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Mutually exclusive events

Research article

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Abstract:
Background:
Why should and how could the occurrence of an event A_t at a certain (period of) time / Bernoulli trial t **exclude** the occurrence of another event B_t at the same (period of) time / Bernoulli trial t and vice versa? Can this be described somehow mathematically?
Methods:
Basic methods of classical logic, probability theory and statistics were used to analyse the interior logic of an exclusion relationship.
Results:
Mutually exclusive events are mathematized, the relationship to the relative risk and the odds' ratio is worked out.
Conclusion:
Mathematically, it is possible to recognize mutually exclusive events.

Keywords: Exclusion; Study design; Bias; Cause; Effect; Causal relationship k

1. Introduction

What is going to keep us moving while facing circumstances which forces us more and more to succumb to difficulties? Nowadays, it cannot be overlooked that under the extremely high pressure of the non-ending severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) pandemics not only human diamonds, but heroes are formed. Even if our world is changing including us humans too, we

humans are still far from finished. "**We are not now that strength which in old days Moved earth and heaven; that which we are, we are; One equal temper of heroic hearts, Made weak by time and fate, but strong in will To strive, to seek, to find, and not to yield** "(see Lord Tennyson, 1842, Ulysses, p. 91). Indeed, at the end we are what we are. But after all, because we are what we are, it is our natural privilege to look for sustainable [1] , [2] , [3] , [4] , [5] , [6] , [7] , [8] , [9] , [10] , [11] , [12] solutions [13] , [14] , [15] , [16] , [17] , [18] while eye in eye with a threat, who backs away, exposes himself to the danger of getting stuck on the track. This historical crisis and natural disaster, contains in itself the germ of the new and the solution needed. In our human endeavours, to bring a great danger quickly under control, we can rely on logic, probability theory and mathematics too. In logic and probability theory, there are events which are mutually exclusive or disjoint. In other words, there are circumstances, for whatever reason, where i. e. two events cannot both occur at the same time. A example in the extreme is the coin-tossing example. However, being pregnant and being a male human being is another example. As known, it is not possible for a men to be pregnant. Being pregnant and being a men are excluding each other. As known, under normal circumstances, the sample space is a collection or a set of all possible outcomes of an experiment. However, what if emptiness, nothingness or no outcome is the outcome? Can we identify somehow which event and to what extent does an event excludes at the same time the occurrence of a Covid - 19 infection or Covid-19 death?

2. Material and methods

2.1. Data

2.2. Study design

A study design should be fair and representative as much as possible in order to assure data, which we can rely on and work with. The index of unfairness(Barukčić, 2019b) and the index of independence(Barukčić, 2019a) are indicating to which extent data could be biased due to study design. Under ideal conditions, it is desirable that p(IOU) = 0 or p(IOI) = 0 or that even p(IOU) = p(IOI) = 0.

[1] Sputnik V, Russia
[2] AZD1222, AstraZeneca/University Oxford, Sweden/Great Britain
[3] Ad26.COV2.S, Janssen-Cilag International NV
[4] BNT162b2 (Comirnaty™), BioNTech/Pfizer, Germany
[5] Spikevax™, Moderna Biotech Spain, S.L
[6] DNA SARS-CoV-2 vaccine (ZyCoV-D), India
[7] NVX-CoV2373 Covid-19 Vaccine (PREVENT-19 trial)
[8] Valneva's vaccine
[9] Soberana, Cuba
[10] and other vaccines
[11] Molnupiravir (LAGEVRIO™)
[12] Ritonavir (PF-07321332), PAXLOVID ™
[13] Regdanvimab (Regkirona ™)
[14] Casirivimab and Imdevimab (Ronapreve™)
[15] Bamlanivimab
[16] Etesevimab
[17] Sotrovimab
[18] Tixgevimab and Cilgavimab (AZD7442)

Table 1. The quality of data (see Barukčić, 2019b, p. 25)

p(IOI)	Quality of study design
$0 < p(IOI) \leq 0,25$	Unfair study design
$0,25 < p(IOI) \leq 0,5$	Very unfair study design
$0,5 < p(IOI) \leq 0,75$	Highly unfair study design
$0,75 < p(IOI) \leq 1,0$	Extremely unfair study design

2.2.1. Causality and induction

Yet ever since thousands of years the topic of causality has been subject to intense controversy. Nonetheless, causality is still a central concept of any philosophy and of any science as a whole. At this point it is desirable but impossible to go into the details of David Hume's famous sceptical view on the relationship between causality and induction (David Hume, A Treatise of Human Nature, 1739, Book 1, part iii, section 6) and Kant's response to Hume's view in his Prolegomena to Any Future Metaphysics (1783). Anyway, the quality of data has influence on the quality of the conclusion drawn. Decisions should be based on high quality of data. However, we are continually asking ourselves can we and to which extent are we allowed to rely on the data published. Data integrity issues in Pfizer's vaccine trial [19] have become public. Poor practices at clinical trials of different kind [20] raise questions about the quality of data published and can be suitable to diminish the public trust in a safe and effective Covid-19 vaccine. It is unavoidable that the data provided to the public have to be treated with some caution. However, it has to be ascertained that the quality of the data will be further improved. Besides of these improvements, data should be made freely publicly available in detail while following a certain standard. There are accurate mathematical methods to check data for integrity, while few of them are already discussed [21] in the literature.

[19] Paul D Thacker, investigative journalist PMID: 34728500
[20] The British Medical Journal PMID: 34728500
[21] Hong Chen et al. PMID: 25087521

2.2.2. The study of Grange et al.

Vaccines are effective in preventing COVID-19 deaths. Grange et al. (see Grange et al., 2021) investigated COVID-19-related deaths in 3 273 336 individuals in Scotland who were fully vaccinated by Aug 18, 2021. Scotland's 2011 census population has been 5 313 600. Grange et al. are writing: "Of the 3 273 336 individuals in Scotland who were fully vaccinated by Aug 18, 2021 (73.6% of the eligible population), 1 205 642 individuals received two doses of BNT162b2 and 2 026 198 individuals received two doses of ChAdOx1 nCoV-19. As there were no deaths among the 41 496 individuals who received two doses of mRNA-1273 (Moderna) vaccine during the study period, they were not further considered in this analysis.

236 deaths in fully vaccinated people were recorded (0·007% of the total vaccinated): 47 (0·004%) of those individuals had received BNT162b2 (median age 74·0 years [IQR 69·0–89·0]), and 188 (0·009%) individuals had received ChAdOx1 nCoV-19 (80·0 years [73·0–86·0])."(see Grange et al., 2021).

What is the true placebo group in order to analyse the efficacy of a certain vaccine? Compared one vaccine with other vaccines might lead to bias. In other words, it is appropriate to proof what happens if people are not vaccinated. In the group of non-vaccinated the death rate due to Covid-19 is about 2 per cent.

2.2.3. Moderna

Table 2. Moderna vaccine and Covid-19 death (Study Grange et al., 2021).

		Covid-19 death		
		YES	NO	
Moderna vaccine	YES	0	41496	41496
	NO	236	3231634	3231870
		236	3273130	3273366

Statistical analysis.	
Causal relationship k =	-0,0009621672
p Value left tailed (HGD) =	0,0492438
p (SINE) =	0,9999279030
$\tilde{\chi}^2$ (SINE — B_t) =	236,0000
$\tilde{\chi}^2$ (SINE — $\underline{A}_t$) =	0,0172
p Value (SINE) =	0,0000720944
p (IMP) =	0,9873231408
$\tilde{\chi}^2$ (IMP — A_t) =	41.496,0000
$\tilde{\chi}^2$ (IMP — $\underline{B}_t$) =	526,0769
p Value (IMP) =	0,0125968463
p (SINE∩IMP) =	0,9872510437
$\tilde{\chi}^2$ $(SINE \cap IMP)_1$ =	41.496,0172
$\tilde{\chi}^2$ $(SINE \cap IMP)_2$ =	762,0769
p Value (SINE∩IMP) =	0,0127
p (EXCL) =	1,0000000000
p (EXCL) approx.=	1,0000000000
$\tilde{\chi}^2$ (EXCL— A_t) =	0,0000
$\tilde{\chi}^2$ (EXCL— B_t) =	0,0000
p Value (EXCL) =	0,0000000000
Relative risk (RR).	
RR (nc) =	0,0000
RR (sc) =	0,0000
Additional measures.	
OR =	0,9873
IOR =	-1,0000
Study design.	
p(IOU)=	0,987251044
p(IOI)=	0,012604762
$\tilde{\chi}^2$ (p(IOU) = p(IOI)) =	1.636.211,0340

2.2.4. BionTech

Table 3. BionTech vaccine and Covid-19 death (Study Grange et al., 2021).

		Covid-19 death		
		YES	NO	
BionTech vaccince	YES	47	1246979	1247026
	NO	189	2026151	2026340
		236	3273130	3273366

Statistical analysis.	
Causal relationship k =	-0,0031789961
p Value left tailed (HGD) =	0,0000000
p (SINE) =	0,9999422613
$\tilde{\chi}^2$ (SINE — B_t) =	151,3602
$\tilde{\chi}^2$ (SINE — $\underline{A}_t$) =	0,0176
p Value (SINE) =	0,0000577371
p (IMP) =	0,6190529870
$\tilde{\chi}^2$ (IMP — A_t) =	1.246.932,0018
$\tilde{\chi}^2$ (IMP — $\underline{B}_t$) =	475.067,1762
p Value (IMP) =	0,3167859098
p (SINE∩IMP) =	0,6189952483
$\tilde{\chi}^2$ (SINE∩IMP)$_1$ =	1.246.932,0194
$\tilde{\chi}^2$ (SINE∩IMP)$_2$ =	475.218,5364
p Value (SINE∩IMP) =	0,3168
p (EXCL) =	0,9999856417
p (EXCL) approx.=	0,9999623103
$\tilde{\chi}^2$ (EXCL— A_t) =	0,0018
$\tilde{\chi}^2$ (EXCL— B_t) =	9,3602
p Value (EXCL) =	0,0000143582
Relative risk (RR).	
RR (nc) =	0,4041
RR (sc) =	0,5227
Additional measures.	
OR =	0,6190
IOR =	-0,4772
Study design.	
p(IOU)=	0,618966532
p(IOI)=	0,380889274
$\tilde{\chi}^2$ (p(IOU) = p(IOI)) =	1.636.211,0340

2.2.5. AstraZeneca

Table 4. AstraZeneca vaccine and Covid-19 death (Study Grange et al. , 2021).

		Covid-19 death		
		YES	NO	
AstraZeneca vaccine	YES	188	2026010	2026198
	NO	48	1247120	1247168
		236	3273130	3273366

Statistical analysis.	
Causal relationship k =	0,0031055961
p Value left tailed (HGD) =	1,0000000
p (SINE) =	0,9999853362
$\tilde{\chi}^2$ (SINE — B_t) =	9,7627
$\tilde{\chi}^2$ (SINE — $\underline{A}_t$) =	0,0018
p Value (SINE) =	0,0000146637
p (IMP) =	0,3810621849
$\tilde{\chi}^2$ (IMP — A_t) =	2.025.822,0174
$\tilde{\chi}^2$ (IMP — $\underline{B}_t$) =	1.254.064,6171
p Value (IMP) =	0,4614838624
p (SINE∩IMP) =	0,3810475211
$\tilde{\chi}^2$ (SINE∩IMP)$_1$ =	2.025.822,0193
$\tilde{\chi}^2$ (SINE∩IMP)$_2$ =	1.254.074,3798
p Value (SINE∩IMP) =	0,4615
p (EXCL) =	0,9999425668
p (EXCL) approx.=	0,9999072154
$\tilde{\chi}^2$ (EXCL— A_t) =	0,0174
$\tilde{\chi}^2$ (EXCL— B_t) =	149,7627
p Value (EXCL) =	0,0000574316
Relative risk (RR).	
RR (nc) =	2,4108
RR (sc) =	1,2870
Additional measures.	
OR =	0,3810
IOR =	0,2869
Study design.	
p(IOU)=	0,380932655
p(IOI)=	0,618923151
$\tilde{\chi}^2$ (p(IOU) = p(IOI)) =	1.636.211,0340

2.2.6. ... and again AstraZeneca Covid-19 vaccine

The study design (see table 4) has been adopted. About 3288252 people in Scotland were not vaccinated at all or at least not by the Covid-19 vaccine of AstraZeneca. The data of able 4 provide evidence that the Covid-19 vaccine of AstraZeneca protects against the Covid-19 death.

Table 5. AstraZeneca vaccine and Covid-19 death (Study Grange et al. , 2021).

		Covid-19 death		
		YES	NO	
AstraZeneca vaccine	YES	188	2026010	2026198
	NO	48662	3239590	3288252
		48850	5265600	5314450

Statistical analysis.	
Causal relationship k =	-0,0748446286
p Value left tailed (HGD) =	0,0000000
p (SINE) =	0,9908434551
$\tilde{\chi}^2$ (SINE — B_t) =	48.474,7235
$\tilde{\chi}^2$ (SINE — $\underline{A}_t$) =	720,1365
p Value (SINE) =	0,0091147514
p (IMP) =	0,6187733444
$\tilde{\chi}^2$ (IMP — A_t) =	2.025.822,0174
$\tilde{\chi}^2$ (IMP — $\underline{B}_t$) =	779.534,4348
p Value (IMP) =	0,3169769389
p (SINE∩IMP) =	0,6096167995
$\tilde{\chi}^2$ $(SINE \cap IMP)_1$ =	2.026.542,1539
$\tilde{\chi}^2$ $(SINE \cap IMP)_2$ =	828.009,1584
p Value (SINE∩IMP) =	0,3232
p (EXCL) =	0,9999646247
p (EXCL) approx.=	0,9999072154
$\tilde{\chi}^2$ (EXCL— A_t) =	0,0174
$\tilde{\chi}^2$ (EXCL— B_t) =	0,7235
p Value (EXCL) =	0,0000353746
Relative risk (RR).	
RR (nc) =	0,0063
RR (sc) =	0,0100
Additional measures.	
OR =	0,6096
IOR =	-0,9899
Study design.	
p(IOU)=	0,609546049
p(IOI)=	0,372070111
$\tilde{\chi}^2$ (p(IOU) = p(IOI)) =	2.560.423,0506

The data are presented by table 5 are assuming very conservatively, to the detriment of the AstraZeneca Covid-19 vaccination, that about 1,5 per cent or 48662 of these 3288252 people will die because of a Covid-19 infection. The study design with p(IOI)=0,372070111 is not very convincing but of use to work out the basic relationship between AstraZeneca vaccine and Covid-19 death.

2.3. Methods

The nature of definitions is discussed by scientist since ancient times. Many times, several different kinds of definitions are necessary to solve a scientific issue properly. Even if often in play, inappropriate definitions can lead to logical fallacies too.

2.3.1. The number +0

Definition 2.1 (**The number +0**). *Let i denote the imaginary number (Bombelli, 1579). The imaginary number i is known to be defined solely by the property that its square is −1. According to today valid rules of algebra, the number +0 is defined as the expression*

$$+0 \equiv +1 \times +0 \equiv +0 \times +1 \equiv +1 - 1 \equiv +1 + i^2 \equiv +1 + e^{i\pi} \equiv \neg(+1) \tag{2.1}$$

while '= 'or ≡ denotes the equals sign (Recorde, 1557) or equality sign (Rolle, 1690) used to indicate equality and '- '(Pacioli, 1494, Widmann, 1489) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and '+ 'denotes the plus (Recorde, 1557) signs used to represent the operations of addition and the notions of positive as well. Negation is denoted by ¬.

Remark 2.1. *Roger Cotes (1682 – 1716) (Cotes and Halley, 1714) or Leonhard Euler's (1707 – 1783) identity (Euler, 1748) is regarded as one of the most beautiful equations (Wilson, 2018). In this context, it is provisionally presumed, that Euler's identity (Euler, 1748) is logically sound and correct.*

2.3.2. The number +1

Definition 2.2 (**The number +1**). *According to today valid rules of algebra, the number +1 is defined as the expression*

$$+1 \equiv +1 + 0 \equiv +1 - 0 \equiv \neg(+0) \tag{2.2}$$

while again '= 'or ≡ may denote the equals sign (Recorde, 1557) or equality sign (Rolle, 1690) used to indicate equality and '- '(Pacioli, 1494, Widmann, 1489) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and '+ 'denotes the plus (Recorde, 1557) signs used to represent the operations of addition and the notions of positive as well.

2.3.3. Single event distribution

Let a **random variable**(Gosset, 1914) X denote something like a function defined on a probability space, which itself maps from the sample space(Neyman and Pearson, 1933) to the real numbers. A single event distribution is more or less a discrete probability distribution of any random variable X which takes a certain (observer independent) single value X_t at a **Bernoulli trial**(Uspensky, 1937, p. 45) (period of time) t with the probability $p(X_t)$. The same random variable X takes a certain single anti value $\underline{X}_t$ at a Bernoulli trial (period of time) t with the probability 1-$p(X_t)$. There are conditions in

nature where a random variable X can take only the values either +0 or +1. Under these conditions the random variable X takes the value 1 with probability $p(X_t = +1)$ and the value 0 with probability $q(X_t = +0) = 1 - p(X_t = +1)$ while the single event distribution passes over into the **Bernoulli distribution**, named after Swiss mathematician Jacob Bernoulli(Bernoulli, 1713). Less formally, many times, the Bernoulli distribution is represented by a (possibly not biased) coin toss where 1 and 0 would represent 'heads'and 'tails'(or vice versa), respectively. However, the relationship between random variables (Gosset, 1914) can be investigated by many (Gosset, 1908) methods, including the tools of probability theory, too.

Definition 2.3 (Two by two table of single event random variables).

The two by two or contingency table which has been introduced by Karl Pearson(Pearson, 1904) in 1904 harbours still a large variety of topics and debates. Central to this is the problem to apply the laws of classical logic on data sets, which concerns the justification of inferences which extrapolate from sample data to general facts. Nevertheless, a contingency table is still an appropriate theoretical model too for studying the relationships between random variables, including *Bernoulli(Bernoulli, 1713) (i.e. +0/+1) distributed random variables* existing or occurring at the same *Bernoulli trial* (Uspensky, 1937) (period of time) t.

In this context, let a random variable A at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by A_t, indicate a risk factor, a condition, a cause et cetera and occur or exist with the probability $p(A_t)$ at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let $E(A_t)$ denote the expectation value of A_t. In general it is

$$p(A_t) \equiv p(a_t) + p(b_t) \tag{2.3}$$

The expectation value $E(A_t)$ follows as

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv A_t \times (p(a_t) + p(b_t)) \\ &\equiv (A_t \times p(a_t)) + (A_t \times p(b_t)) \\ &\equiv E(a_t) + E(b_t) \end{aligned} \tag{2.4}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv (+0 + 1) \times p(A_t) \\ &\equiv p(A_t) \\ &\equiv p(a_t) + p(b_t) \end{aligned} \tag{2.5}$$

Furthermore, it is

$$p(\underline{A}_t) \equiv p(c_t) + p(d_t) \equiv (1 - p(A_t)) \tag{2.6}$$

The expectation value $E(\underline{A}_t)$ is given as

$$\begin{aligned} E(\underline{A}_t) &\equiv A_t \times (1 - p(A_t)) \\ &\equiv A_t \times (p(c_t) + p(d_t)) \\ &\equiv (A_t \times p(c_t)) + (A_t \times p(d_t)) \\ &\equiv E(c_t) + E(d_t) \end{aligned} \tag{2.7}$$

Under conditions of +0/+1 distributed Bernoulli random variables we obtain

$$\begin{aligned} E(\underline{A}_t) &\equiv A_t \times (1 - p(A_t)) \\ &\equiv (+0 + 1) \times (1 - p(A_t)) \\ &\equiv (1 - p(A_t)) \\ &\equiv p(c_t) + p(d_t) \end{aligned} \tag{2.8}$$

Let a random variable B at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by B_t, indicate an outcome, a conditioned, an effect et cetera and occur or exist with the probability $p(B_t)$ at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let $E(B_t)$ denote the expectation value of B_t. In general it is

$$p(B_t) \equiv p(a_t) + p(c_t) \tag{2.9}$$

The expectation value $E(B_t)$ is given by the equation

$$\begin{aligned} E(B_t) &\equiv B_t \times p(B_t) \\ &\equiv B_t \times (p(a_t) + p(c_t)) \\ &\equiv (B_t \times p(a_t)) + (B_t \times p(c_t)) \\ &\equiv E(a_t) + E(c_t) \end{aligned} \tag{2.10}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(B_t) &\equiv B_t \times p(B_t) \\ &\equiv (+0 + 1) \times p(B_t) \\ &\equiv p(B_t) \\ &\equiv p(a_t) + p(c_t) \end{aligned} \tag{2.11}$$

Furthermore, it is

$$p(\underline{B}_t) \equiv p(b_t) + p(d_t) \equiv (1 - p(B_t)) \tag{2.12}$$

The expectation value $E(\underline{B}_t)$ is given by the equation

$$\begin{aligned} E(\underline{B}_t) &\equiv B_t \times (1 - p(B_t)) \\ &\equiv B_t \times (p(b_t) + p(d_t)) \\ &\equiv (B_t \times p(b_t)) + (B_t \times p(d_t)) \\ &\equiv E(b_t) + E(d_t) \end{aligned} \tag{2.13}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(\underline{B}_t) &\equiv B_t \times (1 - p(B_t)) \\ &\equiv (+0 + 1) \times (1 - p(B_t)) \\ &\equiv (1 - p(B_t)) \\ &\equiv p(b_t) + p(d_t) \end{aligned} \tag{2.14}$$

Let $p(a_t)= p(A_t \wedge B_t)$ denote the joint probability distribution of A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(a_t) &\equiv E(A_t \wedge B_t) \\ &\equiv (A_t \times B_t) \times p(A_t \wedge B_t) \\ &\equiv (A_t \times B_t) \times p(a_t) \end{aligned} \tag{2.15}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(a_t) &\equiv E(A_t \wedge B_t) \\ &\equiv (A_t \times B_t) \times p(A_t \wedge B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(A_t \wedge B_t) \\ &\equiv p(A_t \wedge B_t) \\ &\equiv p(a_t) \end{aligned} \tag{2.16}$$

Let $p(b_t)= p(A_t \wedge \neg B_t)$ denote the joint probability distribution of A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(b_t) &\equiv E(A_t \wedge \neg B_t) \\ &\equiv (A_t \times \neg B_t) \times p(A_t \wedge \neg B_t) \\ &\equiv (A_t \times \neg B_t) \times p(b_t) \end{aligned} \tag{2.17}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(b_t) &\equiv E(A_t \wedge \neg B_t) \\ &\equiv (A_t \times \neg B_t) \times p(A_t \wedge \neg B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(A_t \wedge \neg B_t) \\ &\equiv p(A_t \wedge \neg B_t) \\ &\equiv p(b_t) \end{aligned} \tag{2.18}$$

Let $p(c_t)= p(\neg A_t \wedge B_t)$ denote the joint probability distribution of not A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(c_t) &\equiv E(\neg A_t \wedge B_t) \\ &\equiv (\neg A_t \wedge B_t) \times p(\neg A_t \wedge B_t) \\ &\equiv (\neg A_t \wedge B_t) \times p(c_t) \end{aligned} \tag{2.19}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(c_t) &\equiv E(\neg A_t \wedge B_t) \\ &\equiv (\neg A_t \times B_t) \times p(\neg A_t \wedge B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(\neg A_t \wedge B_t) \\ &\equiv p(\neg A_t \wedge B_t) \\ &\equiv p(c_t) \end{aligned} \tag{2.20}$$

Let $p(d_t)= p(\neg A_t \wedge \neg B_t)$ denote the joint probability distribution of not A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E(d_t) &\equiv E(\neg A_t \times \neg B_t) \\ &\equiv (\neg A_t \times \neg B_t) \times p(\neg A_t \wedge \neg B_t) \\ &\equiv (\neg A_t \times \neg B_t) \times p(d_t) \end{aligned} \tag{2.21}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E(d_t) &\equiv E(\neg A_t \wedge \neg B_t) \\ &\equiv (\neg A_t \times \neg B_t) \times p(\neg A_t \wedge \neg B_t) \\ &\equiv ((+0+1) \times (+0+1)) \times p(\neg A_t \wedge \neg B_t) \\ &\equiv p(\neg A_t \wedge \neg B_t) \\ &\equiv p(d_t) \end{aligned} \tag{2.22}$$

In general, it is

$$p(a_t) + p(b_t) + p(c_t) + p(d_t) \equiv +1 \tag{2.23}$$

Table 6 provide us with an overview of the definitions above.

Table 6. The two by two table of Bernoulli random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition A_t	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

2.3.4. Binomial random variables

Definition 2.4 (Two by two table of Binomial random variables).

Let a, b, c, d, A, $\underline{A}$, B, and $\underline{B}$ denote expectation values. Under conditions where *the probability of an event, an outcome, a success et cetera is* ***constant*** *from Bernoulli trial to Bernoulli trial t*, it is

$$\begin{aligned} A &= N \times E(A_t) \\ &\equiv N \times (A_t \times p(A_t)) \\ &\equiv N \times (p(A_t) + p(B_t)) \\ &\equiv N \times p(A_t) \end{aligned} \tag{2.24}$$

and

$$\begin{aligned} B &= N \times E(B_t) \\ &\equiv N \times (B_t \times p(B_t)) \\ &\equiv N \times (p(A_t) + p(c_t)) \\ &\equiv N \times p(B_t) \end{aligned} \tag{2.25}$$

where N might denote the population or even the sample size. Furthermore, it is

$$a \equiv N \times (E(A_t)) \equiv N \times (p(A_t)) \tag{2.26}$$

and

$$b \equiv N \times (E(B_t)) \equiv N \times (p(B_t)) \tag{2.27}$$

and

$$c \equiv N \times (E(c_t)) \equiv N \times (p(c_t)) \tag{2.28}$$

and

$$d \equiv N \times (E(d_t)) \equiv N \times (p(d_t)) \tag{2.29}$$

and

$$a + b + c + d \equiv A + \underline{A} \equiv B + \underline{B} \equiv N \tag{2.30}$$

Table 7 provide us again an overview of a two by two table of Binomial random variables.

Table 7. The two by two table of Binomial random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition A_t	TRUE	a	b	A
	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

2.3.5. Independence

Definition 2.5 (**Independence**).

In general, an event A_t at the Bernoulli trial t need not but can be independent of the existence or of the occurrence of another event B_t *at the same* Bernoulli trial t. Mathematically, independence (Kolmogoroff, 1933, Moivre, 1718) in terms of probability theory is defined at the same (period of) time t (i.e. Bernoulli trial t) as

$$\begin{aligned} p(A_t \wedge B_t) &\equiv p(A_t) \times p(B_t) \\ &\equiv \frac{\sum_{t=1}^{N} (A_t \wedge B_t)}{N} \equiv \frac{N \times (p(a_t))}{N} \equiv 1 - p(A_t \mid B_t) \equiv 1 - p(A_t \uparrow B_t) \end{aligned} \tag{2.31}$$

2.3.6. Dependence

Definition 2.6 (**Dependence**).

The dependence of events (Barukčić, 1989, p. 57-61) is defined as

$$p\left(\underbrace{A_t \wedge B_t \wedge C_t \wedge \ldots}_{n}\right) \equiv \sqrt[n]{\underbrace{p(A_t) \times p(B_t) \times p(C_t) \times \ldots}_{n}} \tag{2.32}$$

2.3.7. Exclusion relationship

Definition 2.7 (**Exclusion relationship [*EXCL*]**).

Mathematically, the exclusion (EXCL) relationship, denoted by $p(A_t \mid B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p(A_t \mid B_t) &\equiv p(A_t \uparrow B_t) \\ &\equiv p(b_t) + p(c_t) + p(d_t) \\ &\equiv \frac{N \times (p(b_t) + p(c_t) + p(d_t))}{N} \\ &\equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \vee \underline{B}_t)}{N} \equiv \frac{b+c+d}{N} \\ &\equiv \frac{b+\underline{A}}{N} \\ &\equiv \frac{c+\underline{B}}{N} \\ &\equiv +1 \end{aligned} \tag{2.33}$$

Based on the 1913 Henry Maurice Sheffer (1882-1964) relationship, the Sheffer stroke(Nicod, 1917, Sheffer, 1913) usually denoted by $\uparrow$, it is $p(A_t \wedge B_t) \equiv 1 - p(A_t \mid B_t)$ (see table 8).

Table 8. A_t excludes B_t and vice versa.

		Conditioned (COVID-19) B_t		
		TRUE	FALSE	
Condition (Vaccine)	TRUE	**+0**	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Remark 2.2. *Pfizer Inc. and BioNTech SE announced on Monday, November 09, 2020 - 06:45am results from a Phase 3 COVID-19 vaccine trial with 43.538 participants which provides evidence that their vaccine (BNT162b2) is preventing COVID-19 in participants without evidence of prior SARS-CoV-2 infection. In toto, 170 confirmed cases of COVID-19 were evaluated, with 8 in the vaccine*

group versus 162 in the placebo group. The exclusion relationship can be calculated as follows.

$$\begin{aligned} p(Vaccine : BNT162b2 \mid COVID-19(infection)) &\equiv p(b_t) + p(c_t) + p(d_t) \\ &\equiv 1 - p(a_t) \\ &\equiv 1 - \left(\frac{8}{43538}\right) \\ &\equiv +0,99981625 \end{aligned} \tag{2.34}$$

with a P Value = 0,000184.

2.3.8. The goodness of fit test of an exclusion relationship

Definition 2.8 (The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship).

Under some well known circumstances, testing hypothesis about an exclusion relationship $p(A_t \mid B_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}((A_t \mid B_t) \mid A) &\equiv \frac{(b-(a+b))^2}{A} + \\ &\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\ &\equiv \frac{a^2}{A} + 0 \\ &\equiv \frac{a^2}{A} \end{aligned} \tag{2.35}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}((A_t \mid B_t) \mid B) &\equiv \frac{(c-(a+c))^2}{B} + \\ &\quad \frac{((b+d)-\underline{B})^2}{\underline{B}} \\ &\equiv \frac{a^2}{B} + 0 \\ &\equiv \frac{a^2}{B} \end{aligned} \tag{2.36}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of an exclusion relationship/distribution $p(A_t \mid B_t)$, in which case the null hypothesis has to be accepted. Yate's (Yates, 1934) continuity correction was not used under these circumstances.

2.3.9. The left-tailed p Value of an exclusion relationship

Definition 2.9 (**The left-tailed p Value of an exclusion relationship**).

It is known that as a sample size, N, increases, a sampling distribution of a special test statistic approaches the normal distribution (central limit theorem). Under these circumstances, the left-tailed (lt) p Value (Barukčić, 2019c) of an exclusion relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \mid B_{\mathrm{t}}\right) &\equiv 1 - e^{-\left(1-p\left(A_{\mathrm{t}} \mid B_{\mathrm{t}}\right)\right)} \\ &\equiv 1 - e^{-(a/N)} \end{aligned} \tag{2.37}$$

A low p-value may provide some evidence of statistical significance.

2.3.10. Neither nor conditions

Definition 2.10 (**Neither $\mathbf{A_t}$ nor $\mathbf{B_t}$ conditions [*NOR*]**).

Mathematically, a neither A_t nor B_t condition (or rejection according to the French philosopher and logician Jean George Pierre Nicod (1893-1924), i.e. Jean Nicod's statement (Nicod, 1924)) relationship (NOR), denoted by $p(A_t \downarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) &\equiv p\left(d_{\mathrm{t}}\right) \\ &\equiv \frac{N - \sum_{t=1}^{N}\left(A_{\mathrm{t}} \vee B_{\mathrm{t}}\right)}{N} \equiv \frac{\sum_{t=1}^{N}\left(\underline{A}_{\mathrm{t}} \wedge \underline{B}_{\mathrm{t}}\right)}{N} \equiv \frac{N \times \left(p\left(d_{\mathrm{t}}\right)\right)}{N} \\ &\equiv \frac{d}{N} \\ &\equiv +1 \end{aligned} \tag{2.38}$$

2.3.11. The Chi square goodness of fit test of a neither nor condition relationship

Definition 2.11 (**The $\tilde{\chi}^2$ goodness of fit test of a neither $\mathbf{A_t}$ nor $\mathbf{B_t}$ condition relationship**).

A neither A_t nor B_t condition relationship $p(A_t \downarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution). The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship with degree of freedom (d. f.) of d. f. = 1 may be calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\mathrm{Calculated}}\left(\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) \mid A\right) &\equiv \frac{\left(d - (c+d)\right)^2}{\underline{A}} + \\ &\quad \frac{\left((a+b) - A\right)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}} + 0 \end{aligned} \tag{2.39}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_{\text{t}} \downarrow B_{\text{t}}) \mid B\right) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\ &\quad \frac{((a+c)-B)^2}{B} \\ &\equiv \frac{b^2}{\underline{B}} + 0\end{aligned} \tag{2.40}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.12. The left-tailed p Value of a neither nor B condition relationship

Definition 2.12 (The left-tailed p Value of a neither A_t nor B_t condition relationship).

The left-tailed (lt) p Value (Barukčić, 2019c) of a neither A_t nor B_t condition relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_{\text{t}} \downarrow B_{\text{t}}\right) &\equiv 1 - e^{-\left(1-p\left(A_{\text{t}} \downarrow B_{\text{t}}\right)\right)} \\ &\equiv 1 - e^{-p\left(A_{\text{t}} \vee B_{\text{t}}\right)} \\ &\equiv 1 - e^{-((a+b+c)/N)}\end{aligned} \tag{2.41}$$

where $\vee$ may denote disjunction or logical inclusive or. In this context, a low p-value indicates again a statistical significance. In general, it is $p\left(A_{\text{t}} \vee B_{\text{t}}\right) \equiv 1 - p\left(A_{\text{t}} \downarrow B_{\text{t}}\right)$ (see table 9).

Table 9. Neither A_t nor B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	0	0
	NO	0	1	1
		0	1	1

2.3.13. Necessary condition

Definition 2.13 (Necessary condition [*Conditio sine qua non*]).

Mathematically, the necessary condition (SINE) relationship, denoted by $p(A_t \leftarrow B_t)$ in terms of

statistics and probability theory, is defined (Barukčić, 1989, p. 15-28) as

$$
\begin{aligned}
p(A_t \leftarrow B_t) \equiv p(A_t \vee \underline{B}_t) &\equiv \frac{\sum_{t=1}^{N} (A_t \vee \underline{B}_t)}{N} \equiv \frac{(A_t \vee \underline{B}_t) \times p(A_t \vee \underline{B}_t)}{(A_t \vee \underline{B}_t)} \\
&\equiv p(a_t) + p(b_t) + p(d_t) \\
&\equiv \frac{N \times (p(a_t) + p(b_t) + p(d_t))}{N} \equiv \frac{E(A_t \leftarrow B_t)}{N} \\
&\equiv \frac{a+b+d}{N} \equiv \frac{E(A_t \vee \underline{B}_t)}{N} \\
&\equiv \frac{A+d}{N} \equiv \frac{E(A_t \leftarrow B_t)}{N} \\
&\equiv \frac{a+\underline{B}}{N} \equiv \frac{E(A_t \vee \underline{B}_t)}{N} \\
&\equiv +1
\end{aligned}
\tag{2.42}
$$

where $E(A_t \leftarrow B_t) \equiv E(A_t \vee \underline{B}_t)$ indicates the expectation value of the necessary condition. In general, it is $p(A_t \prec\!\!- B_t) \equiv 1 - p(A_t \leftarrow B_t)$ (see Table 10).

Table 10. Necessary condition.

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
A_t	FALSE	**+0**	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Remark 2.3. *A necessary condition A_t is characterized itself by the property that another event B_t will not occur if A_t is not given, if A_t did not occur (Barukčić, 1989, 1997, 2005, 2016, 2017a,b, 2020a,b,c,d, Barukčić and Ufuoma, 2020).* ***Example***. *A human being cannot live without water. A human being cannot live without gaseous oxygen et cetera. Water itself is a necessary condition of human life. However, gaseous oxygen is a necessary condition of human life too. Thus far, even if water is given and even if water is a necessary condition of human life, without gaseous oxygen there will be no human life. In general, if a conditioned or an outcome B_t depends on the necessary condition A_t and equally on numerous other necessary conditions, an event B_t will not occur if A_t itself is not given independently of the occurrence of other necessary conditions.*

2.3.14. The Chi-square goodness of fit test of a necessary condition relationship

Definition 2.14 (The $\tilde{\chi}^2$ goodness of fit test of a necessary condition relationship).

Under some well known circumstances, hypothesis about the conditio sine qua non relationship $p(A_t \leftarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or χ^2-distribution), first described by the German statistician Friedrich Robert Helmert (Helmert, 1876) and later rediscovered by Karl

Pearson (Pearson, 1900) in the context of a goodness of fit test. The $\tilde{\chi}^2$ goodness of fit test of a conditio sine qua non relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \leftarrow B_{\text{t}} \mid B\right) &\equiv \frac{\left(a-(a+c)\right)^2}{B}+ \\ &\quad \frac{\left((b+d)-\underline{B}\right)^2}{\underline{B}} \\ &\equiv \frac{c^2}{B}+0 \\ &\equiv \frac{c^2}{B}\end{aligned} \tag{2.43}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \leftarrow B_{\text{t}} \mid \underline{A}\right) &\equiv \frac{\left(d-(c+d)\right)^2}{\underline{A}}+ \\ &\quad \frac{\left((a+b)-A\right)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}}+0 \\ &\equiv \frac{c^2}{\underline{A}}\end{aligned} \tag{2.44}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. It has not yet been finally clarified whether the use of Yate's (Yates, 1934) continuity correction is necessary at all.

2.3.15. The left-tailed p Value of the conditio sine qua non relationship

Definition 2.15 (**The left-tailed p Value of the conditio sine qua non relationship**).

The left-tailed (lt) p Value (Barukčić, 2019c) of the conditio sine qua non relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_{\text{t}} \leftarrow B_{\text{t}}\right) &\equiv 1-e^{-\left(1-p\left(A_{\text{t}} \leftarrow B_{\text{t}}\right)\right)} \\ &\equiv 1-e^{-(c/N)}\end{aligned} \tag{2.45}$$

2.3.16. Sufficient condition

Definition 2.16 (**Sufficient condition [*Conditio per quam*]**).

Mathematically, the sufficient condition (IMP) relationship, denoted by $\text{p}(\text{A}_{\text{t}} \rightarrow \text{B}_{\text{t}})$ in terms of

statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$
\begin{aligned}
p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \equiv p(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}) &\equiv \frac{\sum_{t=1}^{N} (\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})}{N} \equiv \frac{(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}) \times p(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})}{(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})} \\
&\equiv p(a_{\mathrm{t}}) + p(c_{\mathrm{t}}) + p(d_{\mathrm{t}}) \\
&\frac{N \times (p(a_{\mathrm{t}}) + p(c_{\mathrm{t}}) + p(d_{\mathrm{t}}))}{N} \\
&\equiv \frac{a+c+d}{N} \equiv \frac{E(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}})}{N} \\
&\equiv \frac{B+d}{N} \equiv \frac{E(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}})}{N} \\
&\equiv \frac{a+\underline{A}}{N} \\
&\equiv +1
\end{aligned}
\tag{2.46}
$$

It is $p(A_{\mathrm{t}} \succ\!- B_{\mathrm{t}}) \equiv 1 - p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}})$ (see Table 11).

Table 11. Sufficient condition.

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	**+0**	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Remark 2.4. *A sufficient condition A_t is characterized by the property that another event B_t will occur if A_t is given, if A_t itself occured (Barukčić, 1989, 1997, 2005, 2016, 2017a,b, 2020a,b,c,d, Barukčić and Ufuoma, 2020).* ***Example***. *The ground, the streets, the trees, human beings and many other objects too will become wet during heavy rain. Especially,* ***if*** *it is raining (event A_t),* ***then*** *human beings will become wet (event B_t). However, even if this is a common human wisdom, a human being equipped with an appropriate umbrella (denoted by R_t) need not become wet even during heavy rain. An appropriate umbrella (R_t) is similar to an event with the potential to counteract the occurrence of another event (B_t) and can be understood something as an* ***anti-dot*** *of another event. In other words, an appropriate umbrella is an antidote of the effect of rain on human body, an appropriate umbrella has the potential to protect humans from the effect of rain on their body. It is a good rule of thumb that the following relationship*

$$p(A_t \rightarrow B_t) + p(R_t \wedge B_t) \equiv +1 \tag{2.47}$$

indicates that R_t is an antidote of A_t. However, taking a shower, swimming in a lake et cetera may make human hair wet too. More than anything else, however, these events does not affect the final outcome, the effect of raining on human body.

2.3.17. The Chi square goodness of fit test of a sufficient condition relationship

Definition 2.17 (**The $\tilde{\chi}^2$ goodness of fit test of a sufficient condition relationship**).

Under some well known circumstances, testing hypothesis about the conditio per quam relationship $p(A_t \rightarrow B_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a conditio per quam relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}\left(A_t \rightarrow B_t \mid A\right) &\equiv \frac{(a-(a+b))^2}{A} + \\ &\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + 0 \\ &\equiv \frac{b^2}{A} \end{aligned} \tag{2.48}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}\left(A_t \rightarrow B_t \mid \underline{B}\right) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\ &\quad \frac{((a+c)-B)^2}{B} \\ &\equiv \frac{b^2}{\underline{B}} + 0 \\ &\equiv \frac{b^2}{\underline{B}} \end{aligned} \tag{2.49}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of the conditio per quam relationship/distribution $p(A_t \rightarrow B_t)$, in which case the null hypothesis is accepted. Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.18. The left-tailed p Value of the conditio per quam relationship

Definition 2.18 (**The left-tailed p Value of the conditio per quam relationship**).

The left-tailed (lt) p Value (Barukčić, 2019c) of the conditio per quam relationship can be calculated as follows.

$$\begin{aligned} pValue_{\text{lt}}\left(A_t \rightarrow B_t\right) &\equiv 1 - e^{-\left(1-p\left(A_t \rightarrow B_t\right)\right)} \\ &\equiv 1 - e^{-(b/N)} \end{aligned} \tag{2.50}$$

Again, a low p-value indicates a statistical significance.

2.3.19. Necessary and sufficient conditions

Definition 2.19 (**Necessary and sufficient conditions [*EQV*]**).

The necessary and sufficient condition (EQV) relationship, denoted by $p(A_t \leftrightarrow B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p\left(A_t \leftrightarrow B_t\right) &\equiv \frac{\sum_{t=1}^{N}\left(\left(A_t \vee \underline{B}_t\right) \wedge \left(\underline{A}_t \vee B_t\right)\right)}{N} \\ &\equiv p\left(a_t\right) + p\left(d_t\right) \\ &\equiv \frac{N \times \left(p\left(a_t\right) + p\left(d_t\right)\right)}{N} \\ &\equiv \frac{a+d}{N} \\ &\equiv +1 \end{aligned} \tag{2.51}$$

2.3.20. The Chi square goodness of fit test of a necessary and sufficient condition relationship

Definition 2.20 (**The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship**).

Even the necessary and sufficient condition relationship $p(A_t \leftrightarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} {\tilde{\chi}^2}_{\text{Calculated}}\left(A_t \leftrightarrow B_t \mid A\right) &\equiv \frac{\left(a-\left(a+b\right)\right)^2}{A} + \\ &\quad \frac{d-\left(\left(c+d\right)\right)^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + \frac{c^2}{\underline{A}} \end{aligned} \tag{2.52}$$

or equally as

$$\begin{aligned} {\tilde{\chi}^2}_{\text{Calculated}}\left(A_t \leftrightarrow B_t \mid B\right) &\equiv \frac{\left(a-\left(a+c\right)\right)^2}{B} + \\ &\quad \frac{d-\left(\left(b+d\right)\right)^2}{\underline{B}} \\ &\equiv \frac{c^2}{B} + \frac{b^2}{\underline{B}} \end{aligned} \tag{2.53}$$

The calculated $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship can be compared with a theoretical chi-square value at a certain level of significance α. Under conditions where the observed values are equal to the expected/theoretical values of a necessary and sufficient condition relationship/distribution $p(A_t \leftrightarrow B_t)$, the $\tilde{\chi}^2$-distribution equals zero. It is to be cleared whether Yate's (Yates, 1934) continuity correction should be used at all.

2.3.21. The left-tailed p Value of a necessary and sufficient condition relationship

Definition 2.21 (**The left-tailed p Value of a necessary and sufficient condition relationship**).

The left-tailed (lt) p Value (Barukčić, 2019c) of a necessary and sufficient condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}\right) &\equiv 1 - e^{-\left(1-p\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}\right)\right)} \\ &\equiv 1 - e^{-((b+c)/N)} \end{aligned} \tag{2.54}$$

In this context, a low p-value indicates again a statistical significance. Table 12 may provide an overview of the theoretical distribution of a necessary and sufficient condition.

Table 12. Necessary and sufficient condition.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	1	0	1
	NO	0	1	1
		1	1	2

2.3.22. Either or conditions

Definition 2.22 (**Either A_t or B_t conditions [*NEQV*]**).

Mathematically, an either A_t or B_t condition relationship (NEQV), denoted by $p(A_t \succ\!\prec B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p\left(A_{\mathrm{t}} \succ\!\prec B_{\mathrm{t}}\right) &\equiv \frac{\sum_{t=1}^{N}\left(\left(A_{\mathrm{t}} \wedge \underline{B}_{\mathrm{t}}\right) \vee \left(\underline{A}_{\mathrm{t}} \wedge B_{\mathrm{t}}\right)\right)}{N} \\ &\equiv p\left(b_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right) \\ &\equiv \frac{N \times \left(p\left(b_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right)\right)}{N} \\ &\equiv \frac{b+c}{N} \\ &\equiv +1 \end{aligned} \tag{2.55}$$

It is $p\left(A_{\mathrm{t}} \succ\!\prec B_{\mathrm{t}}\right) \equiv 1 - p\left(A_{\mathrm{t}} \Leftrightarrow B_{\mathrm{t}}\right)$ (see Table 13).

Table 13. Either A_t or B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	1	1
	NO	1	0	1
		1	1	2

2.3.23. The Chi-square goodness of fit test of an either or condition relationship

Definition 2.23 (**The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship**).

An either or condition relationship $p(A_t \succ\prec B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t \succ\prec B_t) \mid A\right) &\equiv \frac{(b-(a+b))^2}{A} + \\ &\quad \frac{c-((c+d))^2}{\underline{A}} \\ &\equiv \frac{a^2}{A} + \frac{d^2}{\underline{A}}\end{aligned} \tag{2.56}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t \succ\prec B_t) \mid B\right) &\equiv \frac{(c-(a+c))^2}{B} + \\ &\quad \frac{b-((b+d))^2}{\underline{B}} \\ &\equiv \frac{a^2}{B} + \frac{d^2}{\underline{B}}\end{aligned} \tag{2.57}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.3.24. The left-tailed p Value of an either or condition relationship

Definition 2.24 (**The left-tailed p Value of an either or condition relationship**).

The left-tailed (lt) p Value (Barukčić, 2019c) of an either or condition relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_t \succ\prec B_t\right) &\equiv 1 - e^{-\left(1-p\left(A_t>-<B_t\right)\right)} \\ &\equiv 1 - e^{-((a+d)/N)}\end{aligned} \tag{2.58}$$

In this context, a low p-value indicates again a statistical significance.

2.3.25. Causal relationship k

Definition 2.25 (**Causal relationship k**).

Nonetheless, mathematically, the causal(Barukčić, 2011a,b, 2012) relationship (Barukčić, 1989, 1997, 2005, 2016, 2017b) between a cause U_t (German: Ursache) and an effect W_t (German: Wirkung), denoted by $k(U_t, W_t)$, is defined *at each single Bernoulli trial t* in terms of statistics and probability

theory as

$$k(U_t, W_t) \equiv \frac{\sigma(U_t, W_t)}{\sigma(U_t) \times \sigma(W_t)} \equiv \frac{p(U_t \wedge W_t) - p(U_t) \times p(W_t)}{\sqrt[2]{(p(U_t) \times (1 - p(U_t))) \times (p(W_t) \times (1 - p(W_t)))}} \tag{2.59}$$

where σ (U_t , W_t) denotes the co-variance between a cause U_t and an effect W_t *at every single Bernoulli trial t*, σ (U_t) denotes the standard deviation of a cause U_t at the same single Bernoulli trial t, σ (W_t) denotes the standard deviation of an effect W_t at same single Bernoulli trial t. Table 14 illustrates the theoretically possible relationships between a cause and an effect.

Table 14. Sample space and the causal relationship k

		Effect B_t		
		TRUE	FALSE	
Cause A_t	TRUE	$p(a_t)$	$p(b_t)$	$p(U_t)$
	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{U}_t)$
		$p(W_t)$	$p(\underline{W}_t)$	+1

2.3.26. Study design and bias

Systematic observation and experimentation, inductive and deductive reasoning are essential for any formation and testing of hypotheses and theories about the natural world. In one way or another, logically and mathematically sound scientific methods and concepts are crucial constituents of any scientific progress. When all goes well, different scientists at different times and places using the same scientific methodology should be able to generate the same scientific knowledge. However, more than half (52%) of scientists surveyed believe that studies do not successfully reproduce sufficiently similar or the same results as the original studies (Baker, 2016). In a very large study on publication bias in meta-analyses, Kicinski et al. (Kicinski et al., 2015) found evidence of publication bias even in systematic reviews. Therefore, a careful re-evaluation of the study/experimental design, the statistical methods and other scientific means which underpin scientific inquiry and research goals appears to be necessary once and again. While it is important to recognise the shortcoming of today's science, one issue which has shaped debates over studies published is the question: **has a study really measured what it set out to**? Even if studies carried out can vary greatly in detail, the data from the studies itself provide information about the credibility of the data.

Index of unfairness (IOU)

Definition 2.26 (Index of unfairness).

The index of unfairness (Barukčić, 2019b) (IOU) is defined as

$$p(IOU(A, B)) \equiv Absolute\left(\left(\frac{A+B}{N}\right) - 1\right) \tag{2.60}$$

A very good study design should assure as much as possible a p(IOU) = 0. In point of fact, against the background of lacking enough experience with the use of p(IOU), a p(IOU) up to 0.25 could be of use too. An index of unfairness is of use to prove whether sample data are biased and whether sample data can be used for Chi-square based analysis of necessary conditions, of sufficient conditions and of causal relationships.

Index of independence (IOI)

Definition 2.27 (**Index of independence**)**.**

The index of independence(Barukčić, 2019a) (IOI) is defined as

$$p\left(IOI\left(A,\underline{B}\right)\right) \equiv Absolute\left(\left(\frac{A+\underline{B}}{N}\right)-1\right) \tag{2.61}$$

A very good study design which aims to prove **an exclusion relationship or a causal relationship** should assure as much as possible a p(IOI) = 0. However, once again, against the background of lacking enough experience with the use of p(IOI), sample data with a p(IOI) up to 0.25 are of use too. Today, most double-blind placebo-controlled studies are based on the demand that **p(IOU) = p(IOI)** while the value of p(IOU) of has been widely neglected. Such an approach leads to unnecessary big sample sizes, the increase of cost, the waste of time and, most importantly of all, to epistemological systematically biased sample data and conclusions drawn. A change is necessary.

Index of relationship (IOR)

Definition 2.28 (Index of relationship (IOR))**.**

Due to several reasons, it is not always easy to identify the unique characteristics between two events like A_t and B_t. And more than that, it is difficult to decide what to do, and much more difficult to know in which direction one should think and which decision is right. Sometimes it is helpful to know at least something about the direction of the relationship between two events like A_t and B_t. Under conditions where $p(a_t) = p(A_t \wedge B_t)$, the index of relationship(Barukčić, 2021a), abbreviated as IOR, is defined as

$$\begin{aligned} IOR(A_t,B_t) &\equiv \left(\frac{p(A_t \wedge B_t)}{p(B_t)\times p(A_t)}\right)-1 \\ &\equiv \left(\frac{p(a_t)}{p(B_t)\times p(A_t)}\right)-1 \\ &\equiv \left(\left(\frac{N\times N\times p(a_t)}{N\times p(B_t)\times N\times p(A_t)}\right)-1\right) \\ &\equiv \left(\left(\frac{N\times a}{A\times B}\right)-1\right) \end{aligned} \tag{2.62}$$

where $p(A_t)$ denotes the probability of an event A_t at the Bernoulli trial t and $p(B_t)$ denotes the probability of another event B_t at the same Bernoulli trial t while $p(a_t)$ denotes the joint probability of $p(A_t$ AND $B_t)$ at the same Bernoulli trial t and a, A and B may denote the expectation values.

2.3.27. Relative risk (RR)

Relative risk (RR_{nc})

Definition 2.29 (Relative risk (RR_{nc}))**.**

The degree of association between the two binomial variables can be assessed by a number of very different coefficients, the relative (Cornfield, 1951, Sadowsky et al., 1953) risk is one(Barukčić, 2021c) of them. In general, relative risk RR_{nc}, which provides some evidence of a necessary condition, is defined as

$$RR(A_t,B_t)_{nc}=\frac{\frac{p(a_t)}{p(A_t)}}{\frac{p(c_t)}{p(NotA_t)}}=\frac{p(a_t)\times p(NotA_t)}{p(c_t)\times p(A_t)}=\frac{N\times p(a_t)\times N\times p(NotA_t)}{N\times p(c_t)\times N\times p(A_t)}=\frac{a_t\times(NotA_t)}{c_t\times A_t}=\frac{EER(A_t,B_t)}{CER(A_t,B_t)} \tag{2.63}$$

That what scientist generally understand by relative risk is the ratio of a probability of an event occurring with an exposure versus the probability of an event occurring without an exposure. In other words,

relative risk = (probability(event in exposed group)) / (probability(the same event in not exposed group)).

A $RR(A_t,B_t) = +1$ means that exposure does not affect the outcome or both are independent of each other while $RR(A_t,B_t)$ less than +1 means that the risk of the outcome is decreased by the exposure. In this context, an $RR(A_t,B_t)$ greater than +1 denotes that the risk of the outcome is increased by the exposure. Widely known problems with odds ratio and relative risk are already documented in literature.

Relative risk (RR (sc))

Definition 2.30 (Relative risk (RR (sc)))**.**

The relative risk (sc), which provides some evidence of a sufficient condition, is calculated from the point of view of an outcome and is defined as

$$RR(A_t,B_t)_{sc}=\frac{\frac{p(a_t)}{p(B_t)}}{\frac{p(b_t)}{p(NotB_t)}}=\frac{p(a_t)\times p(NotB_t)}{p(b_t)\times p(B_t)}=\frac{N\times p(a_t)\times N\times p(NotB_t)}{N\times p(b_t)\times N\times p(B_t)}=\frac{a_t\times(NotB_t)}{b_t\times B_t}=\frac{OPR(A_t,B_t)}{CPR(A_t,B_t)} \tag{2.64}$$

Relative risk reduction (RRR)

Definition 2.31 (Relative risk reduction (RRR))**.**

$$\begin{aligned} RRR(A_t, B_t) &\equiv \frac{CER(A_t, B_t) - EER(A_t, B_t)}{CER(A_t, B_t)} \\ &= 1 - RR(A_t, B_t) \end{aligned} \tag{2.65}$$

Vaccine efficacy (VE)

Definition 2.32 (Vaccine efficacy (VE))**.**

Vaccine efficacy is defined as the percentage reduction of a disease in a vaccinated group of people as compared to an unvaccinated group of people.

$$\begin{aligned} VE(A_t, B_t) &\equiv 100 \times (1 - RR(A_t, B_t)) \\ &\equiv 100 \times \left(\frac{CER(A_t, B_t) - EER(A_t, B_t)}{CER(A_t, B_t)} \right) \end{aligned} \tag{2.66}$$

Historically, vaccine efficacy has been designed to evaluate the efficacy of a certain vaccine by Greenwood and Yule in 1915 for the cholera and typhoid vaccines(Greenwood and Yule, 1915) and best measured using double-blind, randomized, clinical controlled trials. However, the calculated vaccine efficacy is depending too much on the study design, can lead to erroneous conclusions and is only of very limited value.

Experimental event rate (EER)

Definition 2.33 (Experimental event rate (EER))**.**

$$EER(A_t, B_t) \equiv \frac{p(a_t)}{p(A_t)} = \frac{a_t}{a_t + b_t} \tag{2.67}$$

Definition 2.34 (Control event rate (CER))**.**

$$CER(A_t, B_t) \equiv \frac{p(c_t)}{p(\underline{A}_t)} = \frac{c_t}{c_t + d_t} \tag{2.68}$$

Absolute risk reduction (ARR)

Definition 2.35 (Absolute risk reducation (ARR))**.**

$$\begin{aligned} ARR(A_t, B_t) &\equiv \frac{p(c_t)}{p(\underline{A}_t)} - \frac{p(a_t)}{p(A_t)} \\ &= \frac{c_t}{c_t + d_t} - \frac{a_t}{a_t + b_t} \\ &= CER(A_t, B_t) - EER(A_t, B_t) \end{aligned} \tag{2.69}$$

Absolute risk increase (ARI)

Definition 2.36 (Absolute risk increase (ARI))**.**

$$\begin{aligned} ARI(A_t, B_t) &\equiv \frac{p(a_t)}{p(A_t)} - \frac{p(c_t)}{p(\underline{A}_t)} \\ &= EER(A_t, B_t) - CER(A_t, B_t) \end{aligned} \tag{2.70}$$

Number needed to treat (NNT)

Definition 2.37 (Number needed to treat (NNT))**.**

$$NNT(A_t, B_t) \equiv \frac{1}{CER(A_t, B_t) - EER(A_t, B_t)} \tag{2.71}$$

An ideal number needed to treat(Cook and Sackett, 1995, Laupacis et al., 1988), mathematically the reciprocal of the absolute risk reduction, is NNT = 1. Under these circumstances, everyone improves with a treatment, while no one improves with control. A higher number needed to treat indicates more or less a treatment which is less effective.

Number needed to harm (NNH)

Definition 2.38 (Number needed to harm (NNH))**.**

$$NNH(A_t, B_t) \equiv \frac{1}{EER(A_t, B_t) - CER(A_t, B_t)} \tag{2.72}$$

The number needed to harm (Massel and Cruickshank, 2002), mathematically the inverse of the absolute risk increase, indicates at the end how many patients need to be exposed to a certain factor, in order to observe a harm in one patient that would not otherwise have been harmed.

Outcome prevalence rate (OPR)

Definition 2.39 (Outcome prevalence rate (OPR))**.**

$$OPR(A_t, B_t) \equiv \frac{p(a_t)}{p(B_t)} = \frac{a_t}{a_t + c_t} \tag{2.73}$$

Control prevalence rate (CPR)

Definition 2.40 (Control prevalence rate (CPR))**.**

$$CPR(A_t, B_t) \equiv \frac{p(b_t)}{p(\underline{B}_t)} = \frac{b_t}{b_t + d_t} \tag{2.74}$$

Bias and confounding is present to some degree in all research. In order to assess the relationship of exposure with a disease or an outcome, a fictive control group (i.e. of newborn or of young children et cetera) can be of use too. Under certain circumstances, even a CPR = 0 is imaginable.

Absolute prevalence reduction (APR)

Definition 2.41 (Absolute prevalence reduction (APR))**.**

$$APR(A_t, B_t) \equiv CPR(A_t, B_t) - OPR(A_t, B_t) \tag{2.75}$$

Absolute prevalence increase (API)

Definition 2.42 (Absolute prevalence increase (API))**.**

$$API(A_t, B_t) \equiv OPR(A_t, B_t) - CPR(A_t, B_t) \tag{2.76}$$

Relative prevalence reduction (RPR)

Definition 2.43 (Relative prevalence reduction (RPR))**.**

$$\begin{aligned} RPR(A_t, B_t) &\equiv \frac{CPR(A_t, B_t) - OPR(A_t, B_t)}{CPR(A_t, B_t)} \\ &= 1 - RR(A_t, B_t)_{sc} \end{aligned} \tag{2.77}$$

The index NNS

Definition 2.44 (The index NNS)**.**

$$NNS\,(A_t, B_t) \equiv \frac{1}{CPR(A_t, B_t) - OPR(A_t, B_t)} \tag{2.78}$$

Mathematically, the index NNS is the reciprocal of the absolute prevalence reduction.

The index NNI

Definition 2.45 (The index NNI)**.**

$$NNI(A_t, B_t) \equiv \frac{1}{OPR(A_t, B_t) - CPR(A_t, B_t)} \tag{2.79}$$

Mathematically, the index NNI is the reciprocal of the absolute prevalence increase.

2.3.28. Odds ratio (OR)

Definition 2.46 (Odds ratio (OR))**.**

Odds ratios as an appropriate measure for estimating the relative risk have become widely used in medical reports of case-control studies. The odds ratio(Fisher, 1935, p. 50) is defined(Cox, 1958) as the ratio of the odds of an event occurring in one group with respect to the odds of its occurring in another group. Odds(Yule and Pearson, 1900b, p. 273) ratio (OR) is a measure of association which quantifies the relationship between two binomial distributed random variables (exposure vs. outcome) and is related to Yule's (Yule and Pearson, 1900b, p. 272) Q(Yule, 1912, p. 585/586). Two events A_t and B_t are regarded as independent if $(A_t, B_t) = 1$. Let

a_t = number of persons exposed to A_t and with disease B_t
b_t = number of persons exposed to A_t but without disease $\underline{B}_t$
c_t = number of persons unexposed $\underline{A}_t$ but with disease B_t
d_t = number of persons unexposed $\underline{A}_t$: and without disease $\underline{B}_t$
a_t+c_t = total number of persons with disease B_t (case-patients)
b_t+d_t = total number of persons without disease $\underline{B}_t$ (controls).
Hereafter, consider the table 15. The odds' ratio (OR) is defined as

$$\begin{aligned} OR(A_t, B_t) &\equiv \left(\frac{a_t}{b_t}\right) / \left(\frac{c_t}{d_t}\right) \\ &\equiv \left(\frac{a_t \times d_t}{b_t \times c_t}\right) \end{aligned} \tag{2.80}$$

Remark 2.5. *Odds ratios can support logical fallacies and cause difficulties in drawing logically consistent conclusions. The chorus of voices is growing, which demand the immediate ending(Knol, 2012, Sackett et al., 1996) of any use of Odds ratio.*

Table 15. The two by two table of random variables

		Conditioned/Outcome B_t		
		TRUE	FALSE	
Condition/Exposure	TRUE	a_t	b_t	A_t
A_t	FALSE	c_t	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	N_t

Under conditions where (b = 0), the measure of association odds ratio will collapse, because we need to divide by zero, as can be seen at eq. 2.80. However, according to today's rules of mathematics, a division by zero is neither allowed nor generally accepted as possible. It does no harm to remind ourselves that in the case b = 0 the event A_t is a sufficient condition of B_t. In other words, odds ratio is not able to recognize elementary relationships of objective reality. In fact, it would be a failure not to recognize how dangerous and less valuable odds ratio is.

Under conditions where (c = 0) odds ratio collapses too, because we need again to divide by zero, as can be seen at eq. 2.80. However, and again, today's rules of mathematics don't allow us a division by zero. In point of fact, in the case c = 0 it is more than necessary to point out that A_t is a necessary condition of B_t. In other words, odds ratio or the cross-product ratio is not able to recognize elementary relationships of nature like necessary conditions. We can and need to overcome all the epistemological obstacles as backed by odds ratio entirety. Sooner rather than later, we should give up this measure of relationship completely.

2.4. Statistical methods

The probability of the necessary (Barukčić, 2021b) condition p(SINE) has been calculated and tested for statistical significance. The probability of the sufficient (Barukčić, 2021b) condition p(IMP) has been calculated, the statistical significance of this relationship has been proofed. The chi-square goodness of fit test with one degree of freedom has been used to test whether the sample data published fit a certain theoretical distribution in the population. The causal relationship k (Barukčić, 2021b) has been calculated to evaluate a possible causal relationship between the events/factors analysed. The hyper-geometric(Fisher, 1922, Gonin, 1936, Huygens and van Schooten, 1657, Pearson, 1899) distribution (HGD) has been used to test the one-sided significance of the causal relationship k. The study (design) bias has been controlled by IOI, the index of independence(Barukčić, 2019a) and IOU, the index of unfairness(Barukčić, 2019b). All the data were analysed using MS Excel (Microsoft Corporation, USA). The p values less than 0.05 were considered to indicate a statistically significant difference.

2.5. Axioms

2.5.1. Axiom I. Lex identitatis

In this context, we define axiom I as the expression

$$+1 = +1 \tag{2.81}$$

2.5.2. Axiom II. Lex contradictionis

In this context, axiom II or **lex contradictionis**, the negative of lex identitatis, or

$$+0 = +1 \tag{2.82}$$

and equally the most simple form of a contradiction formulated.

2.5.3. Axiom III. Lex negationis

$$\neg(0) \times 0 = 1 \tag{2.83}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990). In this context, there is some evidence that $\neg(1) \times 1 = 0$. In other words, it is $(\neg(1) \times 1) \times (\neg(0) \times 0) = 1$

3. Results

Theorem 3.1. *In general, the mutually exclusive relationship is defined as*

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \equiv \frac{b+\underline{A}}{N} \equiv \frac{c+\underline{B}}{N} \equiv +1 \tag{3.1}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{3.2}$$

is true. In the following, we rearrange the premise. We obtain

$$N \equiv N \tag{3.3}$$

and equally

$$a+b+c+d \equiv a+b+c+d \equiv N \tag{3.4}$$

Mutually exclusive events (at a certain (period of) time t) or the exclusion relationship is defined by the condition that the joint distribution function $p(A_t \cap B_t) = 0$ or that a = 0. Equation 3.4 becomes

$$b+(c+d) \equiv c+(b+d) \equiv N \tag{3.5}$$

or

$$b+\underline{A} \equiv c+\underline{B} \equiv N \tag{3.6}$$

or

$$\frac{b+\underline{A}}{N} \equiv \frac{c+\underline{B}}{N} \equiv \frac{N}{N} \equiv +1 \tag{3.7}$$

In general, the exclusion relationship or mutually exclusive events are determined by the equation

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \equiv \frac{b+\underline{A}}{N} \equiv \frac{c+\underline{B}}{N} \equiv +1 \tag{3.8}$$

□

Table 8 illustrated this relationship in more detail.

3.1. Mutually exclusive events and relative risk

Theorem 3.2 (Mutually exclusive events and relative risk)**.** *In general, a relative risk(see Cornfield, 1951, Sadowsky et al., 1953) which is equal to*

$$RR(A_t, B_t)_{nc} \equiv 0 \tag{3.9}$$

is indicating mutually exclusive events (at a certain (period of) time t) or the exclusion relationship.

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{3.10}$$

is true. In the following, we rearrange the premise. The relative risk $RR(A_t,B_t)_{nc}$ is equal to itself. It is

$$RR(A_t, B_t)_{nc} \equiv RR(A_t, B_t)_{nc} \tag{3.11}$$

or

$$RR(A_t, B_t)_{nc} \equiv \frac{p(a_t) \times p(NotA_t)}{p(A_t) \times p(c_t)} \tag{3.12}$$

Mutually exclusive events (at a certain (period of) time t) or the exclusion relationship itself are determined by the condition that the joint distribution function $p(A_t \cap B_t) = 0$ or that $a_t = 0$. Equation 3.12 becomes

$$RR(A_t, B_t)_{nc} \equiv \frac{0 \times p(NotA_t)}{p(A_t) \times p(c_t)} \tag{3.13}$$

Mutually exclusive events (at a certain (period of) time t) or the exclusion relationship are indicated by a relative risk

$$RR(A_t, B_t)_{nc} \equiv 0 \tag{3.14}$$

□

3.2. Mutually exclusive events and odds ratio

Theorem 3.3. *In general, an*

$$OR(A_t, B_t) \equiv 0 \tag{3.15}$$

is indicating mutually exclusive events (at a certain (period of) time t) or an exclusion relationship between events which occur at the same (period of) time / Bernoulli trial (see Uspensky, 1937) t.

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{3.16}$$

is true. $OR(A_t,B_t)$, the odds(see also Fisher, 1935) ratio, is another measure of association(see also Yule and Pearson, 1900a, p. 273) which quantifies the relationship between two (binomial distributed) random variables (exposure vs. outcome) and is related to Yule's (see also Yule and Pearson, 1900a, p. 272) Q(see also Yule, 1912, p. 585/586). In the following, we rearrange the premise. We obtain

$$OR(A_t, B_t) \equiv OR(A_t, B_t) \tag{3.17}$$

or

$$OR(A_t, B_t) \equiv \left(\frac{a_t \times d_t}{b_t \times c_t} \right) \tag{3.18}$$

Mutually exclusive events (at a certain (period of) time t) or the exclusion relationship itself are determined by the condition that the joint distribution function $p(A_t \cap B_t) = 0$ or that $a_t = 0$. Equation 3.18 becomes

$$OR(A_t, B_t) \equiv \left(\frac{0 \times d_t}{b_t \times c_t} \right) \tag{3.19}$$

Mutually exclusive events (at a certain (period of) time t) or the exclusion relationship are indicated by an odds ratio

$$OR(A_t, B_t) \equiv 0 \tag{3.20}$$

□

3.3. Approximation of mutually exclusive events

Placebo controlled randomized trials are controversial, ethically and mathematically. Withholding a valuable treatment poses unnecessary risks and serious harm to participants, while the study design of placebo controlled randomized trials requires a sample size which seems pointless. Mathematically, it is possible to estimate the effect of an event A_t on an outcome B_t.

Theorem 3.4. *In general, the exclusion relationship follows approximately as*

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \approx 1 - \frac{p(a_t)}{p(A_t)} \tag{3.21}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{3.22}$$

is true. In the following, we rearrange the premise. We obtain

$$p(A_t) \equiv p(A_t) \tag{3.23}$$

or

$$p(a_t) + p(b_t) \equiv p(A_t) \tag{3.24}$$

Rearranging equation 3.24, it is

$$p(b_t) \equiv p(A_t) - p(a_t) \tag{3.25}$$

Simplifying equation 3.25, we obtain

$$\frac{p(b_t)}{p(A_t)} \equiv \frac{p(A_t)}{p(A_t)} - \frac{p(a_t)}{p(A_t)} \tag{3.26}$$

Equation 3.26 becomes

$$\frac{p(b_t)}{p(A_t)} \equiv 1 - \frac{p(a_t)}{p(A_t)} \tag{3.27}$$

Mutually exclusive events demand that $\frac{p(b_t)}{p(A_t)} \equiv 1$ which is not given under any circumstances. Therefore, the exclusion relationship can only be estimated roughly by the relationship

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \approx 1 - \frac{p(a_t)}{p(A_t)} \tag{3.28}$$

□

In reality, the exclusion relationship will be stronger than suggested by the equation 3.28. Therefore, equation 3.28 is of particular value under conditions where a placebo group is absent or appears to be completely unsuitable.

3.4. Approximation of mutually exclusive events

Case-control studies or retrospective[22] studies are observational studies which can contribute to the identification of risk factors, conditions and causes of disease or changes in general. Due to the study design, these studies are particularly susceptible to generate various forms of bias. Therefore, a rough and reliable estimate of a certain relationship like the exclusion relationship which is independent of the study design bias appears to be helpful.

Theorem 3.5. *In general, the exclusion relationship follows approximately as*

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \approx 1 - \frac{p(a_t)}{p(B_t)} \tag{3.29}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{3.30}$$

is true. In the following, we rearrange the premise. We obtain

$$p(B_\mathrm{t}) \equiv p(B_\mathrm{t}) \tag{3.31}$$

where $p(B_\mathrm{t})$ is the probability of an outcome $\mathrm{B_t}$ within a sample or a population. It is

$$p(a_\mathrm{t}) + p(c_\mathrm{t}) \equiv p(B_\mathrm{t}) \tag{3.32}$$

Rearranging equation 3.32, it is

$$p(c_\mathrm{t}) \equiv p(B_\mathrm{t}) - p(a_\mathrm{t}) \tag{3.33}$$

Simplifying equation 3.33, we obtain

$$\frac{p(c_\mathrm{t})}{p(B_\mathrm{t})} \equiv \frac{p(B_\mathrm{t})}{p(B_\mathrm{t})} - \frac{p(a_\mathrm{t})}{p(B_\mathrm{t})} \tag{3.34}$$

Equation 3.34 becomes

$$\frac{p(c_\mathrm{t})}{p(B_\mathrm{t})} \equiv 1 - \frac{p(a_\mathrm{t})}{p(B_\mathrm{t})} \tag{3.35}$$

Mutually exclusive events demand that $\frac{p(c_\mathrm{t})}{p(B_\mathrm{t})} \equiv 1$ which is not given under any circumstances. Therefore, the exclusion relationship can only be estimated roughly by the relationship

$$p(A_\mathrm{t} \mid B_\mathrm{t}) \equiv p(A_\mathrm{t} \uparrow B_\mathrm{t}) \approx 1 - \frac{p(a_\mathrm{t})}{p(B_\mathrm{t})} \tag{3.36}$$

□

[22] PMID: 20299809

3.5. Very conservative approximation of mutually exclusive events

In reality, the exclusion relationship will be stronger than suggested by the equation 3.36. Therefore, equation 3.36 is of particular value under conditions where a control group is absent or appears to be completely unsuitable.

Theorem 3.6 (Very conservative approximation of mutually exclusive events)**.** *In general, a rough and only approximate estimate of the mutually exclusive relationship is given by the equation*

$$p(A_t \mid B_t) \geq +1 - \frac{p(a_t)}{p(A_t)} \tag{3.37}$$

Proof by direct proof. The premise

$$+1 \geq p(A_t) \tag{3.38}$$

is true. In the following, we rearrange the premise. We obtain

$$p(a_t) \geq p(a_t) \times p(A_t) \tag{3.39}$$

Dividing equation 3.39 by $p(A_t)$, it is

$$\frac{p(a_t)}{p(A_t)} \geq \frac{p(a_t) \times p(A_t)}{p(A_t)} \tag{3.40}$$

or

$$\frac{p(a_t)}{p(A_t)} \geq p(a_t) \tag{3.41}$$

while $p(a_t) = p(A_t \cap B_t)$ is the joint probability or distribution et cetera of $A_t \cap B_t$. Adding $p(b_t)$ to equation 3.41, it is

$$p(b_t) + \frac{p(a_t)}{p(A_t)} \geq p(a_t) + p(b_t) \tag{3.42}$$

or

$$p(b_t) + \frac{p(a_t)}{p(A_t)} \geq p(A_t) \tag{3.43}$$

Changing equation 3.43, it is

$$p(b_t) - p(A_t) \geq -\frac{p(a_t)}{p(A_t)} \tag{3.44}$$

Adding +1 to equation 3.44, it is

$$\underbrace{\boldsymbol{p(b_t) + 1 - p(A_t)}}_{Exclusion} \geq +1 - \frac{p(a_t)}{p(A_t)} \tag{3.45}$$

According to equation 2.33, the 1913 Henry Maurice Sheffer (1882-1964) stroke(Nicod, 1917, Sheffer, 1913) or equation 3.45 becomes

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \equiv \underbrace{\boldsymbol{p(b_t) + 1 - p(A_t)}}_{Exclusion} \geq +1 - \frac{p(a_t)}{p(A_t)} \tag{3.46}$$

In other words, a rough and an approximate estimate of the mutually exclusive relationship is given by the equation

$$p(A_t \mid B_t) \geq +1 - \frac{p(a_t)}{p(A_t)} \tag{3.47}$$

□

In reality, the mutually exclusive relationship $+p(A_t \mid B_t)$ is much stronger than estimated by the relationship $+1 - \frac{p(a_t)}{p(A_t)}$. At the end, it is $p(A_t \mid B_t) \geq +1 - \frac{p(a_t)}{p(A_t)}$. Many times, the efficacy of treatment or prevention interventions is judged by randomized, placebo [23] , [24] , [25] controlled trials (PCT). A key point of view of PCT is the use of placebo controls in trials, even under conditions with no effective treatment. However, withholding a treatment might pose negligible risks or even serious harm to participants of a placebo group. An inappropriate placebo group in PCT increases the cost and might induce study design caused bias, and therefore requires more than ethical justification. Mathematically, equation 3.47 or

$$p(A_t \mid B_t) \geq +1 - \frac{p(a_t)}{p(A_t)} \tag{3.48}$$

offers the possibility to test the efficacy of a drug or a treatment even without a placebo group.

Example.

The 1995 West of Scotland Coronary Prevention Study(see Barukčić, 2019d), also known as WOSCOPS[26], compared 40 mg pravastatin to placebo. In about 3302 men aged 45–64 years used 40 mg pravastatin while 106 of these participants died due to any cause. Table 16 might provide a review of the data of this study.

Table 16. Pravastatin 40 mg and death (WOSCOPS Study, 1995).

		Death		
		YES	NO	
Pravastatin 40 mg	YES	106	3196	3302
	NO	135	3158	3293
		241	6354	6595

Independent of any study design, 40 mg pravastatin excludes death due to any cause in men aged 45–64 years with the probability

$$p(A_t \mid B_t) \geq +1 - \frac{p(a_t)}{p(A_t)} = +1 - \frac{21}{3302} = 0,9934721790 \tag{3.49}$$

which indicates a positive result. In reality, the relationship is better than this estimation (see equation 3.49). In 1990 Germany, the death rate in men aged 45–64 years has been about 0.3 to 2 % [27]. In

[23] PMID: 24035802
[24] PMID: 29510711
[25] PMID: 27703733
[26] WOSCOPS study, 1995
[27] Wikipedia, Mortality in Germany 1990 (before use of pravastatin)

Germany 1990, under normal circumstances, about 10 up to 66 of 3302 men aged 45–64 years would have died independent of any use of the drug pravastatin. A use of the drug pravastatin should improve this situation. **During the trial period** (average follow-up period was 4.9 years) 106 pravastatin participants died. In other words, per year (((106/4,9)=21) / 3302)*100 = 0,655137888 % pravastatin participants died. However, the evidence of a decreased death rate in the group of patients treated with pravastatin compared to the death rate of the population of men aged 45–64 years (0,655137888 % v.s 0.3 to 2 %) does not follow for sure. The relative risk reduction (per year) can be calculated as

$$\begin{aligned} RRR(Pravastatin\ 40\ mg, Death(Per\ Year)) &\equiv \left(1-\left(\frac{a_{\mathrm{t}}\times \underline{A}_{\mathrm{t}}}{A_{\mathrm{t}}\times c_{\mathrm{t}}}\right)\right)\times 100 \\ &\equiv \left(1-\left(\frac{21\times 3185}{3217\times 27}\right)\right)\times 100 \\ &\equiv 22,99588989 \end{aligned} \tag{3.50}$$

or as 22,99588989 %. However, whether a therapy with 40 mg pravastatin and to what extent is really of any benefit for men aged 45–64 years remains to be proven. Pravastatin posses anti-viral (human cytomegalovirus) effects. The WOSCOPS study did not answer the question whether the lowering of the blood cholesterol levels or the control of a human cytomegalovirus infection reduces the risk of coronary heart disease or both or none. We still have to gain some experience with the use of the exclusion relationship. Nonetheless, the example before clearly demonstrates the dangers emanating from unsuitable statistical methods and the importance of a logically consistent study design. Otherwise, we run the risk of reaching conclusions which have nothing or too little in common with objective reality, while humans will suffer unnecessary harm.

3.6. Mutually exclusive events and study design

A study design has different tasks and functions. It should be noted, however, that regardless of whether the data are achieved by a placebo controlled randomized trial or a case-control study design, data need to be provided by the studies which enable us to draw the same conclusions. Therefore, the reduction of study design bias is of far-reaching and extraordinary importance.

Theorem 3.7. *In general, a study design of a study which investigates an exclusion relationship between the events A_t and B_t should assure as much as possible (see equation 2.61) an index of independence(Barukčić, 2019a) given as*

$$IOI(A_t, B_t) \equiv \frac{A_t + \underline{B}_t}{N} - 1 \equiv 0 \tag{3.51}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{3.52}$$

is true. In the following, we rearrange the premise. We obtain

$$p(A_{\mathrm{t}} \mid B_{\mathrm{t}}) \equiv p(A_{\mathrm{t}} \mid B_{\mathrm{t}}) \tag{3.53}$$

In the following, we would like to analyse this relationship independent of any type of study. From the point of view of a placebo controlled randomized trial (see equation 3.28), this relationship is given

approximately as

$$p(A_t \mid B_t) \equiv 1 - \frac{p(a_t)}{p(A_t)} \tag{3.54}$$

Ideally, the type of study should not have any influence on the conclusions drawn. From the point of view of a case control study, it is (see equation 3.36)

$$1 - \frac{p(a_t)}{p(B_t)} \equiv 1 - \frac{p(a_t)}{p(A_t)} \tag{3.55}$$

or

$$\frac{p(a_t)}{p(B_t)} \equiv \frac{p(a_t)}{p(A_t)} \tag{3.56}$$

or

$$\frac{1}{p(B_t)} \equiv \frac{1}{p(A_t)} \tag{3.57}$$

and at the end

$$p(A_t) \equiv p(B_t) \tag{3.58}$$

Equation 3.58 becomes

$$p(A_t) \equiv 1 - p(\underline{B}_t) \tag{3.59}$$

Multiplying equation 3.59 by the sample size N, it is

$$(N \times p(A_t)) \equiv N - (N \times p(\underline{B}_t)) \tag{3.60}$$

Equation 3.60 becomes

$$A_t \equiv N - \underline{B}_t \tag{3.61}$$

and

$$A_t + \underline{B}_t \equiv N \tag{3.62}$$

Dividing equation by the sample size N, it is

$$\frac{A_t}{N} + \frac{\underline{B}_t}{N} \equiv \frac{N}{N} \equiv +1 \tag{3.63}$$

Rearranging equation 3.63, it is

$$\frac{A_t + \underline{B}_t}{N} - 1 \equiv 0 \tag{3.64}$$

The study design of a study which investigates an exclusion relationship between the events A_t and B_t should assure as much as possible (see equation 2.61) an index of independence(Barukčić, 2019a) given as

$$IOI(A_t, B_t) \equiv \frac{A_t + \underline{B}_t}{N} - 1 \equiv 0 \tag{3.65}$$

□

Assumed that certain condition like the one detailed above cannot be met, there is a risk of completely unworldly conclusions, which could be without any meaning or understanding. A consequence of equation 3.65 is the need for the study design to assure conditions as much as possible where

$$b_t \equiv c_t \tag{3.66}$$

3.7. *Moderna*

How effective is Moderna's Covid-19 vaccine in preventing COVID-19 deaths? Grange et al. (see Grange et al., 2021) investigated COVID-19-related deaths in Scotland with respect to Moderna's Covid-19 vaccine.The data and the statistical analysis is presented by table 2. As can be seen, an IOR = -1 indicates correctly that an exclusion relationship is given. This position is supported by the relative risk RR = 0, which indicates, that an exclusion relationship is given. The causal relationship k is negative and significant and demands an exclusion relationship. The index of independence with p(IOI) = 0,012604762 allows some conclusions about an exclusion relationship between Moderna's Covid-19 vaccine and Covid-19 death. In other words, Moderna's Covid-19 vaccine is preventing very effectively against COVID-19 death. The Moderna vaccine efficacy (Greenwood and Yule, 1915) can be calculated as

$$\begin{aligned} VE\left(Moderna\ vaccine, Covid-19death\right) &\equiv \left(1-\left(\frac{a_{\mathrm{t}}\times \underline{A}_{\mathrm{t}}}{A_{\mathrm{t}}\times c_{\mathrm{t}}}\right)\right)\times 100 \\ &\equiv \left(1-\left(\frac{0\times 3231870}{41496\times 236}\right)\right)\times 100 \\ &\equiv 100 \end{aligned} \tag{3.67}$$

or as 100 % .

3.8. BionTech

How effective is BionTech/Pfizer's Covid-19 vaccine in preventing COVID-19 deaths? Grange et al. (see Grange et al., 2021) investigated COVID-19-related deaths in Scotland with respect to BionTech/Pfizer's Covid-19 vaccine Covid-19 vaccine. The data and the statistical analysis is presented by table 3. Based on the data of the study of Grange et al. (see Grange et al., 2021), the probability of an exclusion relationship (see table 3) has been calculated very conservatively, approximately as p = 0,9999623103. Let us assume that the rest of Scotland's population (about 4090424 inhabitants) is not vaccinated against Covid-19 while about 1 to 2 % would die due to Covid-19 virus infection (see table 17).

Table 17. BionTech vaccine and Covid-19 death (Study Grange et al. , 2021).

		Covid-19 death		
		YES	NO	
BionTech vaccine	YES	47	1246979	1247026
	NO	48803	4041621	4090424
		48850	5288600	5337450

Under these assumptions, the BionTech vaccine efficacy (Greenwood and Yule, 1915) can be calculated as

$$\begin{aligned} VE(BionTech\ vaccine, Covid-19 death) &\equiv \left(1-\left(\frac{a_t \times \underline{A}_t}{A_t \times c_t}\right)\right)\times 100 \\ &\equiv \left(1-\left(\frac{47\times 4090424}{1247026\times 48803}\right)\right)\times 100 \\ &\equiv 99,68410398 \end{aligned} \tag{3.68}$$

or as 99,68410398 % .

In Germany about 83129285 inhabitants are living. If all inhabitants of Germany were vaccinated by the BioNTech vaccine, then less than (1- 0,9999623103) * 83129285 = 3133 inhabitants would die because of Covid-19.

3.9. ... and again AstraZeneca Covid-19 vaccine

Let us assume that the rest of Scotland's population (about 3288252 inhabitants) is not vaccinated against Covid-19 while about 1 to 2 % of these inhabitants would die due to Covid-19 virus infection (see table 18).

Table 18. AstraZeneca vaccine and Covid-19 death (Study Grange et al. , 2021).

		Covid-19 death		
		YES	NO	
AstraZeneca vaccine	YES	188	2026010	2026198
	NO	48662	3239590	3288252
		48850	5265600	5314450

Under these assumptions, the AstraZeneca vaccine vaccine efficacy (Greenwood and Yule, 1915) can be calculated as

$$\begin{aligned} VE\,(AstraZeneca\ vaccine, Covid-19 death) &\equiv \left(1-\left(\frac{a_t \times \underline{A}_t}{A_t \times c_t}\right)\right)\times 100 \\ &\equiv \left(1-\left(\frac{188\times 3288252}{2026198\times 48662}\right)\right)\times 100 \\ &\equiv 99,37302373 \end{aligned} \tag{3.69}$$

or as 99,37302373 % . Contrary to discussion, the AstraZeneca vaccine itself is highly effective. In reality, more than about 1 to 2 % of these inhabitants not vaccinated against a Covid-19 infection will die due to Covid-19 with the consequence that AstraZeneca's vaccine efficacy is much better than the calculated (99,37302373 %) one. However, as can be seen (see table 4) the study design, which is completely without any sense (p(IOI) = 0,618923151; p(IOU) = 0,380932655) can lead to erroneous conclusions. Looking very closely, based on this study design, we would have to accept the hypothesis that AstraZeneca Covid-19 vaccine has been the cause of the Covid-19 death of the people vaccinated.

Reasons.

The causal relationship is positive, the risk ratio RR is RR > 1, IOR is not negative et cetera. In toto, the data of Grange et al. (see Grange et al., 2021) demand us to accept the Null-hypothesis: without vaccination with AstraZeneca Covid-19 no Covid-19 death (p (SINE)=0,9999853362, p Value (SINE)=0,0000146637). Such a conclusion is of course beyond any conceivable logic. Unfortunately, the data of the study of Grange et al. (see Grange et al., 2021) do support and require such a conclusion. Therefore, where is the error?

Firstly.

The study design does not allow such a conclusion. A p(IOU) = 0,380932655 it too high for such a sample size. The data presented should not be used for the analyses of conditio sine qua non et cetera.

Secondly.

Based on the data of the study of Grange et al. (see Grange et al., 2021), the probability of an exclusion relationship (see table 4) has been calculated very conservatively, approximately as p = 0,9999072154. In Germany about 83129285 inhabitants are living. If all inhabitants of Germany were

vaccinated by AstraZeneca Covid-19 vaccine, then less than (1- 0,9999072154) * 83129285 = 7713 inhabitants would die because of Covid-19. In contrast to these 7713 cases calculated according to the data of the study of Grange et al. (see Grange et al., 2021), in Germany today (December 1, 2021) more than 100000 thousand of inhabitants died because of Covid-19 infection. This supports a vaccine efficacy of AstraZeneca's Covid-19 vaccine of about ((7713/56007713)/(92287/27121572))*100 = 95,953 %. A critic may note that these 7713 patients died because of the AstraZeneca Covid-19 vaccination and not because of the Covid-19 virus infection. However, this topic has been discussed to the negative in great detail somewhere elsewhere.

Thirdly.

Great care and caution is required when analysing data. As can be seen from the table 4 and table 5, seriously wrong conclusions can follow if statistical methods are blindly applied to a data set without any meaning and without any understanding. Before data can be re-analysed, great attention should be paid to the facts, whether these data allow us at all to analyse the same. What is the design of the study, et cetera?

4. Discussion

Logically consistent statistical methods alone are not sufficient to reliably and automatically recognize conditions or cause-effect relationships, et cetera. In this context, a very careful assessment of the quality of the data and an evaluation of the quality of the study design of a study is of very great importance too. Nonetheless, to some extent, it is possible to rely on the index of unfairness(Barukčić, 2019b) and the index of independence(Barukčić, 2019a). However, additional tools are necessary in order to help scientist to improve the quality of scientific publications.

5. Conclusion

Mutually exclusive events can be recognized with a probability bordering on certainty.

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6. Patient consent for publication

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Conflict of interest statement

No conflict of interest to declare.

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