theorem only to certain situations. The question is justified why the Riemannian geometry should be oppressed by the quadratic restriction. In this context, **Finsler geometry**, named after Paul Finsler (1894 - 1970) who studied it in his doctoral thesis (see Finsler, 1918) in 1918, appears to be a kind of metric generalization of Riemannian geometry without the quadratic restriction and justifies the attempt to systematize and to extend the possibilities of general relativity.

Theorem 3.25 (**Pythagorean theorem and probablitiy of an event**).

Under conditions of special theory of relativity (see also Einstein, 1905), the probability that ${}_{R}E_{t}$ is determined by ${}_{0}E_{t}$ is given by

$$p\left({}_{R}E_{t}\right) \equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right) \tag{254}$$

by direct proof. According to equation 213 on page 46 it is

$${}_{R}a_{t}{}^{2} \equiv E\left({}_{R}X_{t}\right) \times {}_{R}X_{t} \tag{255}$$

equation 255 is equivalent with

$${}_{R}a_{t}{}^{2} \equiv p\left({}_{R}X_{t}\right) \times {}_{R}X_{t} \times {}_{R}X_{t} \tag{256}$$

Dividing equation 256 by ${}_{R}X_{t}{}^{2}$ it is

$$\frac{{}_{R}a_{t}{}^{2}}{\left({}_{R}X_{t} \times {}_{R}X_{t}\right)} \equiv \frac{p\left({}_{R}X_{t}\right) \times \left({}_{R}X_{t} \times {}_{R}X_{t}\right)}{\left({}_{R}X_{t} \times {}_{R}X_{t}\right)} \equiv p\left({}_{R}X_{t}\right) \tag{257}$$

Let us consider conditions of the special theory of relativity where ${}_{R}a_{t}{}^{2} \equiv {}_{0}E_{t}{}^{2} \equiv {}_{R}E_{t}{}^{2} \times \left(+1-\left(\frac{v^2}{c^2}\right)\right)$. Furthermore, there are conditions where ${}_{R}X_{t} \equiv {}_{R}E_{t}$ and it follows that equation 257 changes to

$$\begin{aligned} p\left({}_{R}E_{t}\right) &\equiv \frac{{}_{R}a_{t}{}^{2}}{\left({}_{R}X_{t} \times {}_{R}X_{t}\right)} \\ &\equiv \frac{{}_{0}E_{t}{}^{2}}{\left({}_{R}E_{t}{}^{2}\right)} \equiv \frac{{}_{R}E_{t}{}^{2} \times \left(+1-\left(\frac{v^2}{c^2}\right)\right)}{\left({}_{R}E_{t}{}^{2}\right)} \\ &\equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right) \end{aligned} \tag{258}$$

Under conditions of special theory of relativity, the probability that total energy (relativistic energy et cetera) ${}_{R}E_{t}$ is determined by the rest-energy ${}_{0}E_{t}$ is given by

$$p\left({}_{R}E_{t} = {}_{0}E_{t}\right) \equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right) \tag{259}$$

□

Remark 3.1. *It need not be noisy to consider whether there exist any circumstances which might permit us to conclude that* $p\left({}_{R}E_{t} = {}_{0}E_{t}\right) \equiv \left(+1-\left(\frac{v^2}{c^2}\right)\right)$ *indicates the probability to which a quantum mechanical entity can be regarded as being local.*

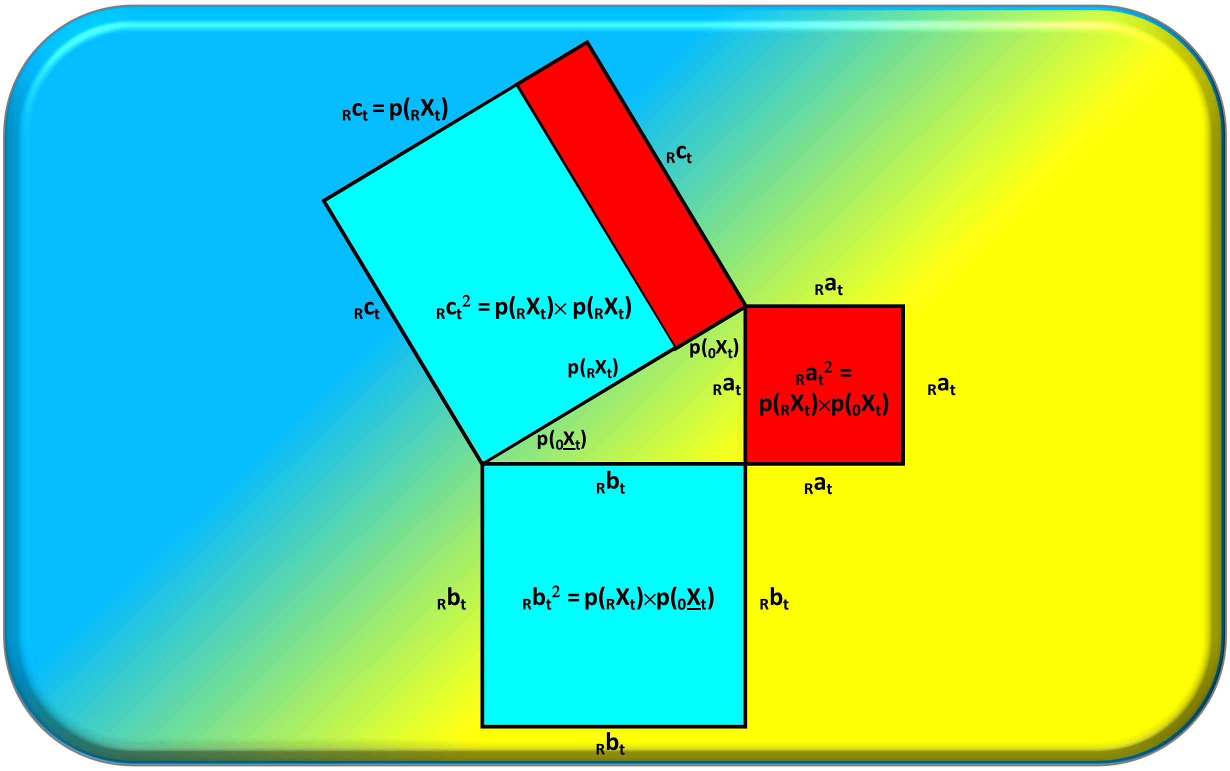

Figure 12. Pythagorean theorem and right-angled triangle. ${}_{R}c_{t}$ is hypotenuse, ${}_{R}a_{t}$ and ${}_{R}b_{t}$ are called triangles legs.

3.22. Slope (i.e. Gradient)

Theorem 3.26 (Slope (i.e. Gradient))**.** *A straight line between any two points can be expressed by probability theory too. Usually, a difference between the y-coordinates between two points (A,B) is called the rise. At the same time, the difference between the x-coordinates of the same two points (A,B) is called the run. The slope (i.e. gradient) can be calculated by dividing the rise (${}_{R}a_{t}$) by run ((${}_{R}b_{t}$)). In general, it is*

$$Slope \equiv \frac{Rise}{Run} \equiv \frac{{}_{R}a_{t}}{{}_{R}b_{t}} \tag{260}$$

Figure 13 explores the rise over run relationship (or slope formula) in more detail.

Proof by direct proof. It is

$$Slope \equiv \frac{Rise}{Run} \equiv \frac{{}_{\mathrm{R}}a_{\mathrm{t}}}{{}_{\mathrm{R}}b_{\mathrm{t}}} \equiv \sqrt[2]{\frac{{}_{\mathrm{R}}a_{\mathrm{t}}{}^{2}}{{}_{\mathrm{R}}b_{\mathrm{t}}{}^{2}}} \tag{261}$$

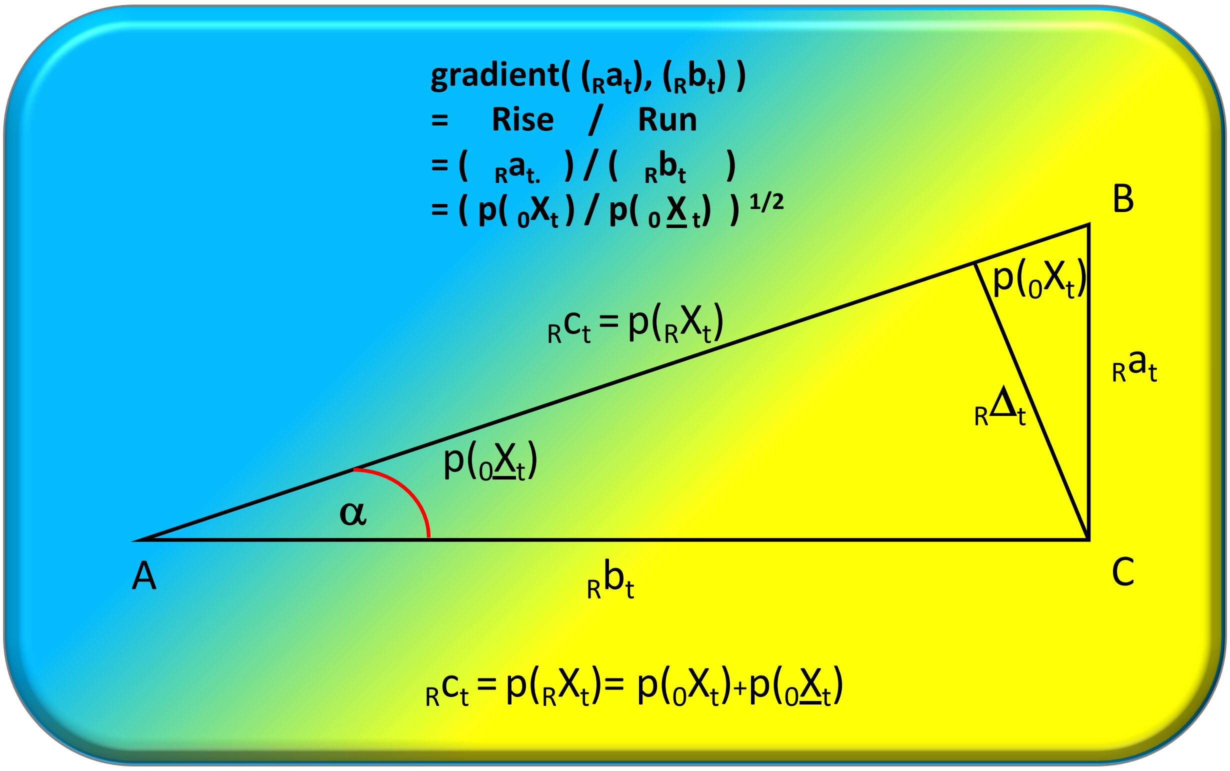

Figure 13. Right-angled triangle. p($_{R}X_{t}$) is hypotenuse, $_{R}a_{t}$ and $_{R}b_{t}$ are called triangles legs.

It is

$$_{R}a_{t}{}^{2} \equiv E\left({}_{R}X_{t}\right) \times E\left({}_{0}X_{t}\right) \tag{262}$$

and

$$_{R}b_{t}{}^{2} \equiv E\left({}_{R}X_{t}\right) \times E\left({}_{0}\underline{X}_{t}\right) \tag{263}$$

Equation 261 becomes

$$Slope \equiv \frac{Rise}{Run} \equiv \frac{{}_{R}a_{t}}{{}_{R}b_{t}} \equiv \sqrt[2]{\frac{{}_{R}a_{t}{}^{2}}{{}_{R}b_{t}{}^{2}}} \equiv \sqrt[2]{\frac{E\left({}_{R}X_{t}\right) \times E\left({}_{0}X_{t}\right)}{E\left({}_{R}X_{t}\right) \times E\left({}_{0}\underline{X}_{t}\right)}} \tag{264}$$

or

$$Slope \equiv \sqrt[2]{\frac{E\left({}_{R}X_{t}\right) \times E\left({}_{0}X_{t}\right)}{E\left({}_{R}X_{t}\right) \times E\left({}_{0}\underline{X}_{t}\right)}} \tag{265}$$

and simplifies finally as

$$Slope \equiv \sqrt[2]{\frac{E\left({}_{0}X_{t}\right)}{E\left({}_{0}\underline{X}_{t}\right)}} \tag{266}$$

As found before, equation 266 changes slightly to

$$Slope \equiv \sqrt[2]{\frac{{}_{R}X_{t} \times p\left({}_{0}X_{t}\right)}{{}_{R}X_{t} \times p\left({}_{0}\underline{X}_{t}\right)}} \tag{267}$$

Under conditions of probability theory, the slope is given by the equation

$$Slope \equiv \sqrt[2]{\frac{p\left({}_{0}X_{t}\right)}{p\left({}_{0}\underline{X}_{t}\right)}} \tag{268}$$

□

The slope is either increasing, decreasing, horizontal or vertical. In general, it is necessary to consider negative probabilities too. The mathematics of general relativity theory is very complex. However, the slope is one of the foundations of the metric tensor of general relativity theory. Equation 268 enables as a final consequence a complete description of the curvature of space-time of general relativity by the tools of probability theory, as indicated to some extent by figure 14.

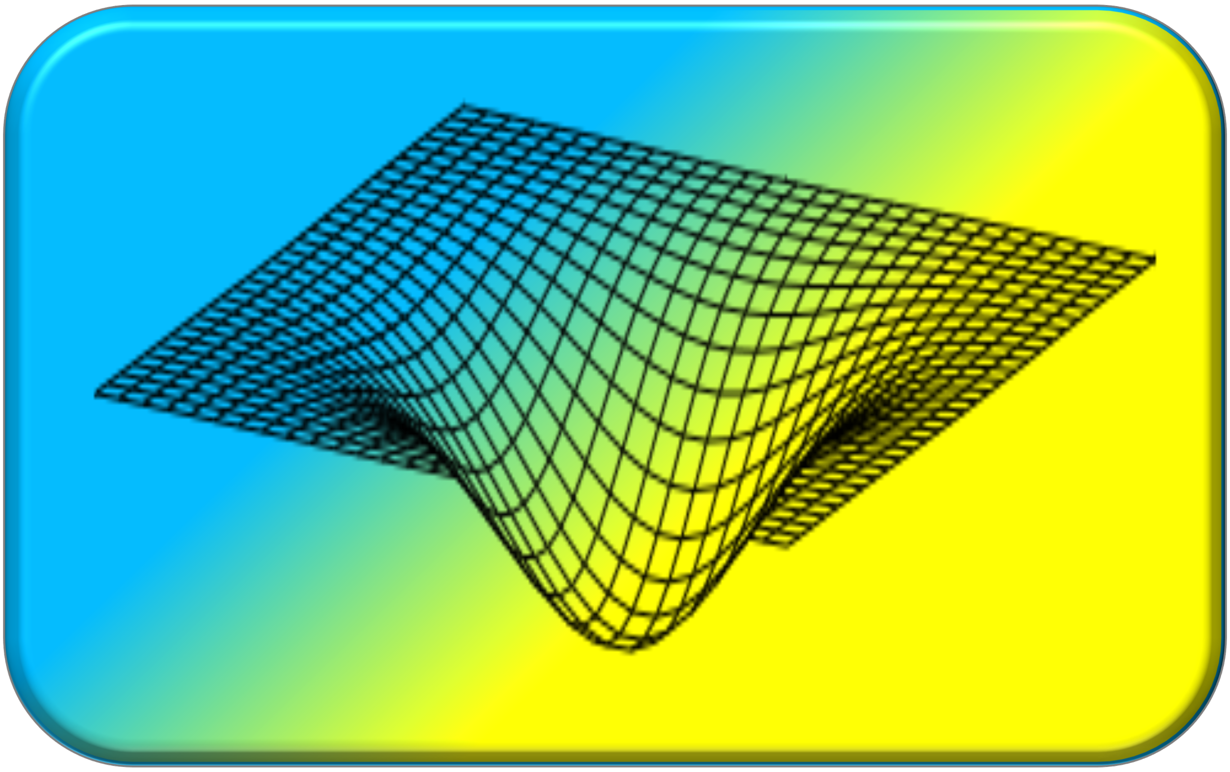

Figure 14. General relativity theory and probability theory.

4. Discussion

Today, Kolmogorov's (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) axiomatization of probability theory works fine as a mathematical theory under many circumstances. Unfortunately, there are some seemingly simple situations which cannot be described (see also Wenmackers, 2011, p. 131) within Kolmogorov's axiomatization of probability theory. It comes as no surprise that various efforts on notions like geometric probability, especially by Gian Carlo Rota (see Holik et al., 2016, Klain and Rota, 1997) or probabilistic geometry (see Menger, 2003) have been made and are published or discussed in literature. Typical probabilistic topics are discussed in a geometric framework (see Milman, Vitali D., 2008) too. The interaction between geometry and probability theory and as a consequence between quantum mechanics and relativity theory is already on the theoretical map established by several respectable authors. However, several attempts to find a unified mathematical framework for geometry and probability theory have not been successful up to now. By looking at its conceptual, methodological and philosophical dimensions a unification of geometry and probability into an unique scientific theory is both a key major intellectual challenge and a huge opportunity at the same time too. So it is now high time for scientist to take the next step.

5. Conclusion

The heart of this enterprise, and the insistence on the possibility and desirability of a unified theory of geometry and probability cannot be understood without fully acknowledging the meaning of Euclid's theorem and the Pythagorean theorem. Based solely on these two theorems, **geometry and probability theory are unified**.

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6. Patient consent for publication

Not required.

Conflict of interest statement

No conflict of interest to declare.

Erratum

Nil.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

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